PHY-851 QUANTUM MECHANICS I
Homework 7, 30 points
October 17 -24, 2001
Review.

Reading: *Merzbacher*, Chapters 1-4, 6; *Homeworks* 1-6.

1. /7/ Consider an operator $\hat{F}$ that is a linear combination of the coordinate and momentum components along the same axis, $\hat{F} = \alpha\hat{x} + \beta\hat{p}$ where α and β are complex numbers. Find eigenfunctions and eigenvalues of $\hat{F}$. What values of α and β give physically acceptable wave functions (check for example the combinations $\hat{x} \pm i\hat{p}$)? Does $\hat{F}$ correspond to an observable? If yes: is the spectrum of eigenvalues continuous or discrete? is the spectrum degenerate? Check the orthogonality and completeness of the wave functions and normalize them.

2. /6/ Define the three-dimensional operators $\hat{\mathcal{P}}$ of inversion and $\hat{\mathcal{D}}(\mathbf{a})$ of displacement according to $\hat{\mathcal{P}}\psi(\mathbf{r}) = \psi(-\mathbf{r})$ and $\hat{\mathcal{D}}(\mathbf{a})\psi(\mathbf{r}) = \psi(\mathbf{r} - \mathbf{a})$.
   a. Find the transformation of the operators $\hat{\mathbf{r}}$, $\hat{\mathbf{p}}$ and orbital momentum $\hat{\mathbf{L}}$ under inversion $\hat{\mathcal{P}}$.
   b. Find the action of $\hat{\mathcal{P}}$ and $\hat{\mathcal{D}}(\mathbf{a})$ in the momentum representation.
   c. Show that parity of a wave function (with respect to its corresponding argument) is the same in the coordinate and momentum representations.

3. /7/ Consider a particle of mass m in the n-th stationary state in an infinitely deep one-dimensional potential box of width a.
   a. Find the distribution function for the coordinate of the particle, the mean value and the dispersion of the coordinate.
   b. Find the distribution function for the momentum of the particle, the mean value and the dispersion of the momentum, the uncertainty product $(\Delta x)(\Delta p)$, the mean value and dispersion of kinetic energy.

4. /10/ A particle of mass m is confined in a box $-a \le x \le +a$ with infinitely high walls. In the middle of the box a partition is erected described by the additional potential $U = g\delta(x)$, $g > 0$.
   a. Find the energy levels of the particle (help yourself with a graphical solution).
   b. In a special case of a strong barrier, $mga/\hbar^2 \gg 1$, show that the lowest part of the spectrum consists of a sequence of pairs of very close levels and find the energy splitting inside a pair.
   c. What is the structure of the spectrum for highly excited states?