PHY 471 Friday, Nov 16

Exam 3

1. A particle moves in 2 dimensions attracted to the origin by a linear force. The potential is shown in the Figure. The Hamiltonian is

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{2}m\omega^2\left(x^2 + y^2\right).$$

(a) Describe the energy eigenstates — eigenfunctions and eigenvalues. [Hint: Use separation of variables, $u(x, y) = f(x)g(y)$.]
(b) For the 4 lowest energy levels, give the energy and degeneracy.

2. An electron moves freely in a spherical cavity of radius $a =$ 1 angstrom $= 1 \times 10^{-8}$ cm. ($V = \infty$ for $r > a$.) What is the energy if the electron is in the ground state? Give the answer in eV.

Useful:
$j_0(\xi) = \dfrac{\sin\xi}{\xi}$

3. In copper, the density of conduction electrons is $8.5\times10^{22}\,\mathrm{cm}^{-3}$. The *Fermi temperature* T_F is defined by $kT_F = E_F$, where $E_F =$ Fermi energy. Calculate T_F.

4. Consider a particle that moves freely in a box with dimensions $\ell_1 \times \ell_2 \times \ell_3$. Compare the ground state energies for these dimensions:

(a) $(\ell_1, \ell_2, \ell_3) = (a, a, a)$

(b) $(\ell_1, \ell_2, \ell_3) = (2a, a/2, a)$

Information
electron mass $= 0.511\,\mathrm{MeV}/c^2 = 0.911 \times 10^{-27}$ g
$\hbar c = 2 \times 10^{-5}$ eV cm
$k = 1.38 \times 10^{-16}$ erg/K = Boltzmann constant
$\hbar = 1.055 \times 10^{-27}$ erg s
$1\,\mathrm{eV} = 1.6 \times 10^{-12}$ erg