Problem Set #2c

PHY 854, Spring Semester, 2000

Chip Brock, brock@chip.pa.msu.edu

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Typos fixed in Problem 30 (the Weinberg angle does not fit here...it's a scattering angle - we talked about that in class); Problem 31 (the neutrino scattering cross section needed G_F^2); Problem 37 (the big term in brackets is cubed - we talked about that in class); and Problem 40 (the σ is really the 4d version, namely Σ, which has Pauli 2×2 matrices on the diagonal). Thanks for letting me know about these. whew.

Problems 27 through 33 at least are due on April 28, 2000 6:30 p.m. at my home for an informal dinner. Please bring a companion, if you would like. We would appreciate an rsvp and I'll post a map on the web site. The balance of the problems are due at 5:00 p.m. on the following Wednesday, May 3 (of finals week) to Lisa Ruess. Note, $\hbar = c = 1$.

Problem 27 In class we derived the differential cross section, $d\sigma/d\Omega$, for electrons which scatter from a Coulomb field. In that presentation, I treated the spinors directly. Derive the same thing under the same conditions, except this time use the trace-techniques for averaged and summed electron helicities.

Problem 28 The invariant amplitude for Compton scattering which we derived in class was the following:

$$T_{fi} = \bar{u}^{(f)}(p) \left[\not{\varepsilon}' \frac{\not{p} + \not{k}' + m}{2p \cdot k} \not{\varepsilon} - \not{\varepsilon} \frac{\not{p} + \not{k} + m}{2p \cdot k'} \not{\varepsilon}' \right] u^{(i)}(p)$$

where each term came from one of the two possible Feynman diagrams. Show that both diagrams are necessary in order to insure Gauge Invariance.

Problem 29 For the Compton problem, we found that the summed/averaged invariant amplitude-squared becomes

$$\bar{\sum_i} \sum_f |T|^2 = \frac{e^4}{2} Tr \left\{ (\not{p}' + m) \left[\frac{1}{2p \cdot k} \not{\varepsilon}' \not{k} \not{\varepsilon} + \frac{1}{2p \cdot k'} \not{\varepsilon} \not{k}' \not{\varepsilon}' \right] (\not{p} + m) \left[\frac{1}{2p \cdot k} \not{\varepsilon} \not{k} \not{\varepsilon}' + \frac{1}{2p \cdot k'} \not{\varepsilon}' \not{k}' \not{\varepsilon} \right] \right\}.$$

Show that the term proportional to $\frac{1}{(2p \cdot k')^2}$ is:

$$8(p \cdot k')^2 \left[p' \cdot k' - 2 \left(k' \cdot \varepsilon \right)^2 \right].$$

Problem 30 For the decay of the W in its rest frame, show that the differential decay rate for a longitudinally polarized W is

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta$$

Recall that the relationship between the Fermi constant and the W-fermion coupling strength is $g^2 = \frac{8M_W^2 G_F}{\sqrt{2}}$.

Problem 31 **Do either this problem:**
Consider the scattering of electron-neutrinos from electrons through the exchange of an intermediate W boson, the so-called "charged current" process.

(a) What is the diagram?

(b) Show that the differential cross section, in the center of momentum frame is given by $\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2}$. Here the coupling is through an intermediate W boson with the vertex element which is the same as that for the W decay, namely $\frac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$. Neglect the mass of the electron and presume the neutrino to be (still) massless.

(c) In actuality, there is a neutral current process for this same final state with the exchange of a massive, neutral intermediate vector boson, the Z^0. What is the diagram for this process? How does it combine with the charged current process?

or this problem:
The top quark semileptonic decay ($t \to b + \ell + \nu_\ell$) looks a lot like regular muon decay ($\mu \to e + \nu_e + \nu_\mu$). However, since the top quark mass of approximately 173 GeV is larger than the real W boson mass, it actually decays, unlike muon decay, into a physical W, which subsequently can decay into lepton pairs.

(a) Show that the total rate for $t \to W + b$ in the rest frame of the top quark is

$$\Gamma = \frac{G_F}{8\pi\sqrt{2}} \left(1 - \frac{M_W^2}{m_t^2}\right)^2 \left(1 + \frac{2M_W^2}{m_t^2}\right)$$

where the bottom quark is presumed to have $m_b^2 << m_t^2$. In order to simplfy grading use p for the top quark 4-momentum, k for the bottom quark 4-momentum, and P for the W 4-momentum.

(b) There is something peculiar about this decay rate, in particular the coupling. What is it?

Problem 32 For the Bhabha process $e^+e^- \to e^+e^-$:

(a) How many diagrams are there? Draw them and label the positron 4-momenta p and p' and the electron 4-momenta k and k'.

(b) Show that the square of the invariant amplitude for highly relativistic electrons and positrons is proportional to:

$$2e^4 \left[\frac{s^2+u^2}{t^2} + \frac{u^2+t^2}{s^2} + \frac{2u^2}{ts} \right].$$

Problem 33 In class we calculated the total cross section for the process $e^+e^- \to \mu^+\mu^-$ in which the muon was presumed to be massless. In terms of the β of the outgoing fermion and s, what is the total cross section for the process $e^+e^- \to \tau^+\tau^-$ in which the mass of the only the tau leptons cannot be neglected? Do the same thing for c quarks and b quarks and plot them on one graph. Perhaps you can see why there was such confusion in 1974-5 when both charm and the tau were nearly simultaneously discovered.

Problem 34 How many and what are the independent graphs for photon-photon scattering in fourth order (called Delbruck scattering)?

Problem 35 One more time... Show that a redefinition of the gradient operator (to the "Covariant derivative"

$$\frac{\partial}{\partial x^\mu} \to D_\mu \equiv \frac{\partial}{\partial x^\mu} - iqA$$

leads to an invariance in this theory if one also allows for the A_μ to have a gauge arbitrariness. Start with the free Dirac Lagrange Density and derive the consequences of demanding this invariance under a local $U(1)$ local gauge transformation.

Problem 36 Calculate the ratio of leptonic decay widths for the charged pion for two assumptions about the spacetime nature of its matrix element. That is, calculate:

$$R \equiv \frac{\Gamma(\pi \to \mu\nu_\mu)}{\Gamma(\pi \to e\nu_e)}$$

for two choices of possible physics,

a) vector-axial vector: $T = i\frac{G}{\sqrt{2}} P^\mu \bar{u}_{e,\mu}(k_1)\gamma_\mu(1-\gamma_5)v_{\nu_e,\nu_\mu}(k_2)$ and
b) scalar-pseudoscalar: $T = ig\bar{u}_{e,\mu}(k_1)(1-\gamma_5)v_{\nu_e,\nu_\mu}(k_2)$.

Call the 4-momenta of the electron (muon) and its neutrino k_1 and k_2, respectively. Look up the experimental results for the decay rates and decide which model works best.

Problem 37 There is a theorem in weak interaction physics called the "Conserved Vector Current Hypothesis". It says that the divergence of the vector weak interaction current is zero (there's also a theorem about the not-so-conserved axial vector current called the, um ... , "Partially Conserved Axial Vector Current Hypothesis". The finiteness of the pion mass is what destroys the conservation of the axial current ... and therein lies a different story of symmetry breaking told originally by Nambu at Chicago …) Anyhow, in general terms, the vector current can be described by form factors, functions of the momentum transfer which play the same role as the electric form factor we encountered in electron scattering. The general vector current is the following:

$$J_\mu = f_1(q^2)\gamma_\mu + if_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q^\mu$$

You can see that our electron term is the first one and might be able to guess that the second one is related to the magnetic moment distribution that I alluded to in class. The vector current transition for the hadronic transition of the Sigma baryon to the Lambda baryon is represented generally as

$$V^\mu = \left\langle \Lambda \,|J^\mu|\, \Sigma^\pm \right\rangle = \bar{u}_\Lambda(p_2)J^\mu u_\Sigma(p_1)$$

Where J is above. So, CVC says that V^μ is a conserved current. This is relevant for, say, the beta decay of the Sigma, $\Sigma^\pm \to \Lambda e^\pm \nu$.

(a) Use CVC to show that for Σ beta decay that only f_1 and f_3 are related and argue that f_1 vanshes at $q^2 = 0$, where $q = p_1 - p_2$.

(b) For the decay $\Sigma \to \Lambda\gamma$, that is, the production of a real photon, why does only the f_2 term contribute to V? In the rest frame of the Σ, using the T matrix element of

$$T = -ie\varepsilon^\mu_{(\lambda)}(q)\left\langle \Lambda \,|J^\mu|\, \Sigma^\pm \right\rangle$$

show that the rate is

$$\Gamma = \frac{e^2 f_2^2(0)}{2\pi}\left[\frac{M^2_{\Sigma^\pm} - M^2_\Lambda}{2M_{\Sigma^\pm}}\right]^3$$

(c) From this result and the good presumption that the branching fraction is nearly 100%, show that $f_2(0) \sim 1.17\ \mathrm{GeV}^{-1}$. Notice that this can be compared with the strict $SU(3)$ predictions of:

$$\begin{aligned} f_2(0) =& \sqrt{2}\frac{\mu_{\Sigma N}}{2M_N} \\ \mu_{\Sigma N} =& \frac{-\sqrt{6}\mu_N}{2} \end{aligned} .$$

Indeed, make this comparison … is $SU(3)$ badly broken in this decay?

Problem 38 **Do either this problem:**
Show that for the beta decay of the pion,

$$\Gamma(\pi^\pm \to \pi^0 e\nu) = \frac{G_F^2\Delta^5}{30\pi^2}$$

where $\Delta \equiv m_{\pi^\pm} - m_{\pi^0}$. Feel free to use the kinematics of neutron beta decay for the phase space results, but actually calcuate that $\tau = 2.5$ s with proper units.
or this problem:
The Higgs boson is presumed to decay with a coupling which is proportional to the mass of the decay products. So, it will decay to the heaviest objects. Presume that leptonic decay is of interest. Then, for the interaction Lagrangian of

$$L = -\left(\sqrt{2}G_F\right)^{\frac{1}{2}} m_\tau h^0(x)\bar{\psi}_\tau(x)\psi_\tau(x),$$

show that the decay rate is

$$\Gamma = \frac{G_F m_\tau^2 m_h}{4\pi\sqrt{2}}\left(1 - \frac{4m_\tau^2}{m_h^2}\right)^{\frac{3}{2}}.$$

Problem 39 Calculate the relevant term in the Bremsstrahlung cross section, namely, work out the integral

$$\int d^3k \frac{1}{\omega(p\cdot k)(p'\cdot k)}.$$

You will have to introduce a pair of Feynman parameters. Show that it has the value

$$2\pi \int dx \frac{1}{P^2}\ln\left(\frac{P^2\Delta E^2}{\lambda^2 P_0^2}\right)$$

where $P = p' + (p - p')x$ and the energy integration goes from a lower limit of λ to an upper limit of ΔE.

Problem 40 Show that $\frac{1}{2}\frac{e}{2m}\sigma^{\mu\nu}F_{\mu\nu} = \mu_0\vec{\Sigma}\cdot\vec{B} - i\mu_0\vec{\alpha}\cdot\vec{E}$.