Solutions — Exam 1

① $$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\,\psi$$

② $$\psi(x,t) = A\, e^{i(kx-\omega t)}$$

(a) wavelength $= 2\pi/k$

(b) momentum $= \hbar k$

(c) energy $= \dfrac{\hbar^2 k^2}{2m}$

③ $$\psi(x) = N e^{-\alpha x^2}$$

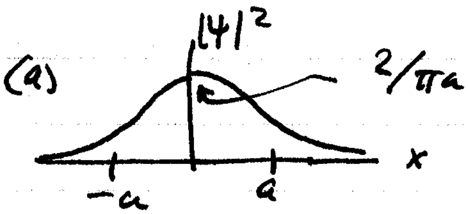
Require $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \implies N = \left(\frac{2\alpha}{\pi}\right)^{1/4}$

④ 

Bohr model : $r_n = \dfrac{a_0 n^2}{Z}$

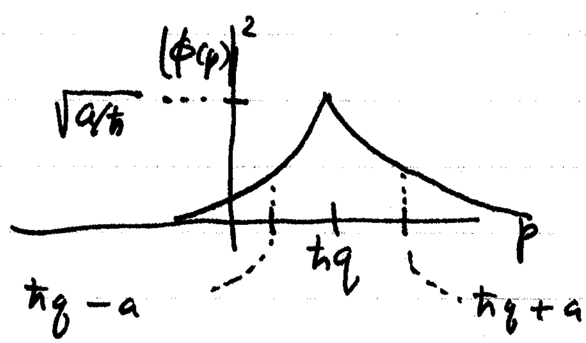
$$a_0 = \frac{\hbar^2}{me^2} = 0.53 \times 10^{-8}\ \text{cm}$$

⑤ $$\psi(x) = \sqrt{\frac{2a^3}{\pi}}\, \frac{e^{iqx}}{(x^2+a^2)}$$

(a) 

(b) Obviously $\langle x \rangle = 0$

(c) $$\phi(p) = \sqrt{\frac{a}{\hbar}}\; e^{-a|q - p/\hbar|}$$

(d) 

(e) Obviously $\langle p \rangle = \hbar q$.