PHY 471 Friday, Sept 21

Exam 1

1. Write the Schroedinger equation for $\psi(x,t)$.

2. Suppose a free particle of mass m has the wave function

$$\psi(x,t) = Ae^{i(kx-\omega t)}.$$

In terms of k, ω, and m, what are (a) the wavelength, (b) the momentum, and (c) the energy?

3. At some time the wave function of a particle is

$$\psi(x) = Ne^{-\alpha x^2}.$$

What must N be?

4. In the Bohr model of the atom, what are the possible radii of electron orbits? What is the numerical value of the smallest orbit radius? Answer as precisely as you can.

5. At some time the wave function of a particle is

$$\psi(x) = \sqrt{\frac{2a^3}{\pi}}\frac{e^{iqx}}{x^2+a^2}.$$

(a) Sketch a graph of $|\psi(x)|^2$ versus x; include scales on the axes, in terms of a.

(b) Determine $\langle x\rangle$, e.g. from the graph in (a).

(c) Find the momentum-space wave function $\phi(p)$.

(d) Sketch a graph of $|\phi(p)|^2$ versus p; include scales on the axes.

(e) Determine $\langle p\rangle$, e.g. from the graph in (d).

Information

$\hbar c = 2 \times 10^{-5}\,\mathrm{eV\,cm}$

$mc^2 = 0.5 \times 10^{6}\,\mathrm{eV}$

$$\frac{e^2}{\hbar c} = \frac{1}{137}$$

$$\int_{-\infty}^{\infty} e^{-bx^2} dx = \sqrt{\frac{\pi}{b}}$$

$$\int_{-\infty}^{\infty} \frac{e^{iux} dx}{x^2 + a^2} = \frac{\pi}{a}\, e^{-a|u|}$$