PHY-851: QUANTUM MECHANICS I
Quiz 5
November 30, 2001

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PROBLEM. A harmonic oscillator is prepared in the initial state $\Psi(0) = A(\psi_1 + \psi_2 + \psi_3), \quad A \neq 0$, where ψ_n are the normalized wave functions of the stationary n-quantum states. Will the wave function $\Psi(t)$, at some moment $t > 0$, take a form

a. $\Psi = B\Big[\psi_1 + \frac{1}{\sqrt{2}}(\psi_2 + \psi_3)\Big]$;
b. $\Psi = C(\psi_1 + \psi_2 + \psi_3 + \psi_4)$;
c. $\Psi = D(\psi_1 - \psi_2 + \psi_3)$;
d. $\Psi = F(\psi_1 - \psi_2 - \psi_3)$;
e. $\Psi = G(\psi_1 + i\psi_2 - \psi_3)$?

The coefficients in those examples are some complex constants.

SOLUTION. The wave function

$$\Psi(0) = \sum_n c_n \psi_n \tag{1}$$

evolves in time according to

$$\Psi(t) = \sum_n c_n e^{-(i/\hbar)E_n t} \psi_n. \tag{2}$$

With the energy levels of the harmonic oscillator

$$E_n = \hbar\omega \left(n + \frac{1}{2}\right), \tag{3}$$

we find

$$\Psi(t) = e^{-(i/2)\omega t} \sum_n c_n e^{-i\omega n t} \psi_n. \tag{4}$$

New amplitudes c_n (not present at $t = 0$) do not appear, and the magnitudes $|c_n|^2$ do not change with time. Therefore we can immediately reject the proposals a and b. To discriminate between the remaining three cases, we can write explicitly

$$\Psi(t) = e^{-(i/2)\omega t} A \left(e^{-i\omega t}\psi_1 + e^{-2i\omega t}\psi_2 + e^{-3i\omega t}\psi_3\right). \tag{5}$$

This shows that at any moment t the second phase differs from the first one by the same factor $\exp(-i\omega t)$ as the third phase differs from the second one (an exceptional case of the harmonic oscillator). This eliminates the proposal d. The cases c and e are possible, the corresponding moments are determined by

$$e^{-i\omega t_c} = -1 \quad \rightarrow \quad \omega t_c = \pi,\, 3\pi, ... \tag{6}$$

and

$$e^{-i\omega t_e} = i \quad \rightarrow \quad \omega t_e = \frac{3}{2}\pi,\, \frac{7}{2}\pi, ... \tag{7}$$