Lecture 11

Chapter 26

Capacitance
Set #3, Problem #4

• Let $Q_1, Q_3 > 0$ and $Q_2 = -Q_1$ so
  - $Q_1$ is $+$
  - $Q_3$ is $+$
  - $Q_2$ is $-$
  - $Q_1 = Q_2 = Q$
Set #3, Problem #4

True or false: $V$ at (0,0) is zero

False

$V = \sum_{1}^{n} V = k (V_1 + V_2 + V_3)$

$V = k \left( \frac{Q_1}{a} + \frac{Q_2}{a} + \frac{Q_3}{d} \right) = k \left( \frac{Q}{a} - \frac{Q}{a} + \frac{Q_3}{d} \right)$

$V = k \left( \frac{Q_3}{d} \right)$

$V$ is + for any position along y-axis
Set #3, Problem #4

Where does $F$ point at $Q_3$?

It is TRUE that:

- $Q_3$ will accelerate to right
- $F_y$ on $Q_3$ due to other charges is zero
- $W$ to move $Q_3$ to origin is zero

$$W = \vec{F} \cdot \vec{d} = Fd \cos(90)$$
Set #3, Problem #4

Where does $F$ point at 0?

E field at origin does not point directly away from $Q_1$
True or false: External work to bring charges together is negative

True

\[ W = \Delta U = k \frac{qq'}{r} \]

\[ U = U_{12} + U_{13} + U_{23} \]

\[ U_{13} = -U_{23} \]

\[ U_{12} = k \frac{Q(-Q)}{2a} \]
Capacitance (23)

- Parallel-plate capacitor charged to potential $V$ by battery
- Disconnect battery to have isolated system
- If decrease distance, $d$, between the plates what happens to $C$? LARGER
- What happens to $V$? Isolated system $q$ stays same so $V$ decreases if $C$ increases
Capacitance (24)

• What happens if put material between the plates?

• Will the capacitance of the plates increase or decrease?

\[ C = \frac{q}{V} \]

\( V \) decreases so \( C \) increases
Capacitance (25)

- Why does $C$ increase if add material?
- Material made up of molecules which are dipoles
- Molecules align with $E$ field from capacitor
Capacitance (26)

- Dipoles set up $E$ field which opposes capacitors $E$ field

- Total $E$ field is weakened by adding material

- Material is called a dielectric
Capacitance (27)

\[ V = Ed \]

- \( E \) field is weaker so \( V \) decreases

\[ C = \frac{q}{V} \]

- \( q \) is constant so \( C \) INCREASES
Capacitance (28)

- Place a dielectric in capacitor its capacitance increases by numerical factor
  - Called dielectric constant, \( \kappa \)

\[ C_{\text{dielectric}} = \kappa C_{\text{air}} \]

- Modify all electrostatic equations by replacing \( \varepsilon_0 \) with \( \kappa \varepsilon_0 \)

\[ E = \frac{1}{4\pi \kappa \varepsilon_0} \frac{q}{r^2} \]

\[ \varepsilon_0 \int \kappa \vec{E} \cdot d\vec{A} = q \]
Capacitance (29)

- If system connected to battery, $V$ is a constant
- $C$ increases with dielectric so $q$ must increase

$$q = CV$$

- If system isolated, $q$ is a constant
- $C$ increases with dielectric so $V$ must decrease
Capacitance (30)

• What if we have more than one capacitor in a circuit?
  – Replace combination with equivalent capacitor $C_{eq}$

• 2 basic combinations
  – Parallel
  – Series
Capacitance (31)

- Capacitors in parallel
- Capacitors are directly wired together at each plate and $V$ applied across the group of plates
- $V$ is same across all capacitors

$$V_1 = V_2 = V_3 = V$$
• Capacitors in parallel
• Total $q$ stored on capacitors is sum of the charges of all capacitors

$$q = q_1 + q_2 + q_3$$

• $C_{eq}$ has total charge $q$ and same $V$ as original capacitors

$$C_{eq} = \frac{q}{V}$$
Capacitance (33)

- Capacitors in parallel

\[ q = q_1 + q_2 + q_3 \]

\[ q_1 = C_1V \quad q_2 = C_2V \quad q_3 = C_3V \]

\[ q = (C_1 + C_2 + C_3)V \]

\[ C_{eq} = C_1 + C_2 + C_3 \]

\[ C_{eq} = \sum_{i=1}^{n} C_i \]
Capacitance (34)

- Capacitors in series
- Capacitors are wired one after the other and \( V \) is applied across the two ends of the series
- Capacitors have identical \( q \)

\[ q_1 = q_2 = q_3 = q \]

- Battery produces \( q \) only on top and bottom plates, induced \( q \) on other plates
Capacitance (35)

- Capacitors in series
- Sum of $V$ across all capacitors is equal to applied $V$

$$ V = V_1 + V_2 + V_3 $$

- $C_{eq}$ has same $q$ and total $V$ as original capacitors

$$ C_{eq} = \frac{q}{V} $$
Capacitance (36)

- Capacitors in series

\[ V = V_1 + V_2 + V_3 \]

\[
V_1 = \frac{q}{C_1} \quad V_2 = \frac{q}{C_2} \quad V_3 = \frac{q}{C_3}
\]

\[ V = q \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \]

\[ \frac{1}{C_{eq}} = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \]
Capacitance (37)

- Capacitors in series
- Charge can only be shifted from one capacitor to another
- If alternate routes not in series

\[
\frac{1}{C_{eq}} = \sum_{i}^{n} \frac{1}{C_i}
\]

- \( C_{eq} \) is always less than smallest capacitance
Capacitance (38)

- Capacitors in parallel
  - $V$ across each is equal
  - Total $q$ is sum

- Capacitors in series
  - $q$ is equal on each
  - Total $V$ is sum
Capacitance (39)

- Checkpoint #3 – A battery with $V$ stores charge $q$ on 2 identical capacitors
- A) What is $V$ across and $q$ on either capacitor if they are in parallel?
- $V$ is same for each and equal to $V$ of battery
- Total charge conserved so

\[ q = q_1 + q_2 = 2q_1 \quad \text{and} \quad q_{\text{cap}} = \frac{q}{2} \]
Capacitance (40)

• Checkpoint #3 – A battery with $V$ stores charge $q$ on 2 identical capacitors

• A) What is $V$ across and $q$ on either capacitor if they are in series?
  
  • $q$ is same for each
  
  • $V$ is sum of $V$ across capacitors

\[ V = V_1 + V_2 = 2V_1 \]

\[ V_{cap} = \frac{V}{2} \]