Problem Set #3

PHY 853, Fall Semester, 1999

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These problems are due on December 15, 1999 5:00 p.m. to Lisa Ruess in Rm 207!

Problem 13 The infinitesimal interval is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where, for Cartesian coordinates, the metric is

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Derive the form of the metric for spherical coordinates.

Problem 14 Prove that for 4-vectors a^μ, b^μ, and c^μ that two explicit Lorentz transformations $a \to b$ followed by $b \to c$ is the same thing as adding the imaginary angles for the Lorentz transformation viewed as a rotation in the complex plane. Assume that the boost is along the 3-direction for simplicity.

Problem 15 From Newton's Second Law, $\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = \frac{d}{dt}\left(m_0 \gamma \boldsymbol{u}\right)$ and the Work-Energy Theorem show that, for the time-rate-of-change of kinetic energy, T, is

$$\frac{dT}{dt} = mc^2 \frac{d\gamma}{dt}$$

Further, prove Einstein's famous relation that $E = m_0 c^2 \gamma = T + m_0 c^2$, where E is the total energy and m_0 is the rest mass.

Problem 16 For the tensor $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$, show that the space components reduce to

$$\Sigma_{ij} = \frac{1}{2} \varepsilon_{ijk} \sigma^k \mathbf{I}$$

where $\mathbf{I}$ is the identity matrix.

Problem 17 Show that the helicity of a positive energy particle spinor is $+1$ or -1. What is the form of the spinor for the $+1$ helicity?

Problem 18 (a) Show that the Lorentz character of

$$\bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$$

is that of a vector and that the parity character is that of an axial vector.

(b) Show that the Lorentz character of

$$\bar{\psi}(x)\gamma^5\psi(x)$$

is that of a scalar and that the parity character is that of a pseudoscalar.

Problem 19 (a) Work out the 4×4 matrix representation for the infinitesimal generators K_j for pure Lorentz transformations. Show that the commutation relations,

$$[K_i, K_j] = -i\varepsilon_{ijk}J_k$$
$$[J_i, K_i] = 0$$
$$[J_i, K_j] = i\varepsilon_{ijk}K_k$$

hold where the J_i are the familiar generators of rotations.

(b) Show that the Casimir operators $\mathbf{J} \cdot \mathbf{K}$ and $\mathbf{J}^2 - \mathbf{K}^2$ commute with all of the generators.

Problem 20 The matrix element for the elastic scattering of an electron from a Coulomb potential is

$$\bar{u}(k')\frac{\gamma^0}{q^2}u(k)$$

where the k momentum is along the 3-axis and is the initial momentum of the electron and the k' momentum is the final momentum and is inclined at an angle of θ with respect to the 3-axis. q is the 4-momentum transfer, $q = k - k'$. Since the scattering is elastic, the particle *in* is the same electron as the particle *out*, so $E = E'$. Show that for

$$(+\text{helicity}) \to (-\text{helicity})$$

scattering that the amplitude is

$$A_{\uparrow\downarrow} = \frac{2E}{q^2}\cos\frac{\theta}{2}.$$