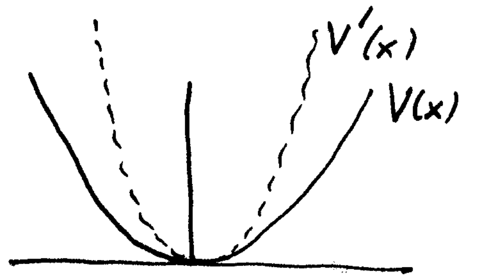
HOMEWORK SET 7

①



$$V(x) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2 x^2 \quad \text{where } \omega = \sqrt{\tfrac{k}{m}}$$

$$V'(x) = 2kx^2 = \tfrac{1}{2}m\omega'^2 x^2 \quad \text{where } \omega' = \sqrt{\tfrac{4k}{m}} = 2\omega.$$

The initial wave function is the ground state of $V(x)$,

$$u_0(x) = \left(\frac{\alpha^2}{\pi}\right)^{1/4} e^{-\frac{1}{2}\alpha^2 x^2} \quad \text{where } \alpha = \sqrt{\frac{m\omega}{\hbar}}.$$

After the change, the ground state is

$$u_0'(x) = \left(\frac{\alpha'^2}{\pi}\right)^{1/4} e^{-\frac{1}{2}\alpha'^2 x^2} \quad \text{where } \alpha' = \sqrt{\frac{m\omega'}{\hbar}} = \sqrt{2}\,\alpha.$$

The overlap integral is

$$\langle u_0' | u_0 \rangle = \int_{-\infty}^{\infty} u_0'(x)\, u_0(x)\, dx$$

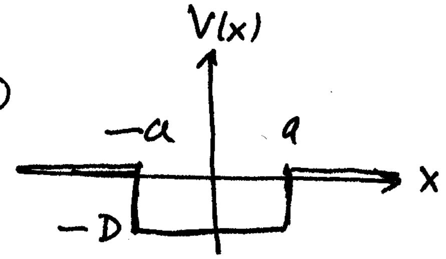
$$= \left(\frac{\alpha\alpha'}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\alpha^2+\alpha'^2)x^2} dx$$

$$= \left(\frac{\alpha\alpha'}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} e^{-y^2} \frac{dy}{\sqrt{\frac{1}{2}(\alpha^2+\alpha'^2)}} \quad \text{letting } y^2 = \tfrac{1}{2}(\alpha^2+\alpha'^2)x^2$$

$$= \left(\frac{\alpha\alpha'}{\pi}\right)^{1/2} \frac{\sqrt{2\pi}}{\sqrt{\alpha^2+\alpha'^2}} = \left(\frac{2\alpha\alpha'}{\alpha^2+\alpha'^2}\right)^{1/2} = \left(\frac{2\sqrt{2}}{3}\right)^{1/2}$$

(Note that $\alpha' = \sqrt{2}\,\alpha$.) The probability that the particle is in the ground state after the change is $P_0 = |\langle u_0' | u_0 \rangle|^2 = \frac{2\sqrt{2}}{3} = 0.943.$

②



$a = 1.5$ Å

$D = 2$ eV (= depth of well)

The eigenvalue equation for the ground state is given in equation (5–65)

$$\sqrt{\lambda - y^2} = y \tan y \quad \text{where} \quad \lambda = \frac{2mDa^2}{\hbar^2}.$$

The variable y is related to the energy E in (5–60)

$$y = qa = \sqrt{\frac{2ma^2}{\hbar^2}(D+E)} = \sqrt{\lambda + \frac{2ma^2E}{\hbar^2}};$$

so the energy is

$$E = \frac{\hbar^2}{2ma^2}(y^2 - \lambda).$$

Use Mathematica to calculate λ, solve the eigenvalue equation for y, and evaluate E:

$$\lambda = 1.178$$

$$y = \cancel{0.735} \quad 0.775$$

$$E = \cancel{-1.082 \text{ eV}} \quad -0.980 \text{ eV}$$

In[15]:=
```
(* Set 7 problem 2 *)
a = 1.5 * 10 ^ (-8) cm
Dep = 2 eV
hbar = 6.63 * 10 ^ (-27) erg * s / (2 Pi)
m = 0.911 * 10 ^ (-27) g
erg = 1 eV / (1.6 * 10 ^ (-12))
g = erg * s ^ 2 / cm ^ 2
lambda = 2 * m * Dep * a ^ 2 / hbar ^ 2
```

Out[15]= 1.5×10^{-8} cm

Out[16]= 2 eV

Out[17]= 6.59498×10^{-16} eV s

Out[18]= $\frac{5.69375\times10^{-16}\ \text{eV s}^2}{\text{cm}^2}$

Out[19]= 6.25×10^{11} eV

Out[20]= $\frac{6.25\times10^{11}\ \text{eV s}^2}{\text{cm}^2}$

Out[21]= 1.17819

In[25]:=
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sol = FindRoot[Sqrt[lambda - y ^ 2] == y * Tan[y], {y, .5}]
y0 = y /. sol
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Out[25]= $\{y \to 0.775263\}$

Out[26]= 0.775263

In[27]:=
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Energy = hbar ^ 2 / (2 * m * a ^ 2) (y0 ^ 2 - lambda)
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Out[27]= −0.979731 eV