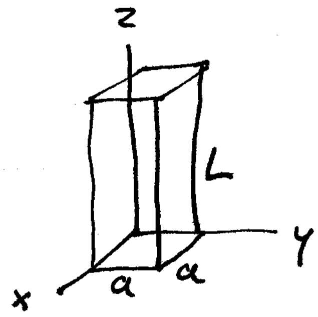
Homework Set 10 (due Nov ~~8~~ 7)

Chapter 9 Problems 3, 6, 7, 8

9-3



Box has dimensions $a \times a \times L$.
The energy levels ~~are~~ depend on 3 quantum numbers, n_1, n_2, n_3

$$E(n_1, n_2, n_3) = \frac{\hbar^2 \pi^2}{2ma^2}(n_1^2 + n_2^2) + \frac{\hbar^2 \pi^2}{2mL^2} n_3^2 .$$

For $a \ll L$, the spacing of ~~with~~ levels with different n_3 values is much less than the spacing for different n_1 or n_2 values.

- For $a = 10$ Å the spacing with different n_1 or n_2 quantum numbers is order
$$\frac{\hbar^2 \pi^2}{2ma^2} = 0.377 \text{ eV}.$$

- For $L = 10^{-4}$ cm the spacing with different n_3 quantum numbers is order
$$\frac{\hbar^2 \pi^2}{2mL^2} = 3.77 \times 10^{-7} \text{ eV}.$$

(Numerical calculations, done with Mathematica, are shown at the end.)

9-6 For Cu, $n = 8.5 \times 10^{22}\ cm^{-3}$.

(1) Fermi energy $E_F = \frac{\hbar^2 \pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3} = 7.06$ eV

(2) Fermi velocity ($\frac{1}{2} m v_F^2 = E_F$) is

$$v_F = \sqrt{\frac{2E_F}{m}} = 1.58 \times 10^8 \text{ cm/s}.$$

9-7 Let $\mathcal{N}$ = # of fermions (= N for neutrons or Z for protons). The Fermi energy is

$$E_F = \frac{\hbar^2 \pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3} \quad \text{where} \quad n = \frac{\mathcal{N}}{\frac{4}{3}\pi R^3}$$

and $R = r_0 A^{1/3}$

$$E_F = \frac{\hbar^2 \pi^2}{2m}\left(\frac{3}{\pi}\right)^{2/3} \frac{\mathcal{N}^{2/3}}{\left(\frac{4}{3}\pi R^3\right)^{2/3}} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{3}{\pi}\right)^{2/3} \frac{\mathcal{N}^{2/3}}{r_0^2 A^{2/3}}\left(\frac{3}{4\pi}\right)^{2/3}$$

$$E_F = \frac{\hbar^2 \pi^2}{2m r_0^2}\left(\frac{9}{4\pi^2}\right)^{2/3} \frac{\mathcal{N}^{2/3}}{A^{2/3}} = \text{const} \times \frac{\mathcal{N}^{2/3}}{A^{2/3}}$$

The constant is

$$\text{const} = \frac{\hbar^2 \pi^2}{2m r_0^2}\left(\frac{9}{4\pi^2}\right)^{2/3} = 66 \text{ MeV}$$

For $Z = 82$ and $N = 126$ (i.e. ^{208}Pb)

E_F (neutron) = 47.3 MeV,

E_F (proton) = 35.5 MeV.

9-8 The Fermi energy is $E_F = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3n}{\pi}\right)^{2/3}$

where n = number density = ρ/m. Thus

$$E_F = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{\rho}{m}\right)^{2/3}$$

$$E_F = K \rho^{2/3}$$

The constant K is

$$K = \frac{\hbar^2 \pi^2}{2m^{5/3}} \left(\frac{3}{\pi}\right)^{2/3} = 2.08 \times 10^{-14} \ \mathrm{cm^4\, g^{-1/3}\, s^{-2}}$$

The nonrelativistic approximation is valid if $E_F \ll m_n c^2$. It is not valid for $E_F \gtrsim m_n c^2$, i.e. for

$$\rho = \left(\frac{E_F}{K}\right)^{3/2} \gtrsim \left(\frac{m_n c^2}{K}\right)^{3/2} = 2.1 \times 10^{16} \ \mathrm{g/cm^3}$$