The initial wave function is the ground state of $V(x)$, 
$$u_0(x) = \left(\frac{2}{\pi} \right)^{1/4} e^{-\frac{1}{2} \alpha^2 x^2} \quad \text{where} \quad \alpha = \sqrt{\frac{m \omega}{\hbar}}.$$ 
After the change, the ground state is 
$$u'_0(x) = \left(\frac{2\alpha'}{\pi} \right)^{1/4} e^{-\frac{1}{2} \alpha'^2 x^2} \quad \text{where} \quad \alpha' = \sqrt{\frac{m \omega'}{\hbar}} = \sqrt{2} \alpha.$$ 

The overlap integral is 
$$\langle u'_0 | u_0 \rangle = \int_{-\infty}^{\infty} u'_0(x) u_0(x) \, dx$$
$$= \left(\frac{2\alpha'}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \alpha'^2 x^2} \, dx$$
$$= \left(\frac{\alpha'}{\pi} \right)^{1/2} \sqrt{\frac{2\pi}{\alpha'^2}} = \left(\frac{2\alpha}{\alpha^2 + \omega'^2} \right)^{1/2} = \left(\frac{\sqrt{2}\alpha}{\alpha^2 + \omega'^2} \right)^{1/2}$$
(Note that $\alpha' = \sqrt{2} \alpha$.) The probability that the particle is in the ground state after the change is 
$$P_0 = |\langle u'_0 | u_0 \rangle|^2 = \frac{2\sqrt{2}}{3} = 0.943.$$
The eigenvalue equation for the ground state is given in equation (5-65)

\[ \sqrt{\lambda - y^2} = y \tan y \quad \text{where} \quad \lambda = \frac{2mDa^2}{\hbar^2}. \]

The variable \( y \) is related to the energy \( E \) via (5-60)

\[ y = \sqrt{2ma^2/\hbar^2} \left(D + E\right) = \sqrt{\lambda + \frac{2ma^2E}{\hbar^2}}; \]

so the energy is

\[ E = \frac{\hbar^2}{2ma^2} \left(y^2 - \lambda\right). \]

Use Mathematica to calculate \( \lambda \), solve the eigenvalue equation for \( y \), and evaluate \( E \):

\[ \lambda = 1.178 \]

\[ y = 0.775 \]

\[ E = -0.980 \text{ eV} \]
\(\textbf{In[15]}:=\) \(\begin{align*}
&\text{(* Set 7 problem 2 *)} \\
&a = 1.5 \times 10^{-8} \text{ cm} \\
&\text{Dep} = 2 \text{ eV} \\
&\hbar = 6.63 \times 10^{-27} \text{ erg} \times \text{s} / (2 \pi) \\
&m = 0.911 \times 10^{-27} \text{ g} \\
&\text{erg} = 1 \text{ eV} / (1.6 \times 10^{-12}) \\
&g = \text{erg} \times \text{s}^2 / \text{cm}^2 \\
&\lambda = 2 \times m \times \text{Dep} \times a^2 / \hbar^2
\end{align*}\)

\(\text{Out[15]}=\) \(1.5 \times 10^{-8} \text{ cm}\)

\(\text{Out[16]}=\) \(2 \text{ eV}\)

\(\text{Out[17]}=\) \(6.59498 \times 10^{-16} \text{ eV s}\)

\(\text{Out[18]}=\) \(\frac{5.69375 \times 10^{-16} \text{ eV s}^2}{\text{cm}^2}\)

\(\text{Out[19]}=\) \(6.25 \times 10^{11} \text{ eV}\)

\(\text{Out[20]}=\) \(\frac{6.25 \times 10^{11} \text{ eV s}^2}{\text{cm}^2}\)

\(\text{Out[21]}=\) \(1.17819\)

\(\text{In[25]}:=\) \(\textbf{sol} = \text{FindRoot}[\text{Sqrt}[\lambda - y^2] = y \times \text{Tan}[y], \{y, 0.5\}]
\)

\(\text{Out[25]}=\) \(\{y \to 0.775263\}\)

\(\text{Out[26]}=\) \(0.775263\)

\(\text{In[27]}:=\) \(\textbf{Energy} = \hbar^2 / (2 \times m \times a^2) (y0^2 - \lambda)\)

\(\text{Out[27]}=\) \(-0.979731 \text{ eV}\)