1.1

Homework Set 1 Wed. Sept 5

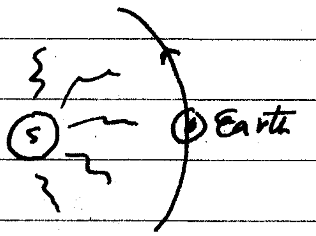
(2) The emissivity from a black body at temp. T is

$$\mathcal{E}(\lambda, T) = \frac{c}{4} u(\lambda, T)$$

where $u(\lambda, T)$ = distribution of energy density in a cavity.
The total energy flux is

$$\mathcal{E}(T) = \int_0^\infty \mathcal{E}(\lambda, T)\, d\lambda = \frac{c}{4} a T^4 = \sigma T^4$$

where $\sigma = 5.42 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}.



The solar constant k is related to $\mathcal{E}$ by

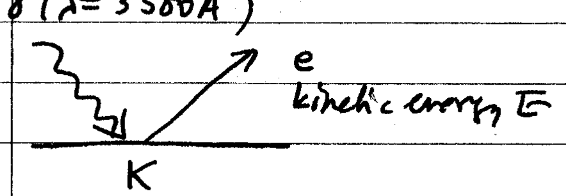
$$\mathcal{E} \cdot 4\pi R_\odot^2 = k \cdot 4\pi d_\odot^2$$

Thus the temperature of the sun is

$$T = \left[\frac{k d_\odot^2}{\sigma R_\odot^2}\right]^{1/4} = \left[\frac{1.4 \times 10^6 \times (1.5 \times 10^{13})^2}{5.42 \times 10^{-5} \times (7 \times 10^{10})^2}\right]^{1/4}$$

$$T = 5870 \text{ K}$$

(4) Photoelectric effect in potassium



By energy conservation,

$$E = h\nu - B$$

where B = binding energy $\geq \phi$.

Thus $E_{max} = h\nu - \phi$

The work function ϕ ($\equiv$ minimum binding energy) is

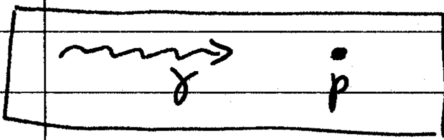
$$\phi = h\nu - E_{max} = \frac{6.63 \times 10^{-27} \text{ erg-s}}{1.602 \times 10^{-12} \text{ erg/eV}} \frac{3 \times 10^{10} \text{ cm/s}}{3.5 \times 10^{-5} \text{ cm}} - 1.6 \text{ eV}$$

$$\phi = 1.94 \text{ eV}$$

1.2

from a proton

⑥ Compton scattering, $\gamma + p \to \gamma' + p'$



The maximum energy loss occurs in a head-on collision, Then the scattered photon has negative momentum $(-p')$

Momentum conservation $\quad p + 0 = -p' + P'$

Energy conservation $\quad cp + mc^2 = cp' + \sqrt{m^2c^4 + P'^2c^2}$

Solve for p'

$$(p - p' + mc)^2 = m^2c^2 + (p+p')^2$$

$$-2pp' + 2mc(p-p') = 2pp'$$

$$p' = \frac{2mcp}{4p+2mc}$$

The energy loss of the photon is

$$E_\gamma - E'_\gamma = cp - cp' = \frac{4cp^2}{4p+2mc}$$

$$= \frac{4\,(100\ \text{MeV})^2}{4\times 100\ \text{MeV} + 2\times 940\ \text{MeV}} = 17.5\ \text{MeV}$$

⑰ $P = \frac{2}{3}\frac{e^2}{c^3}a^2$ (in erg/s) ← radiation power

In the Bohr model, for quantum number n,

$$r = a_B n^2 \quad \text{where} \quad a_B = \hbar^2/me^2$$

$$v = \frac{n\hbar}{mr} = \frac{\hbar}{m a_B n} = \frac{e^2}{n\hbar}.$$

Thus the acceleration is

$$a = \frac{v^2}{r} = \left(\frac{e^2}{n\hbar}\right)^2 \frac{me^2}{\hbar^2 n^2} = \frac{me^6}{\hbar^4 n^4}$$

The power is

$$P = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{me^6}{\hbar^4}\right)^2 \frac{1}{n^8} = \frac{2}{3}\frac{(mc^2)^2\alpha^7}{\hbar}\frac{1}{n^8}.$$

Numerical value: $\qquad \alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$

$$P = 2.91\times 10^{11}\ \frac{\text{eV}}{\text{s}}\ n^{-8}.$$

(18) Decay rate $R = \frac{P}{\Delta E}$ where $P = \frac{2}{3}\frac{(mc^2)^2\alpha^7}{\hbar}\frac{1}{n^8}$.

For a transition from n to $n-1$, energy loss is $\left(\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}\right)$

$$\Delta E = \frac{-E_R}{n^2} + \frac{E_R}{(n-1)^2} = E_R \frac{2n-1}{n^2(n-1)^2}$$

where E_R = Rydberg energy $= \frac{me^4}{2\hbar^2} = \frac{1}{2}mc^2\alpha^2$.
Thus

$$R_{n\to n-1} = \frac{2}{3}\frac{(mc^2)^2\alpha^7}{\hbar n^8}\frac{n^2(n-1)^2}{\frac{1}{2}mc^2\alpha^2(2n-1)}$$

$$\boxed{R_{n\to n-1} = \frac{4}{3}\frac{mc^2\alpha^5}{\hbar}\frac{(n-1)^2}{n^6(2n-1)}}$$

For $n=2$, $R_{2\to 1} = \frac{4}{3}\frac{mc^2\alpha^5}{\hbar}\frac{1}{192} = 1.11\times 10^8\ s^{-1}$

For $n \to n-m$, $R = \frac{4}{3}\frac{mc^2\alpha^5}{\hbar}\frac{(n-m)^2}{n^6(2nm-m^2)}$.

The estimated lifetime for the $n=2$ state is

$$\tau = \frac{1}{R_{2\to1}} = 9\times10^{-9}\,s.$$

(19) $\boxed{\begin{array}{l}\text{Rotor}\\ E = \frac{L^2}{2I}.\end{array}}$ The Bohr quantization rule is $L = n\hbar$,
so $E_n = \frac{\hbar^2 n^2}{2I}$. The frequency in a radiative transition
$n_1 \to n_2$ is $\nu = \frac{E_1 - E_2}{h} = \frac{\hbar}{4\pi I}(n_1^2 - n_2^2) = \frac{\hbar(n_1+n_2)(n_1-n_2)}{4\pi I}$

Correspondence Principle Classically, $L = I\omega$ and the frequency of radiation should be $\nu_{cl} = \frac{\omega}{2\pi} = \frac{L}{2\pi I}$.

To make the connection to quantum mechanics, write $L = n_1\hbar$ and $n_2 = n_1 - \delta$, and let $n_1 \to \infty$.
The frequency is $\nu = \frac{\hbar(2n_1-\delta)\delta}{4\pi I}$. Thus ν tends to $\nu_{cl}\left(=\frac{\hbar n_1}{2\pi I}\right)$ as $n_1 \to \infty$ provided $\delta = 1$.

(15) Harmonic oscillator : $E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2$.

For circular orbits, the Bohr quantization rules are $\frac{mv^2}{r} = m\omega^2 r$ (1)

and $mvr = n\hbar$ (2). where n is an integer.

Solve (2) for v; $v = \frac{n\hbar}{mr}$. Insert into (1):

$$\frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = m\omega^2 r$$

The radius is $r_n = \left(\frac{n\hbar}{m\omega}\right)^{1/2}$.

The energy is

$$E_n = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 r^2 = \frac{m}{2}\left(\frac{n\hbar}{m}\right)^2 \frac{m\omega}{n\hbar} + \frac{m\omega^2}{2}\frac{n\hbar}{m\omega}$$

$$\boxed{E_n = n\hbar\omega}$$

Correspondence Principle Classically the frequency of radiation would be $\nu_{cl} = \frac{\omega}{2\pi}$.

In quantum theory, the frequency for the transition $n \to n-1$ is

$$\nu = \frac{E_n - E_{n-1}}{h} = \frac{\hbar\omega(n) - \hbar\omega(n-1)}{h} = \frac{\omega}{2\pi}.$$

Thus $\nu = \nu_{cl}$ for all transitions $n \to n-1$, which is consistent with the correspondence principle.