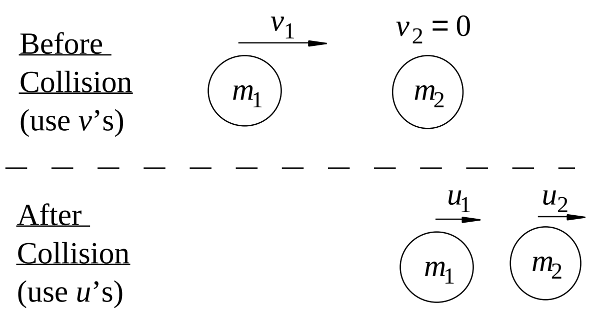
Hooke's law: $F=kx$; Weight: $W = F_G = mg$, $g = 9.81\text{N}/\text{kg}$ (on Earth), Torque($\mathbf{F}\ \ \mathbf{r}$): $\tau = Fr$;
Work: $w = \langle \mathbf{F} \rangle\ \mathbf{s}$; Potential Energy: $PE_S = \frac{1}{2}kx^2$ (spring), $PE_G = mgh$ (gravity on Earth);
Kinetic Energy: $KE = \frac{1}{2}mv^2$; Energy Conservation: $KE + PE = KE_0 + PE_0 + w_{NCF}$;
Momentum: $\mathbf{p} = m\mathbf{v}$; 2nd law (F const.): $\mathbf{p} = \mathbf{p}_0 + \mathbf{F}t$; No $F_{external}$: Sum($\mathbf{p}$) = Sum($\mathbf{p}_0$)

Homework 11 Solutions



Picture for Problems 1-3

Two hockey pucks (masses m_1 and m_2), #1 moving with a speed v_1, and #2 at rest, collide head-on with energy conserved (an elastic collision).

1. What are the expressions for the kinetic energies and momenta of the each puck before the collision and after the collision?

Before collision: $KE_1 = \underline{\frac{1}{2}m_1v_1^2}$, $\mathbf{p}_1 = \underline{+m_1v_1}$, $KE_2 = \underline{0}$, $\mathbf{p}_2 = \underline{0}$

After collision: $KE_1 = \underline{\frac{1}{2}m_1u_1^2}$, $\mathbf{p}_1 = \underline{+m_1u_1}$, $KE_2 = \underline{\frac{1}{2}m_2u_2^2}$, $\mathbf{p}_2 = \underline{+m_2u_2}$

2. What are the expressions for the total momentum, and the total energy,

$\mathbf{p}_{tot} = \mathbf{p}_1 + \mathbf{p}_2$, and $E_{tot} = KE_1 + KE_2$,

of the two pucks before and after the collision?

Before collision: $\mathbf{p}_{tot} = \underline{+m_1v_1}$; After collision: $\mathbf{p}_{tot} = \underline{+m_1u_1 + m_2u_2}$

Before collision: $E_{tot} = \underline{\frac{1}{2}m_1v_1^2}$; After collision: $E_{tot} = \underline{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2}$

3. Momentum conservation and energy conservation can now be used to solve for the two speeds, u_1 and u_2, in terms of the masses and the initial speeds. To simply the problem first consider the two masses to be equal (m_1 and $m_2 = m$) and find the solution for the two speeds, u_1 and u_2, (hint: square the momentum equation.) . Does the solution make sense to you?

Momentum Conservation

$\cancel{m}v_1 = \cancel{m}u_1 + \cancel{m}u_2$

$v_1 = u_1 + u_2$ (now square it)

$v_1^2 = u_1^2 + u_2^2 + 2u_1u_2$

Energy Conservation

$\cancel{\tfrac{1}{2}}\cancel{m}v_1^2 = \cancel{\tfrac{1}{2}}\cancel{m}u_1^2 + \cancel{\tfrac{1}{2}}\cancel{m}u_2^2$

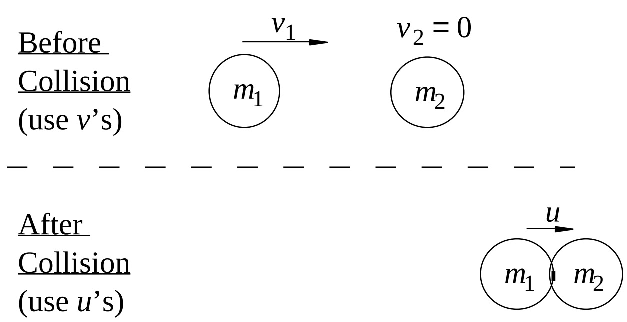
$v_1^2 = u_1^2 + u_2^2$

Combine results

$$u_1^2 + u_2^2 + 2u_1u_2 = u_1^2 + u_2^2$$

$2u_1u_2 = 0$ (u_1 must be zero; it stops)

$\underline{u_2 = v_1}$ (hit puck gets all the momentum)



Picture for problems 4-7

The same two hockey pucks collide again but this time they stick tightly together when they make contact.

4. What is the expression (as in 1 above) for the total momentum <u>vector</u> in the system before and after the collision?

Before collision: $\mathbf{p}_{\text{tot}} = \underline{+m_1v_1}$; After collision: $\mathbf{p}_{\text{tot}} = \underline{+\left(m_1 + m_2\right)u}$

5. Use momentum conservation to determine the speed, u , of the two masses (stuck together) in terms of the masses, m_1 and m_2 , and the initial speed, v_1.

$$(m_1 + m_2)u = m_1v_1$$

$$u = \underline{\frac{m_1}{(m_1 + m_2)} v_1}$$

6. What is the total mechanical energy (*KE* and *PE*, not heat) of the masses before and after the collision in terms of the masses and the initial speeds?

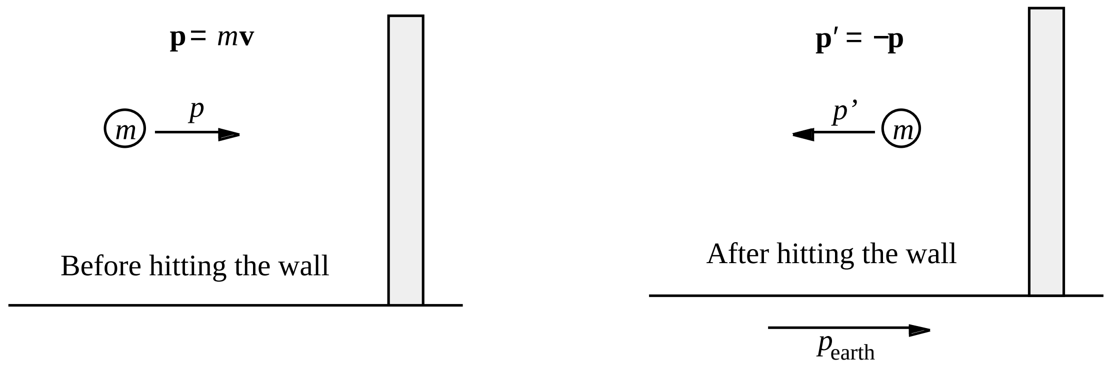
Before collision: $E_{\text{tot}} = E_{\text{tot}} = \underline{\tfrac{1}{2} m_1 v_1^2}$;

After collision: $E_{\text{tot}} = \tfrac{1}{2}(m_1 + m_2)u^2 = \underline{\frac{m_1}{(m_1 + m_2)}\left(\tfrac{1}{2} m_1 v_1^2\right)}$

7. Did the total mechanical energy in the system change? <u>YES, some energy was lost.</u>

Explain why this makes sense.

<u>This makes sense because the lost energy went into heating the sticky stuff.</u>



A mass with momentum vector , **p** , hits a wall (attached to the earth) and bounces off with momentum vector, – **p** (opposite direction) (see section D and E of this chapter).

8. Is the momentum of the small mass alone conserved? Why not!

> Momentum of just the small mass is not conserved. An external force (the force of the wall against the mass) acted on that one object to reverse its direction

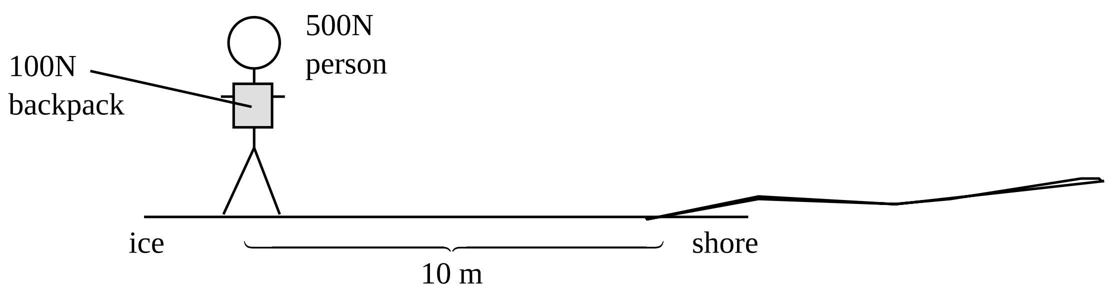
9. What is the other object(s) involved in this collision? the Earth

10. What is the momentum vector of the other object(s) to conserve momentum?

$$\boxed{\begin{array}{l} \text{Before: } \mathbf{p}_{tot} = \mathbf{p}; \quad \text{After: } \mathbf{p}_{tot} = \mathbf{p}_{earth} + (-\mathbf{p}) \\ \mathbf{p}_{earth} + (-\mathbf{p}) = \mathbf{p} \\ \qquad \mathbf{p}_{earth} = \underline{2\mathbf{p}} \end{array}}$$

11. Is this collision “elastic” (see the last section for its definition)?

> Collision is elastic because the energy of the small mass remains the same and the earth get very little energy (but does have momentum!)



Picture for problems 12 and 13.

A person (weight 500N) wearing a backpack filled with CDs (weight 100N) is standing on a frozen lake that is too slippery to walk on but the shore is just 10m away.

12. Which direction should the backpack be thrown to get the person to the shore? (Throwing the CDs on the ice to make a path to the shore is not the answer.) Left !

Backpack’s momentum is to the left and the person’s momentum will be to the right.

13. How long does it take for the person to reach the shore if the backpack is thrown with a speed of 5m/s?

$$m_p = \frac{W_p}{g} = \frac{500\text{ N}}{10\text{ m/s}^2} = 50\text{ kg};\quad m_b = \frac{W_b}{g} = \frac{100\text{ N}}{10\text{ m/s}^2} = 10\text{ kg}$$

$$\mathbf{p}_p = +p_p = +m_p v_p;\quad \mathbf{p}_b = -p_b = -m_b v_b$$

$$\text{(after) } \mathbf{p}_{tot} = \mathbf{p}_p + \mathbf{p}_b = 0\text{(before)}$$

$$+m_p v_p - m_b v_b = 0$$

$$m_p v_p = m_b v_b$$

$$v_p = \frac{m_b}{m_p} v_b = \frac{10\text{ kg}}{50\text{ kg}}(5\text{ m/s}) = 1\text{ m/s}$$

$$t = \frac{s}{v} = \frac{10\text{ m}}{1\text{ m/s}} = \underline{10\text{ s}}$$

14. In which case is momentum **not** conserved?

(a) when two objects collide and stick together. (Only Internal forces)
(b) when an internal force is applied to 2 unequal masses. (Only Internal forces)
(c) when both ends of a force are applied in opposite directions on two objects.
(ends of a force are an Internal pair of forces)
<u>(d) when the effects of a force acting on a single mass are observed.</u>
(e) when masses are affected by an explosion. (explosion forces are internal when the effects on every mass it affects are included)