# Problem Set #1

PHY 853, Fall Semester, 1999

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These problems are due on October 29, 1999, 5:00 p.m., in my mailbox.

October 11, 1999

These problems are due on October 30, 1999 in my mailbox.

**Problem 1** Consider the continuous basis, $\xi_{\mathbf{r}_0}(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}_0)$.

(a) While this function is not square-integrable, so it cannot be used to represent a physical system, it still is a valid basis and any valid function can be expanded in terms of it. What is the proper statement of that fact, in analogy to the discrete case: $\psi(\mathbf{r}) = |\eta_i(\mathbf{r})\rangle\, a^i$?

(b) What ist he analogy for this continuous basis to teh discrete statment of orthogonality: $\langle \eta^i | \eta_j \rangle = \delta^i_j$?

(c) What is the analogy for this continuous basis to the discrete statement of closure, or the statement of a complete set of states:

$$|\eta_i(\mathbf{r})\rangle \left\langle \eta^i(\mathbf{r}')\right| = \delta(\mathbf{r} - \mathbf{r}')?$$

**Problem 2** Show that the momentum space representation of the positon operator is

$$\langle p|\mathbb{X}|p'\rangle = i\hbar\delta'(p - p').$$

**Problem 3** Consider a 25%-75% mixture of two ensembles of particles with spin $\mathbb{S}_{z+}$ and $\mathbb{S}_{x+}$, that is the normal probabilites are $\omega(\mathbb{S}_{z+}) = 0.25$ and $\omega(\mathbb{S}_{x+}) = 0.75$. What is the density operator, $\rho$? Calculate the mean values of $< \mathbb{S}_x >$, $< \mathbb{S}_y >$, and $< \mathbb{S}_z >$.

**Problem 4** Consider the symmetry operations for the covering group of a square, $\mathsf{D}_4$. The order of this group is 8 and consists of the following operations

(a) $e$: the identity

(b) $a, b, c, d$: $\mathsf{C}_2$ operations about the vertical, diagonal and horizontally oriented symmetric axes

(c) $f, g, h$: Counterclockwise rotations in the plane about an axis perpendicular to the surface of the triangle of $\pi/2$, $\pi$, and $3\pi/2$ respectively

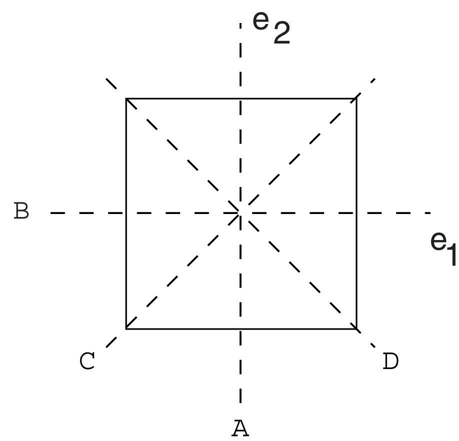



Figure 1: Definition of rotation and coordinate axes for the covering group of a square.

Refer to the system in Figure 1 at the top of the page. In order to help you organize your calculation, I've prepared worksheets for your evaluation of each element, first on the square, then transfering to a relabeling of the axes – you can print the blank ones out 8 times (if you really need help with the identity element!). The blanks are at `square_sheet_1.pdf` . I've also done the first element, $a$, as an example of how to use the sheets. It's at `square_sheet_1_exA.pdf` Have fun.

(a) Construct the multiplication table

(b) Work out the elements which are conjugate to $b$

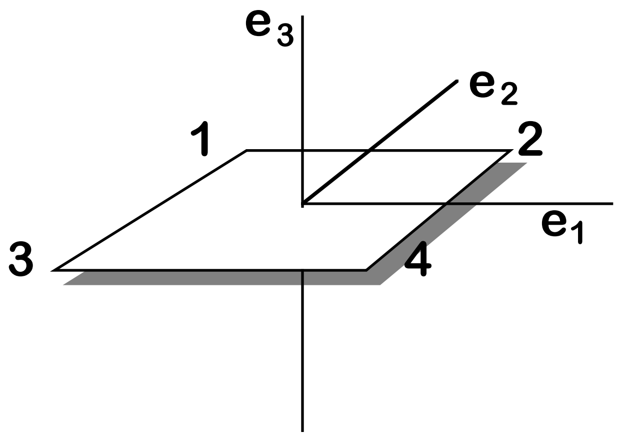
(c) Separate the group into classes

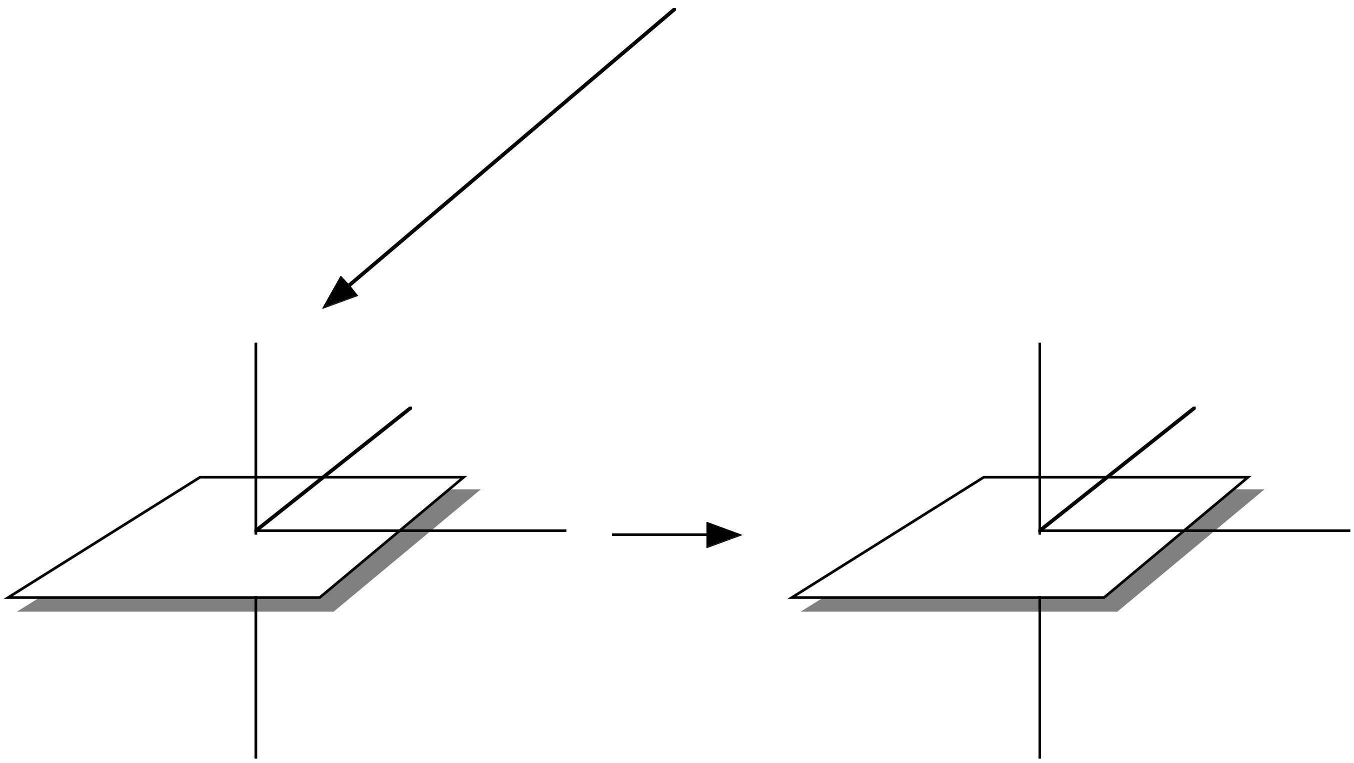
(d) How many IRR will there be?

(e) Work out the 2-d matrix representation by considering the effects on the coordinate bases ($e_1$ to the right from the center, $e_2$ up from the center).

**group element** ________  o

**operating on:**



x →

y →

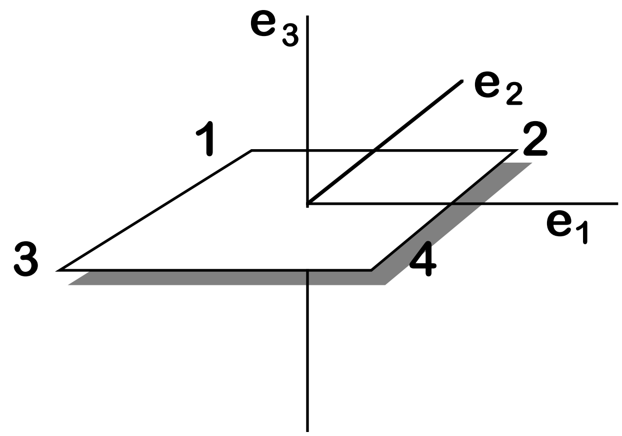
z →

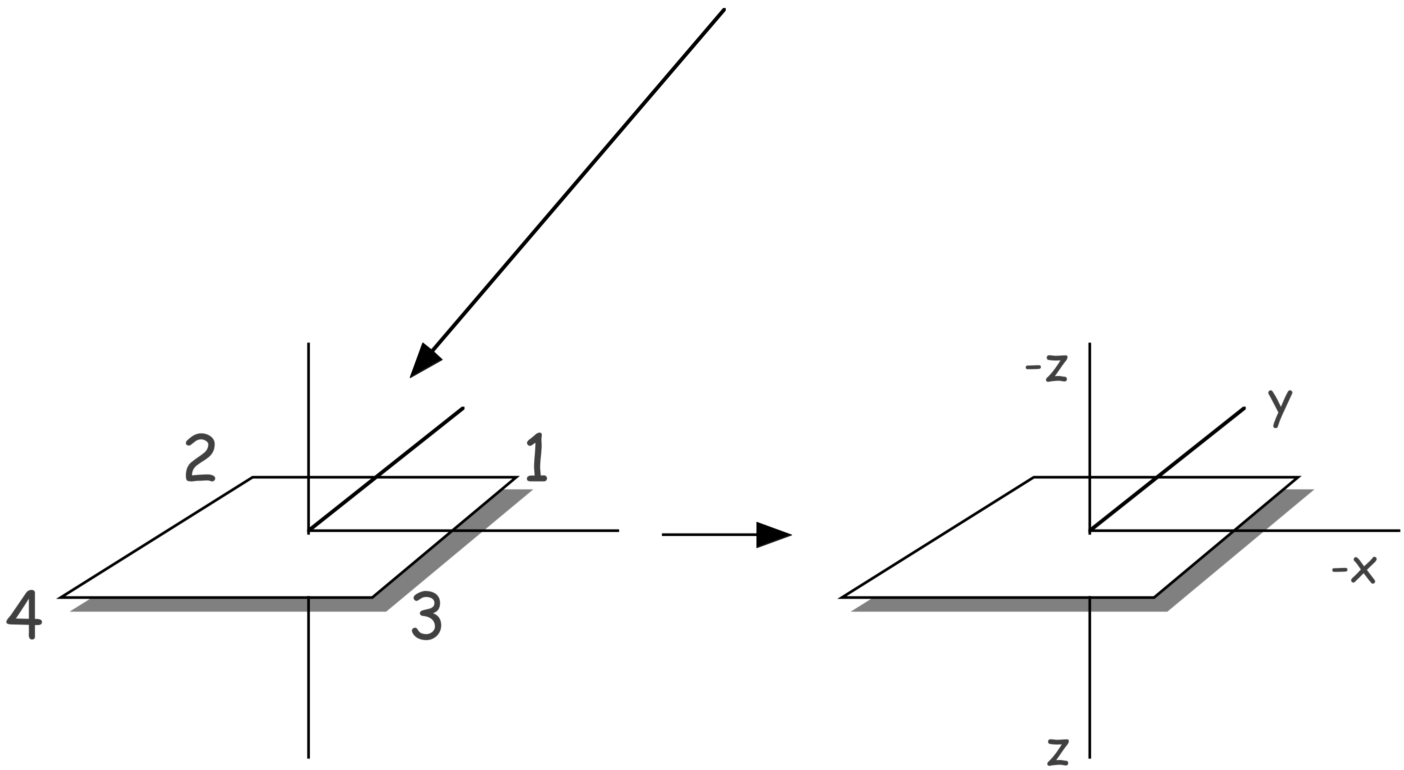
**in matrix form:**

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

**group element** A

**operating on:**




$x \longrightarrow -x$

$y \longrightarrow y$

$z \longrightarrow z$

**in matrix form:**

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x \\ y \\ -z \end{bmatrix}$$