

PHY-851 QUANTUM MECHANICS I

Homework 9, 30 points

November 7 - 14, 2001

Harmonic oscillator.

Reading: *Merzbacher*, Chapter 5.

1. *Merzbacher*, Exercises 5.4, 5.5.
2. *Merzbacher*, Exercises 5.8, 5.9.
3. *Merzbacher*, Problem 5, p. 91. / The normalized probability of populating the n^{th} stationary state with energy E_n in the canonical thermal ensemble is

$$\rho_n = \frac{1}{Z} e^{-E_n/T}, \quad Z = \sum_n e^{-E_n/T}, \quad \sum_n \rho_n = 1, \quad (1)$$

where T is temperature in energy units, $k_B = 1$.

4. Construct the stationary wave functions and find the energy spectrum for a particle of mass m in the potential field ($U_0 > 0$, $x > 0$).

$$U(x) = U_0 \left(\frac{a}{x} - \frac{x}{a} \right)^2. \quad (2)$$

/ Introduce a dimensionless variable $\xi \propto x^2$, find the asymptotic behavior $f(\xi)$ at $\xi \rightarrow \infty$ and $g(\xi)$ at $\xi \rightarrow 0$, look for the full solution $\psi = f(\xi)g(\xi)u(\xi)$, find the equation for $u(\xi)$ and single out its polynomial solutions./