Experiment 6

Rotational Motion

Goals

1. To understand the rotational motion of a rigid body.
2. To study different types of frictional losses in a rotating system that lead to decay.
3. To explore the use of least-squares fitting procedures in analyzing a dynamical system.

Theoretical Introduction

For a rigid body that rotates about a fixed axis, Newton's second law of motion states that

$$t = Ia \tag{1}$$

where is the magnitude of the total torque, *I* is the moment of inertia of the body and is the angular acceleration, measured in radians/s^2. Let us consider some applications of this equation.

1. No frictional torque

Suppose that the rigid body, a rotating disk, is spinning and there is no friction. Then the total torque, τ, is zero and Eq. 1 predicts that $\alpha = 0$. By definition $\alpha = d\omega/dt$, so that one can readily solve for the angular velocity ω,

$$w = \text{constant.} \tag{2}$$

2. Constant frictional torque

Suppose there is a constant frictional torque, τ_f, acting on the disk via the bearings. Then from Eq.1 we have:

$$\tau = -\tau_f = I\alpha = I\frac{d\omega}{dt} \tag{3}$$

$$-\tau_f = I\frac{d\omega}{dt}. \tag{4}$$

Integrating this equation, we obtain:

$$\int d\omega = -\frac{\tau_f}{I}\int dt \tag{5}$$

$$\omega = \omega_0 - \frac{\tau_f}{I}t \tag{6}$$

where ω_0 is the value of ω at t = 0. The minus sign agrees with the notion that friction causes the angular velocity to decrease with time.

3. Frictional torque proportional to ω

Suppose now that $\tau = - C\ \omega$, where C is an arbitrary constant. Using Eq. 1 we now obtain:

$$\tau = -C\omega = I\frac{d\omega}{dt} \tag{7}$$

This can be rewritten as

$$\frac{d\omega}{dt} = -\frac{C}{I}\omega. \tag{8}$$

The solution of this differential equation is

$$\omega = \omega_o \exp(-\gamma\, t) \tag{9}$$

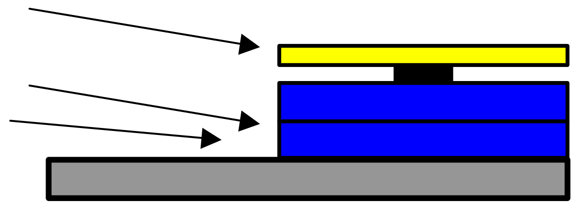
where ω_o is the initial angular frequency and γ is the decay rate (in s^{-1}). We see that the angular frequency now decreases exponentially with time. As we noted in the experiment on damped harmonic motion, $1/\gamma$ is the damping time constant, which is the time for the object to decay to 1/e (0.368) of its initial value. Comparing Eqs. (8) and (9) the damping rate is given by $\gamma = I/C$.

Experimental Procedure

Apparatus

A cross-sectional view of the apparatus is shown on the next page. It consists of a stationary base on which is mounted a rotating platform supported by nearly frictionless air bearings. This rotating disk has alternating black and white bars on its circumference which sweep by a photo-diode detector whose output is amplified to yield standard logic levels (TTL) corresponding to the presence of a white (TRUE) or black (FALSE) bar. Your instructor will show you how to connect this output to the terminal board that is attached to a special data acquisition card residing in your PC. Once in the computer, these signals will be analyzed by various programs written in the LabVIEW language.

In the first part of the experiment, determining N, connect the ground wire (black) to terminal 33 and the signal wire (red) to terminal 47. In the second part, measuring rotational speed, connect the signal wire to terminal 48.



The theoretical relationships discussed at the beginning were derived in terms of the instantaneous angular velocity, ω, but the apparatus used in this lab displays and records the *average rate or frequency*, $\langle n \rangle$, at which the black bars pass by the detector. However, ω and $\langle n \rangle$ are related by the equation $\omega = 2\pi\langle n \rangle/N$, where N is the number of black bars on the circumference of the rotating platform and $\langle n \rangle$ is the average number of black bars passing the detector during the timing interval. A procedure for determining N is given below.

If the acceleration is constant then, as you showed in the gravitational free fall experiment, the average velocity is also the instantaneous velocity at the center of the timing interval. If the frictional torque and hence the angular acceleration depend linearly on the angular frequency then it can be shown that $\langle n \rangle$ is the instantaneous frequency in the middle of the timing interval, but only if the timing interval is short compared to the decay time I/C. Thus, measuring the time dependence of $\langle n \rangle$ yields the time dependence of ω.

Questions for preliminary discussion

1. Substitute Eq. 9 into Eq. 8 to prove that Eq. 9 is a solution of Eq. 8.
2. Show that $\langle n \rangle$ is the instantaneous frequency in the middle of timing interval.
3. Show that you can rewrite Eq. 9 as Eq. 11.
4. Why is Eq. 11 true only if the timing interval is short compared to the decay time?

Experimental Procedures

Determination of N

The objective here is to determine N, the number of black bars on the circumference of the disk. Note that N is an integer.

1. **Make sure the air is flowing through the bearing before you spin the aluminum disk!** Failure to do so could result in irreversible damage to the apparatus. Also, be sure the red magnet is far away from the disk.

2. A computer program is provided that counts the number of black bars which pass by the photocell. Connect the output of the apparatus to the data acquisition card on the PC. Open a program in the Labview: C:\Labview\Vi.lib\vi-for-phy191\counter.llb\simplecounter.vi. Set **device**=1; **counter**=5.

3. Position the platform so the little red light is on and reset the counter to zero. Carefully mark this starting angular position of the platform. Now run the program, and smoothly rotate the platform through exactly 4 or 5 revolutions, being sure to stop at a point where the red light is on. **Do not let the platform reverse direction!** Get the edges counted from the program. If n is the number of revolutions and N_c the total number of counts then $N = N_c / n$.

Part A

The objective of this part of the lab is to determine if there is a significant frictional torque in this apparatus.

1. Open a program in the directory C:\Labview\Vi.lib\vi-for-phy191\Ma-fr-2-.vi. Set **Device**=1, **Total Time**=25 sec, and **Average Time** = 1 sec.
2. Give the disk enough angular momentum so that $\langle \nu \rangle_0$ reads between 250 Hz and 300 Hz on the apparatus. Run the program.
3. Exit the program. Choose **No** to the question " Save changes to Ma-Fr-2-.vi? "
4. Use Kgraph to retrieve the data file as done in Exp 5. The data file contains **Average Time** and **Frequency**. Make a plot of $\langle n \rangle$ vs. time. Find uncertainty for $\langle n \rangle$ and show error bars on the plot. Fit the data taken every second (25 points in all) to the function:

$$\langle n \rangle = \langle n \rangle_o - \frac{N}{2p} at \, . \tag{10}$$

5. With the linear least-squares fitting program in *Kgraph* determine values and uncertainties for $\langle \nu \rangle_0$ and α.

Part B

The objective of this part of the lab is to determine if an external force causes a frictional torque that depends linearly on ω.

1. Remove the "keeper" from the red magnet and place the magnet underneath the aluminum disk. Use the same program as you did in part A, set **Total Time**=70 sec and **Average Time** = 4 sec. Spin the disk so that its initial $\langle n \rangle$ is between 250 and 300 Hz, then run the program. Now **repeat** the experiment, adjusting the program settings so that you can record the motion of the disk until it nearly stops. Mask data points associated with the large fluctuating values that may occur when the disk is rotating very slowly. When done, take the magnet off the apparatus and replace the "keeper" across its pole faces.
2. Make plots of $\langle n \rangle$ vs. time. Fit the data to the function

$$\langle n \rangle = \langle \nu \rangle_0 \exp(-\gamma t). \tag{11}$$

Using the general curve-fit editor in *K-graph* determine the best fit values for parameters $\langle \nu \rangle_0$, γ and their uncertainties.

3. Convert Eq. (11) to a linear form by taking the natural logarithm (ln) of both sides (see Sec. 8.6 of Taylor). Calculate $\ln\langle \nu \rangle$ from your data, then make plots of $\ln\langle \nu \rangle$ vs. t. Fit your data using the general least-squares editor and find parameters $\ln\langle \nu \rangle_o$ and g and their uncertainties.

Questions

1. From your data in Part A deduce if there is a significant frictional torque. If so, do your data imply that this torque is constant? In other words, is Eq. (6) obeyed by your data? Be sure to show clearly how you draw your conclusions.
2. Show how to arrive at Eq. (10) from Eq (6).
3. In Part B, how well does Eq. (11) describe your data? Look for systematic deviations in both experimental data records.
4. Examine your data for $\langle n \rangle$ in the long time run in Part B. If the exponential decay were literally correct, the disc would never stop. Clearly this is at odds with reality. Explain why the disc does stop. Now, make a correction to Eq. (11) to analyze your measurements, taking into account what you have learned in Parts A and B. Hint: The decay is exponential at short times but eventually the linear decay should become more important just before the disk stops.
5. What is the physical mechanism by which the magnet causes the non-magnetic aluminum wheel to slow down? If you don't know, how would you find out for yourself?