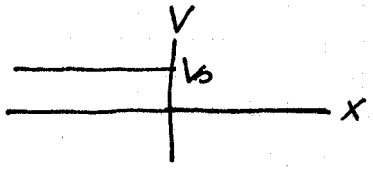
Solutions — Exam 2

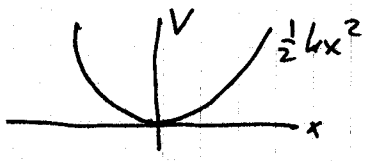
(1) 

For $x<0$, $\psi(x) = e^{ikx} + Re^{-ikx}$

For $x>0$, $\psi(x) = Te^{ilx}$

$$R = \frac{k-q}{k+q} = \frac{\sqrt{E} - \sqrt{E-V_0}}{\sqrt{E} + \sqrt{E-V_0}} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

Reflection probability $= |R|^2 = 2.94 \times 10^{-2}$ (2.94 percent)

(2) 

$u_0(x) = C_0 e^{-\frac{1}{2}\alpha^2 x^2}$ where $\alpha = \sqrt{\frac{m\omega}{\hbar}}$ and $\omega = \sqrt{k/m}$

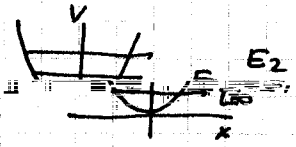
$C_0 = \left(\frac{\alpha^2}{\pi}\right)^{1/4}$ normalization constant

(3) $A = px \Rightarrow A^\dagger = xp$ $\therefore A$ is not Hermitian

$B = xp + px \Rightarrow B^\dagger = px + xp$ $\therefore B$ is Hermitian

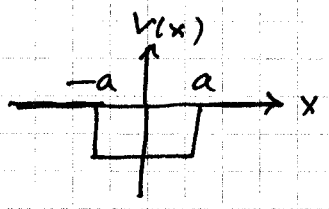
$C = xp - px \Rightarrow C^\dagger = px - xp$ $\therefore C$ is not Hermitian

(4) $E_0 = 10\,eV \Rightarrow E_1 = 30\,eV$ and $E_2 = 50\,eV$



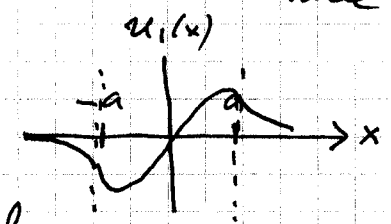
$E_\gamma = \Delta E$

(a) 20 eV
(b) 20 eV
(c) 40 eV

(5) 

$$u_1(x) = \begin{cases} Ce^{-\alpha x} & \text{for } x > a \\ B\sin qx & \text{for } -a < x < a \\ -Ce^{\alpha x} & \text{for } x < -a \end{cases}$$

where $E = -\frac{\hbar^2\alpha^2}{2m}$ and $E = \frac{\hbar^2 q^2}{2m} - D_0$.



← The first excited state is an odd function of x, with 1 node.

← exponential → ← sinusoidal → ← exponential →