Midterm II for phy231, Monday November 5, 2001

Constants: g=9.8 m/s^2.
Moments of inertia: $I(\text{sphere}) = (2/5)MR^2$, $I(\text{disk}) = \frac{1}{2}MR^2$, $I(\text{hoop}) = MR^2$.

1. Two masses are attached to the end of a rod of length ℓ=12 m. Each has a mass of $m = 5$ kg. The rod is rotated around an axis which is 1/4 of the way from one end. What is the moment of inertial I in units of kg m^2? (Ignore the mass of the rod.)

   (a) 720
   (b) 450 ⇐
   (c) 360
   (d) 180
   (e) 90

   Definition of momentum of inertia:
   $I = m(\frac{\ell}{4})^2 + m(\frac{3}{4}\ell)^2$
   $= 5\,kg\,(3^2 + 9^2)\,m^2 = 5 \times 90\,kg\,m^2 = 450\,kg\,m^2$

2. A sphere starts from rest at the top of an inclined plan and rolls down. The height of the inclined plane is $h = 5$ m and it makes an angle of 30° with respect to the horizontal. What is the translational velocity of the sphere when it reaches the bottom in units of m/s?

   (a) 10.0
   (b) 8.4 ⇐
   (c) 7.0
   (d) 4.2
   (e) 2.1

   Use conservation of energy:
   $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad (I = \frac{2}{5}mR^2\ ;\ \omega = v/R)$
   $= \frac{1}{2}mv^2(1 + \frac{2}{5}) = \frac{7}{10}mv^2$
   Therefore $v^2 = \frac{10}{7}gh = \frac{490}{7}\frac{m^2}{s^2}$
   or $v = \sqrt{70}\ m/s = 8.4\ m/s$

3. An ice skater is spinning with his arms stretched out. When the skater brings his arms in, which is the following is **true**? (There are no external torques).

   (a) The moment of inertia is increased.
   (b) The angular momentum is increased.
   (c) The angular velocity is increased. ⇐
   (d) The angular velocity is constant.
   (e) Kinetic energy is conserved.

   Angular momentum is conserved in this process:
   i.e. $I\omega = const.$
   Arms pulled in ⇒ I is reduce
   ⇒ ω is increased.

4. A helium filled blimp is floating at a constant height above the ground. The volume of the balloon is 2000 m^3, the density of helium is 0.18 kg/m^3 and the density of air is 1.2 kg/m^3. What is the buoyancy force in newtons?

   (a) 360
   (b) 2400
   (c) 3500
   (d) 20000
   (e) 23500 ⇐

   Buoyancy force = weight of air replaced:
   force = $m\,g = \rho_{air}\,V\,g$ = 23500 N

5. A fluid which fills a large pipe with a radius of 2 m flows into a smaller pipe with a radius of 1 m. Which of the following is **true**? (Assume laminar flow).

   (a) The volume rate of flow does not change. $\Leftarrow$
   (b) The velocity of the fluid does not change.
   (c) The pressure in the fluid does not change.
   (d) The pressure is smaller in the large pipe.
   (e) The kinetic energy does not change.

	larger A	smaller A
$A\,v$	same	same
v	smaller	larger
$p + \frac{1}{2}\rho v^2$	same	same
p	larger	smaller

6. A pool with a depth of 5 m is filled with water. What is the increase in pressure from the top of the pool to the bottom of the pool in Psi (pounds per square inch)? (The density of water is 1000 kg/m^3, 1 atm = 1.01x10^5 Pa = 14.7 Psi).

   (a) 49000
   (b) 21.9
   (c) 14.7
   (d) 7.1 $\Leftarrow$
   (e) 0.49

Increase in pressure is:
$\Delta p = \rho_w g h = 1000\frac{kg}{m^3}\,9.81\frac{m}{s^2}\,5\,m$
$= 4.91 \times 10^4\,Pa \times \frac{14.7\,Psi}{1.01\times 10^5\,Pa} = 7.1$ Psi

7. The temperature of a hot coal is 2000 °F. What is the temperature in °K?

   (a) 820
   (b) 1093
   (c) 1366 $\Leftarrow$
   (d) 2273
   (e) 3632

$T(K) = T(C) + 273 = \frac{5}{9}(T(F) - 32) + 273 = 1366$

8. A container of an ideal gas starts out with a temperature of 50 °C and a pressure of 1 atm. It is heated to a temperature of 100 °C. The number of atoms and the volume are constant. What it the final pressure in units of atm?

   (a) 0.5
   (b) 0.67
   (c) 0.87
   (d) 1.15 $\Leftarrow$
   (e) 2.0

Ideal gas law: $\frac{P_i}{T_i} = \frac{P_f}{T_f}$
which implies $P_f = P_i(\frac{T_f}{T_i})$ (in $^\circ K$)
Therefore: $P_f = 1\,atm \times \frac{373}{273} = 1.15\,atm.$

9. The temperature of an ideal gas increases from 200 °K to 800 °K. The rms (root mean square) velocity of the atoms in the gas:

   (a) Stays the same.
   (b) Decreases by a factor of two.
   (c) Decreases by a factor of four.
   (d) Increases by a factor of two. ⇐
   (e) Increases by a factor of four.

Since $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$,
increasing T by a factor of $\frac{800\,^\circ K}{200\,^\circ K} = 4$
will increase $\bar{v}$ by a factor of 2.

10. Andy and Bill collide in bumper cars of mass 50 kg each. Andy has a mass of 75 kg, and Bill has a mass of 50 kg. Bill is at rest when Andy strikes him from the rear at $v = 8$ m/s. Andy's speed is reduced to 4 m/s after collision. What is Bill's speed after collision in units of m/s?

    (a) 2
    (b) 3
    (c) 4
    (d) 5 ⇐
    (e) 6

Momentum is conserved in the collision $\Delta P_{Andy} + \Delta P_{Bill} = 0$:
$\Delta P_{Andy} = m_A(v_f - v_i)_A = -125\,kg\,4\,m/s = -500\,kg\,m/s$;
$\Delta P_{Bill} = m_B\,(v_f - v_i)_B = -\Delta P_{Andy} = 500\,kg\,m/s$.
Thus, $v_f^B = \dfrac{500\,kg\,m/s}{100\,kg} = 5\,m/s.$
(Note: masses $m_{A,B}$ include person+car.)

11. When a free-standing rifle is fired:

    (a) Kinetic energy and linear momentum are conserved.
    (b) Kinetic energy is conserved and linear momentum increases.
    (c) Kinetic energy increases and linear momentum is conserved. ⇐
    (d) Neither kinetic energy nor linear momentum are conserved.
    (e) Kinetic energy increases and linear momentum increases.

12. A billiard ball collides in an elastic head-on collision with a second stationary identical ball. After the collision which of the following conditions applies to the first ball?

    (a) maintains the same velocity as before
    (b) has one half its initial velocity
    (c) comes to rest ⇐
    (d) moves in the opposite direction with half the speed
    (e) moves in the opposite direction with the same speed

See the solution of this problem in the textbook.

13. A record drops onto a turntable which turns at a rate of 33 rpm (revolutions per minute) and accelerates up to speed in 0.6 seconds. What is its angular acceleration, α , during this time?

   (a) 3.46 rad/s
   (b) 5.76 rad/s
   (c) 7.57 rad/s^2
   (d) 5.76 rad/s^2 $\Leftarrow$
   (e) 3.46 rad/s^2

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{33\,rpm}{0.6\,s} = \frac{33}{0.6}\frac{rev}{\text{min}\ s}\frac{2\pi\,rad}{rev}\frac{1\ \text{min}}{60\,s}$$
$$= \frac{2\pi \times 33}{36}\frac{rad}{s^2} = 5.76\frac{rad}{s^2}$$

14. Consider a child playing on a swing. As she reaches the lowest point in her swing which is the following is **true**?

   (a) The tension in the rope is equal to her weight.
   (b) The tension in the rope is equal to her mass times her acceleration.
   (c) Her acceleration is downward and equal to g (9.8 m/s^2).
   (d) Her acceleration is zero.
   (e) Her acceleration is equal to v^2/L where L is the length of the swing. $\Leftarrow$

15. The acceleration of gravity on the surface of the earth is $g = 9.8$ m/s^2 . Suppose a planet exists that has 4 times the mass of the earth and twice the radius of the earth. The acceleration of gravity on the surface of this planet would be:

   (a) (1/2)g
   (b) g $\Leftarrow$
   (c) 2g
   (d) 4g
   (e) 8g

Since $g = G\frac{M}{R^2}$ (cf. textbook), quadrupling M and doubling R will change both the numerator and the denominator by a factor of 4. Hence the value of g will stay the same.