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PHY-851: QUANTUM MECHANICS I
Quiz 1
September 7, 2001

A hydrogen atom (at rest) undergoes the transition from the first excited state to the ground state emitting a photon. Find the wave length of the photon for an infinitely heavy nucleus and its change due to the actual atom recoil.

SOLUTION.

The intrinsic energy of the atom is changed by $\epsilon = \hbar\omega = 13.6 \times (3/4)\,\text{eV}$; with no recoil the photon wave length would be

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\pi\hbar c}{\epsilon} = 1216\mathring{A}. \tag{1}$$

As a result of recoil, the photon energy becomes $\epsilon' = \hbar\omega'$, and the atom of mass M acquires the recoil velocity v. From the conservation laws

$$\epsilon = \epsilon' + \frac{Mv^2}{2}, \quad Mv = \frac{\epsilon'}{c}. \tag{2}$$

Solving for ϵ' we obtain

$$\epsilon' = \frac{2\epsilon}{1 + \sqrt{1 + 2(\epsilon/Mc^2)}}, \tag{3}$$

or, since the correction (ϵ/Mc^2) is very small,

$$\epsilon' \approx \epsilon\left(1 - \frac{\epsilon}{2Mc^2}\right). \tag{4}$$

From here we find the shift of the wave length

$$\lambda' - \lambda = 2\pi\hbar c\left(\frac{1}{\epsilon'} - \frac{1}{\epsilon}\right) \approx \frac{\pi\hbar}{Mc} = 6.6 \times 10^{-6}\mathring{A}. \tag{5}$$