PHY-851 QUANTUM MECHANICS I
Homework 6, 30 points
October 10 - 17, 2001
One-dimensional motion.

Reading: *Merzbacher*, Chapter 6.

1. /8/ The nucleus of the deuterium atom (heavy hydrogen isotope) is the deuteron, the only existing bound state of a neutron + proton system, with binding energy $\epsilon = 2.2$ MeV. Consider relative motion in the deuteron as that of a particle with the reduced mass on the semi-axis $x > 0$ in the one-dimensional potential $U(x)$ which has an infinite wall at $x = 0$, the square well $U = -U_0 < 0$ for $0 < x < a,\ a = 1.7\,\text{fm}$, and $U = 0$ for $x > a$.

   a. Calculate the penetration length $1/\kappa$ of the deuteron wave function in the classically forbidden region outside the nuclear potential.

   b. Calculate the critical depth U_0^{crit} for the appearance of a bound state for a given value of a.

   c. For a bound state with a small but nonzero binding energy ϵ, the square potential has to be deeper compared to its critical depth U_0^{crit} by $\delta U_0 = U_0 - U_0^{crit}$. Considering the matching conditions for a well slightly deeper than the critical one, $\delta U_0 \ll U_0^{crit}$, derive the connection between δU_0 and ϵ and calculate the depth of the realistic potential for the deuteron.

   d. Find the probabilities (*numbers*!) to find the particles inside and outside the attractive well.

2. /9/ *Merzbacher*, Problem 3, p. 111.

3. /7/ *a.* Write down the Schrödinger equation in the *momentum representation* for a particle of mass m in a potential $U(x)$.

   b. Find the energy level and the normalized wave function of the bound state in the potential $U(x) = -g\delta(x)$, $g > 0$, solving the Schrödinger equation in the momentum representation.

4. /6/ Consider the potential

$$U(x) = \begin{cases} U_1, & x < 0, \\ 0, & 0 < x < a, \\ U_2, & x > a. \end{cases} \tag{1}$$

   Here $U_1 \geq U_2 > 0$. Find the condition for the existence of bound states. Check your results for the limiting cases $U_1 \to \infty$ and $U_1 = U_2$.