PHY-851 QUANTUM MECHANICS I
Homework 4, 35 points
September 26 - October 3, 2001
Reading: *Merzbacher*, Chapter 3.

1. /5/ *Merzbacher*, Exercises 3.13 and 3.14, p. 39. Show that the orbital momentum operator $\hat{\mathbf{L}}$ commutes with any *scalar* function of $\hat{\mathbf{p}}^2$ or $\hat{\mathbf{r}}^2$.

2. /8/ *Merzbacher*, Problem 5 and 6, p. 49.

3. /8/ Consider following operators $\hat{F}$ acting on the functions $\psi(x)$ defined on the real axis $-\infty < x < +\infty$: (i) inversion $\hat{\mathcal{P}}$, $\hat{\mathcal{P}}\psi(x) = \psi(-x)$; (ii) displacement $\hat{\mathcal{D}}(a)$, $\hat{\mathcal{D}}(a)\psi(x) = \psi(x-a)$; (iii) scale transformation $\hat{\mathcal{M}}(\alpha)$, $\hat{\mathcal{M}}(\alpha)\psi(x) = \sqrt{\alpha}\psi(\alpha x)$, $\alpha > 0$; (iv) $\hat{k} = -i(d/dx)$. For each $\hat{F}$ find a transpose operator $\hat{F}^T$, a complex conjugate operator $\hat{F}^*$, a Hermitian conjugate operator $\hat{F}^\dagger$ and inverse operator $\hat{F}^{-1}$.

4. /4/ Define the Green function $G(x,t;x',t')$ as a propagator for the evolution of the wave function,
$$\Psi(x,t) = \int dx'\, G(x,t;x',t')\Psi(x',t'). \qquad (1)$$
Find the explicit expression for G in the case of a free motion in one- and three-dimensional cases.

5. /10/ Consider one-dimensional motion of a charged particle of mass m and charge e along the axis x of a uniform electric field $\mathcal{E}$.

   a. Write down and solve the Ehrenfest equations of motion for the expectation values of the coordinate, $\langle\hat{x}\rangle$, and momentum, $\langle\hat{p}\rangle$.

   b. Write the Schrödinger equation in the coordinate representation (for $\Psi(x,t)$) and in the momentum representation (for $\Phi(p,t)$). Solve explicitly the Schrödinger equation in the momentum representation for a stationary state $\phi_E(p)$ with energy E. What is the range of possible values of energy?

   c. Show that the functions $\phi_E(p)$ with different energies are orthogonal and normalize them in such a way that
$$\int dp\, \phi_{E'}(p)^*\phi_E(p) = \delta(E'-E). \qquad (2)$$
   Notice that here p is the variable while E serves as a quantum number labeling different states. Show that the set $\{\phi_E(p)\}$ is complete,
$$\int dE\, \phi_E(p)\phi_E^*(p') = 2\pi\hbar\, \delta(p-p'). \qquad (3)$$

   d. Find the Green function $G(p,t;p',t')$ in the momentum representation and interpret the result.