Problem Set #1

PHY 854, Spring Semester, 2000
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These problems are due on March 3, 2000 5:00 p.m. to Lisa Ruess in Rm 207. Note, $\hbar = c = 1$.

Problem 21 Start with the semi-classical Hamiltonian density for the Dirac field:

$$H = \int d^3x \psi_j^\dagger(x) \left(-i\boldsymbol{\alpha} \cdot \nabla + \beta m\right)_{jk} \psi_k(x)$$

and the Fourier expansion fof that field,

$$\psi_j(x) = \sum_{i=1,2} \int dK \left[a^{(i)}(k) u_j^{(i)}(k) e^{-ik\cdot x} + b^{\dagger(i)}(k) v_j^{(i)}(k) e^{ik\cdot x} \right]$$

where $dK \equiv \frac{d^3k}{(2\pi)^3 2\omega_k}$. Quantize the field and show that the Hamiltonian operator becomes

$$H = \int dK \sum_{i=1,2} E \left[a^{\dagger(i)}(k) a^{(i)}(k) + b^{\dagger(i)}(k) b^{(i)}(k) \right].$$

Problem 22

(a) Show that $\frac{1}{2}(1 \pm \gamma_5)$ is a projection operator for right-handed (top sign) or left-handed (bottom sign) Dirac spinors.

(b) Define left-handed and right-handed components and show that the Lagrangian density for free spin 1/2 fields

$$\mathcal{L}(x) = \bar{\psi}(x) \left(i\gamma^\mu \partial_\mu - m\right) \psi(x)$$

can be written

$$\mathcal{L}(x) = \bar{\psi}_L(x) i\gamma^\mu \partial_\mu \psi_L(x) + \bar{\psi}_R(x) i\gamma^\mu \partial_\mu \psi_R(x) - m \left(\bar{\psi}_R(x)\psi_L(x) + \bar{\psi}_L(x)\psi_R(x) \right).$$

(c) Show that the Dirac Equation for $\psi(x)$ is retrieved using the Euler Lagrange equations.

(d) Show that one can retrieve the Dirac equation for $\bar{\psi}(x)$.