1. /7/ Merzbacher, Problem 3, p. 90.

2. /13/ Merzbacher, Problem 6, p. 91.

3. /6/ A particle of mass \( m \) and electric charge \( e \) is placed in the one-dimensional harmonic oscillator potential of frequency \( \omega \) and the uniform electric field \( E \) along the same axis.
   a. Find the wave functions and the energy spectrum of the particle.
   b. With a particle in the ground state of the problem, at time \( t = 0 \) the electric field is suddenly switched off. Find the probability of finding the particle at \( t > 0 \) in the \( n \)th stationary state of the oscillator.
   c. The ground state in the presence of the electric field acquires the nonzero expectation value of the electric dipole moment \( \langle \hat{d} \rangle \) proportional to the applied field \( E \). Find the coefficient of proportionality (static polarizability).

4. /15/ Any linear operator \( \hat{F} \) in the coordinate representation can be defined as an integral operator acting on an arbitrary function \( \psi(x) \) according to

\[
\hat{F}\psi(x) = \int dx' F(x, x')\psi(x'),
\]

where the function \( F(x, x') \) is called the kernel of the operator.
   a. Construct kernels \( F(x, x') \) corresponding to the operators \( \hat{x}, \hat{p}, \hat{P}, \hat{D}(a) \) and scale transformation \( \hat{M}(\alpha) \); the last three operators are defined in Problem 3, Homework 4.
   b. Find the most general form of the kernel \( F(x, x') \) for an operator \( \hat{F} \) which commutes with the coordinate operator \( \hat{x} \).
   c. Find the most general form of the kernel \( F(x, x') \) for an operator \( \hat{F} \) which commutes with the momentum operator \( \hat{p} \).
   d. Find the most general form of the kernel \( F(x, x') \) for an operator \( \hat{F} \) which commutes with \( \hat{x} \) and \( \hat{p} \).
   e. Consider an operator \( \hat{F} \) with the factorized kernel, \( F(x, x') = f(x)g(x') \). At what condition the operator \( \hat{F} \) is Hermitian? For a Hermitian operator of this type find its eigenfunctions and eigenvalues. Find the degeneracies of the eigenvalues (their multiplicities in the spectrum).
5. /9/ a. Let all eigenvalues of a Hermitian operator \( \hat{F} \) in \( N \)-dimensional Hilbert space be different. Show that the \( N^{th} \) power, \( \hat{F}^N \), of this operator can be expressed as a linear combination of the lower powers

\[ \hat{F}^{N-1}, \hat{F}^{N-2}, \ldots, \hat{F}, 1. \]

As an example of the general result consider the inversion operator \( \hat{P} \).

b. Let \( \{f_i\}, i = 1, \ldots, N \), be the set of the eigenvalues of the same operator \( \hat{F} \). Construct the projection operator \( \hat{\Lambda}_i \) which projects out the state \( |i\rangle \) with the value \( f_i \) of \( \hat{F} \).

c. Show that the operator \( \hat{F} \) with the factorized kernel, Problem 3, e, can be transformed into a projection operator \( \hat{\Lambda} = c\hat{F} \) with multiplication by a certain constant \( c \). Which state is projected out by the operator \( \hat{\Lambda} \)?

f. A particle is moving in the external, nonlocal but Hermitian, potential field \( \hat{U} \) which is defined by the kernel \( U(r, r') \). Is the continuity equation valid in this case? Is the normalization of the wave function conserved in time?