EXPERIMENT: THE SPRING II
Subtitle: Time Dependence of Oscillations

OBJECTIVES: A stretched spring is a good example of an oscillator, which oscillates around an equilibrium position (sum of all forces is zero), with a characteristic period (*T*), which will be determined in this lab. During this lab we will record how the position, speed and acceleration change as a function of time.

APPARATUS:

A simple spring with a mass attached to it has been recorded as a videoclip. Several full oscillations of the spring have been recorded. Using the VideoPoint software you will measure the position as a function of time for two full cycles of the spring.

THEORY

1. **Oscillation**

When a mass is attached to a spring, the spring is stretched by a distance given by Hooke's law. The position where the mass is at rest is called the equilibrium position ($x = x_0$). The downward force due to gravity $F_g = mg$. and the upward force due to the spring pulling upward $F_s = kx$ cancel each other. This is shown in the first part of figure 1. If the mass is kept the same and the string is stretched by pulling it down and released, it will accelerate upward ($a > 0$), because the force due to the spring is larger than gravity pulling down. After release it will pass through the equilibrium point and continue to move upward. Once above the equilibrium position gravity will start to exceed the force pulling upward due to the spring and acceleration will be directed downward. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a) and velocities (v) are illustrated in Figure 1 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period T, which is determined by the spring constant and the attached total mass. This period is the time it takes to complete one oscillation i.e. the time necessary to return to the point where the cycle starts repeating (the points where x, v, a are the same). One complete cycle is shown in Figure 1 and the time of each position is indicated in terms of the period T. The period is given by :

$$T = 2\pi\sqrt{\frac{m}{k}}$$

During this lab the position of the mass, attached to the spring, will be measured as a function of time. From these data the speed and the acceleration will be determined as a function of time. The curves obtained will be compared to Figure 1.

HINT on preparation for this class: Be sure you are familiar with the function -*sine*- and -*cosine*-. Before you start measuring or even before class make a graph of *sin(x)* and *cos(x)* for values of x between 0 and 3π. Do this is steps of x=0.1. You will need this graph in order to interpret the data you will record.

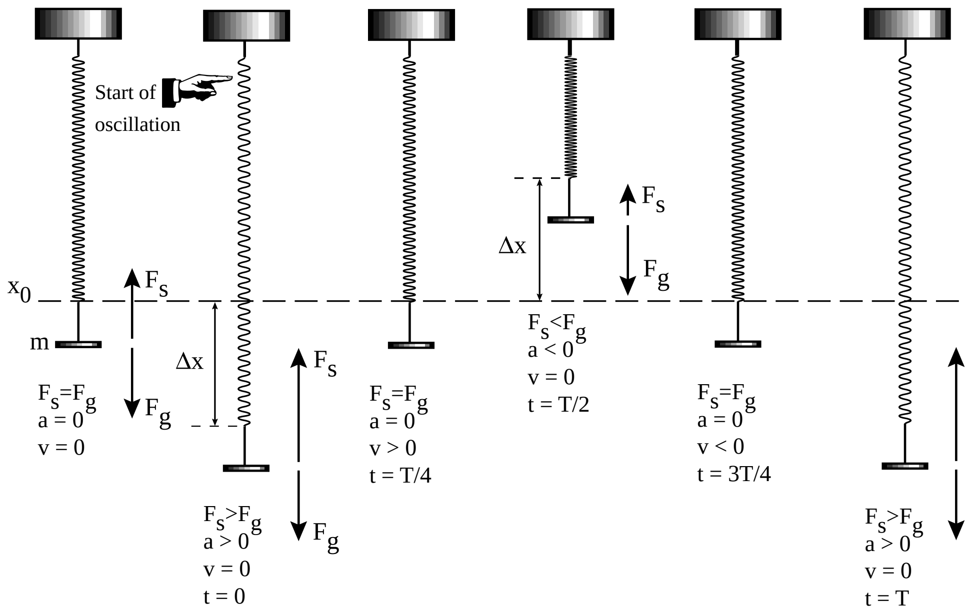

Figure 1: One cycle of an oscillation of the spring.

PROCEDURE

Make a graph of *sin(x)* and *cos(x)* for values of x between 0 and 3π. Do this is steps of x=0.1. You will need this graph in order to interpret the data you will record. The easiest way to do this, is using Excel and calculating the functions. You can also use the graphing calculator and simply draw them on a piece of paper.

Start the application program VideoPoint and open the videoclip containing the motion of the spring. As an initial test simply play the videoclip to see how long it lasts and whether there are gaps in it. Select a set of images where the mass attached to the spring can be measured over **two complete** cycles (two cycles correspond to a time equal to two periods). In principle it does not matter at which point in time you start measuring. Go through the following steps for the measurement:

- determine the conversion of pixel units to meters. For this purpose the videoclip contains a scale and the length is indicated on it (typically a square black object with sides 10 cm). Click the icon for setting the scale of the coordinate system and follow the instructions. Be sure you use the correct units !!
- after completion of this make sure that in the “Coordinate Systems” window the “Origin” box “Scale 1” is given in (pixels) / (m). Also in the “TableWindow” the units should have switched from “pixels” to “meters”.
- Start measuring the position of the mass attached to the spring in the first frame you selected and keep doing this until you have completed two full cycles. View your measurements in the “TableWindow”. You will measure both the *X* and *Y* position of the mass as a function of time. In principle *X* will hardly vary with time, but *Y* will.

- Save your results, so they are not lost in case of computer trouble
- When you are done with your measurements, make a graph (in VideoPoint) of your measurements of *Y* vs. *time* to make sure you did not make any mistakes. Remeasure points that are wrong.
- Once you have checked that your points are correct, select the *time, X* and *Y* columns and copy them to the clipboard for use in the Excel spreadsheet.

After you transfer the data into the empty spreadsheet you will be ready to calculate the speed and acceleration as a function of time. This will only be done for the *Y* direction !!. The determination of the speed (V_y in spreadsheet) and acceleration (a_y in spreadsheet) are described in the following. At any given moment in time the speed is given by:

$$v = \text{distance/ time interval} = \Delta y \,/\, \Delta t$$

Let's consider an example. Your measurements are at several points in time, which we will label t_1, t_2, t_3, t_4 etc. and the positions measured are y_1, y_2, y_3, y_4 etc. In previous experiments we have used a technique to calculate the speed at the point t_3 ,for example, by using the position and time of the adjacent points t_2 and t_4. This technique is not precise enough for this experiment and we will use a similar but slightly different procedure . The procedure is explained in the following example where we use the time points t_2 and t_3. The distance the mass travels between these two points is: $\Delta y=(y_3-y_2)$. The time it takes to do this is : $\Delta t= (t_3-t_2)$. So the average speed between the points t_2 and t_3 is: $v = \Delta y/\Delta t$. The mass had this speed at some point in time t_v which is somewhere between t_2 and t_3 and we will define it as the mean of the two: $t_v = (t_2+t_3)/2$. This time t_v is called time_sp in the spreadsheet. These formulas describe how to calculate the speed as a function of time. Note that t_v will be in the middle between t_2 and t_3.

For the acceleration (a_y in the spreadsheet) we use the same technique. Here the basic equation used is:

$$\text{acceleration} = a = \text{change in speed / time interval} = \Delta v \,/\, \Delta t$$

The same formulas as for the speed are used but now the position *y* is replaced by the speed at a certain point in time. You will also have to calculate the time at which you measure the acceleration (time_a in the spreadsheet). To do this in a similar example as above, we take the speeds v_5 and v_6 at times t_5 and t_6. The acceleration -*a*- is determined by $a = (v_6-v_5)/(t_6-t_5)$ and it is measured at time $t_a=(t_5+t_6)/2$.

After calculating these quantities in the spreadsheet make the following graphs:

I. Position (*y*) of the mass as a function of time showing at least two cycles.
II. Speed of the mass as a function of time in the *y* direction. Make sure you use exactly the same scale for the time axis in all graphs, so the position vs. time and speed vs. time can be compared.
III. Acceleration as a function of time. Again use the same scale for the time axis.

Make sure your graphs are labeled correctly and the axes are labeled and have units.

Questions to be answered:

1) What is the amplitude of the oscillation in the Y direction ? Read this from the graph of Y vs. time.

2) What is the period (T) of the oscillation i.e. after what time does the motion repeat ?

3) What functions describe the dependence of displacement and speed on time ?

4) On *each* of your three graphs indicate the points corresponding to the equilibrium position and the two positions where the excursion from the equilibrium is maximum (lowest point in y and highest point in y).

5)At these points determine the speed and acceleration from your graphs and compare them to the values given in Figure 1.