

## Problem 4

```
Int[k_] :=
```

```
Integrate[Exp[-a*x^2]*x^(2k), {x, -Infinity, Infinity}, Assumptions -> {a > 0}]
```

```
Table[Int[k], {k, 0, 4}]
```

$$\left\{ \frac{\sqrt{\pi}}{\sqrt{a}}, \frac{\sqrt{\pi}}{2 a^{3/2}}, \frac{3 \sqrt{\pi}}{4 a^{5/2}}, \frac{15 \sqrt{\pi}}{8 a^{7/2}}, \frac{105 \sqrt{\pi}}{16 a^{9/2}} \right\}$$

Note:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} x^{2k} dx = \frac{(2k-1)!! \sqrt{\pi}}{2^k \sqrt{\alpha} \alpha^k}$$

## Problem 5

```
Integrate[Exp[-a*x^2/2] Exp[b*x], {x, -Infinity, Infinity}, Assumptions -> {a > 0}]
```

$$\frac{e^{\frac{b^2}{2a}} \sqrt{2\pi}}{\sqrt{a}}$$

Note:

$$\int_{-\infty}^{\infty} e^{-ax^2/2} e^{bx} dx = \sqrt{\frac{2\pi}{a}} e^{b^2/2a}$$

## Problem 6

```
Integrate[x^2*(x^2+a^2)^(-2), {x, -Infinity, Infinity}, Assumptions -> {a > 0}]
```

```
Integrate[u^2 Exp[-b*u], {u, 0, Infinity}, Assumptions -> {b > 0}]
```

$$\frac{\pi}{2a}$$

$$\frac{2}{b^3}$$

Note:

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)^2} = \frac{\pi}{2a}$$

$$\int_0^{\infty} u^2 e^{-bu} du = \frac{2}{b^3}$$