EXPERIMENT :VIBRATION MODES OF A STRING
Subtitle: Standing waves

OBJECTIVE: To observe resonant vibration modes on a string. To determine how resonant frequencies scale with number of the nodes, tension, length and string density. To determine the velocity of transverse waves in the string.

APPARATUS: Variable Frequency oscillator, pulley and weights, string, meter stick, scale.

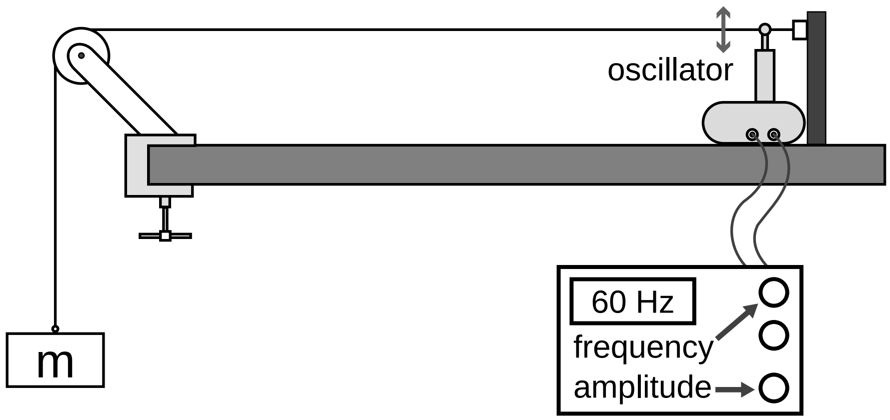

Figure 1

THEORY

A wave in a string can be characterized by its wavelength, λ, just like a sound wave or a light wave. For a string that is fixed on both ends, a standing wave can develop if an integer number of half wavelengths fit into the length, L, of the string:

$$n\frac{\lambda_n}{2} = L. \tag{1}$$

Here n refers to the number of maxima in the wave pattern as demonstrated in the Figure 2.

The resonant frequency, f_n, for wavelength , λ_n, is:

$$f_n = \frac{c}{\lambda_n} = n\frac{c}{2L}. \tag{2}$$

If a force acts on a string with a resonant frequency, the amplitude of the vibration will grow very large. This is a common behavior in many physical systems. An example of such behavior is pushing a child on a swing. A swing oscillates with a characteristic frequency. If someone exerts a push on the child with that frequency, after several cycles the amplitude of the swing becomes large, even if the pushes are gentle. If pushes are given with a different frequency, some of the pushes will be out of phase, meaning that the child will be pushed against his motion and the amplitude will not have a chance to grow. A string differs from the swing example in that the string has many characteristic frequencies and the string's amplitude will grow whenever the driving force has any of the characteristic frequencies.

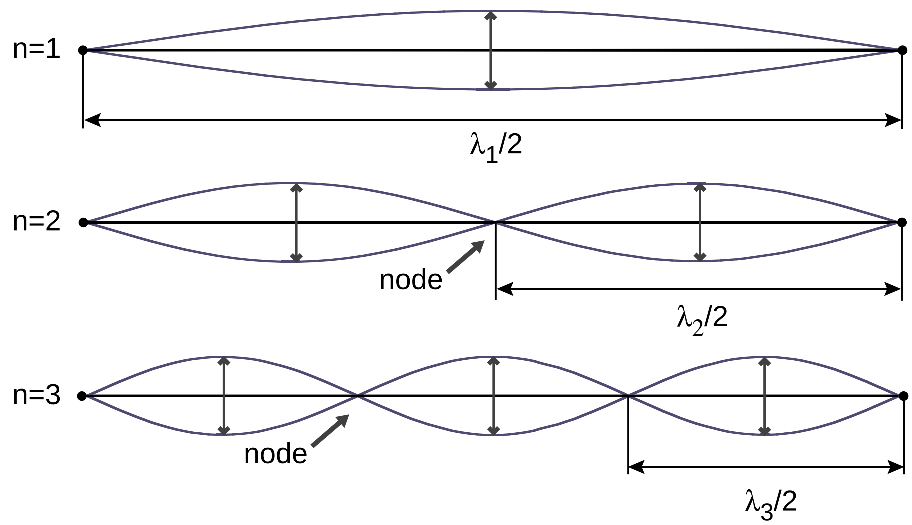

Figure 2

The speed, c, of a transverse wave in a string depends on the string's mass per unit length ρ and the tension T. By setting the tension with the pulley system shown below and by measuring the mass density, one can determine the speed of the transverse wave:

$$c = \sqrt{\frac{T}{\rho}} \,. \tag{3}$$

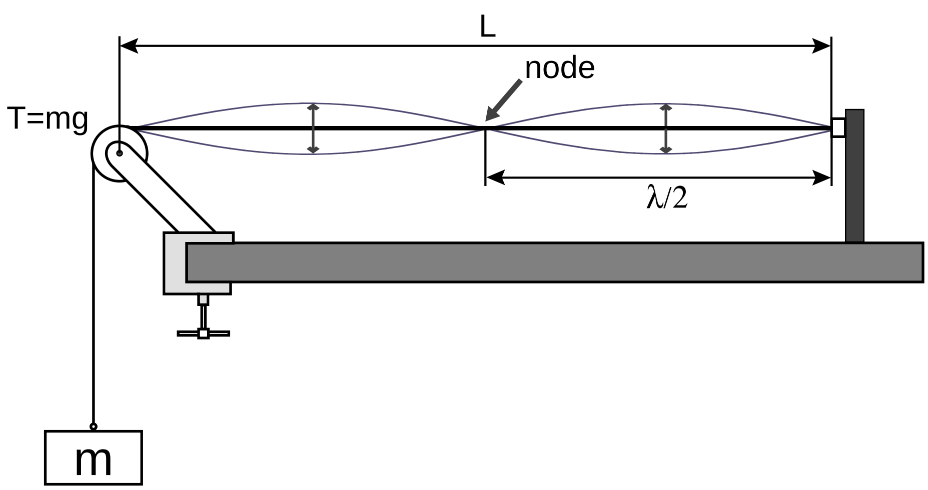

Figure 3

PROCEDURE

1) Set up string and pulley with $m \approx 200$ g and $L \approx 150$ cm. Remember, L is the distance between the top of the wheel and the point where the string is fixed (see Figure 3). Measure this length. NOTE: this is not necessarily the same as the total length of the string you have.

2) Position the vibrator close to the fixed end. Adjust the frequency selector to the range: "Hz 1-100" and start at the highest frequency.
3) Find the resonant frequencies $f_{11}, f_{10}, f_9, f_8 ..... f_1$ for n=11,10,9,8 ...1.
4) Use $n\frac{\lambda_n}{2} = L$ to calculate the wavelength for each n. Using the formula $c = \lambda_n f_n$, calculate the speed for each measurement. Find the average speed from all your measurements.
5) Make a graph of f_n vs. n. From the slope, calculate c, by using the formula $f_n = \frac{c}{2L} n$.
6) The string being used in the lab has a mass density $\rho_o = \frac{m_o}{l_o} = \frac{7.5}{200} = 0.0375$ [gram/cm].From this determine the mass of your string and the mass density of the stretched string. Devise a method to correct for the stretching of the string, so that you determine the mass density of the string when it is stretched with the 200 g mass. Using Equation 3, calculate c and compare with the value of c from part 2.
7) For n=5 answer the questions on the data sheet.

APPENDIX: The pitch of musical instruments is determined by the resonant frequency, whether it is a string instrument, a wind instrument or a percussion instrument. Since instruments are not driven at a fixed frequency, the vibrations are composed of a mixture of several harmonic frequencies.