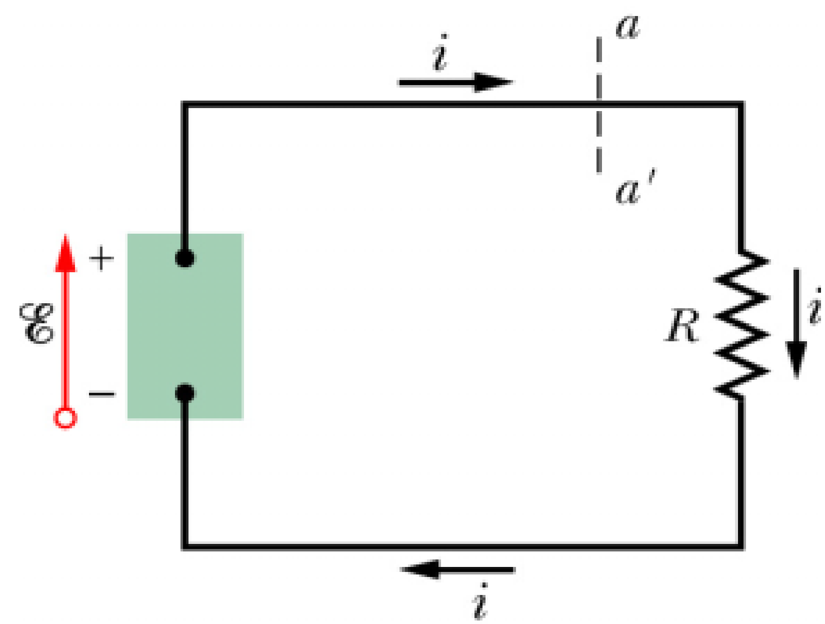
Lecture 15

Chapter 28

Circuits

Review

- emf device label terminal at higher *V* as + and lower *V* as –
- Draw emf, $\mathcal{E}$, arrow from – to + terminal
- + charge carriers move against *E* field in emf device from lower (-) to higher (+) *V*

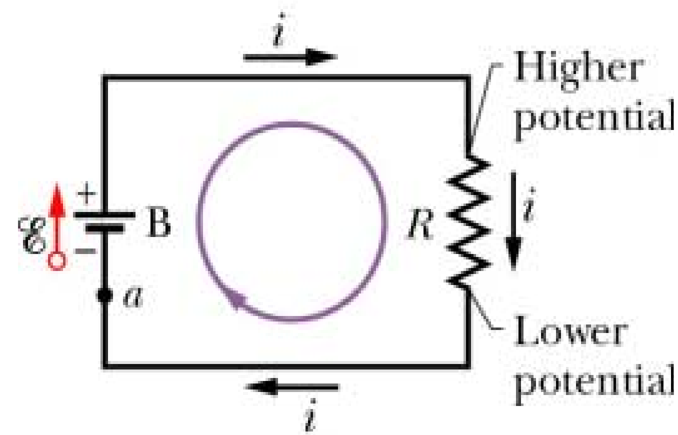


Review

- Kirchhoff's loop rule – in traversing a circuit loop the sum of the changes in *V* is zero, $\Delta V = 0$

- Resistance rule – Move through resistor in direction of current *V* =-*iR*, in opposite direction *V* =+*iR*

- Emf rule – Move through emf device in direction of emf arrow *V* =+E, in opposite direction *V* =-E

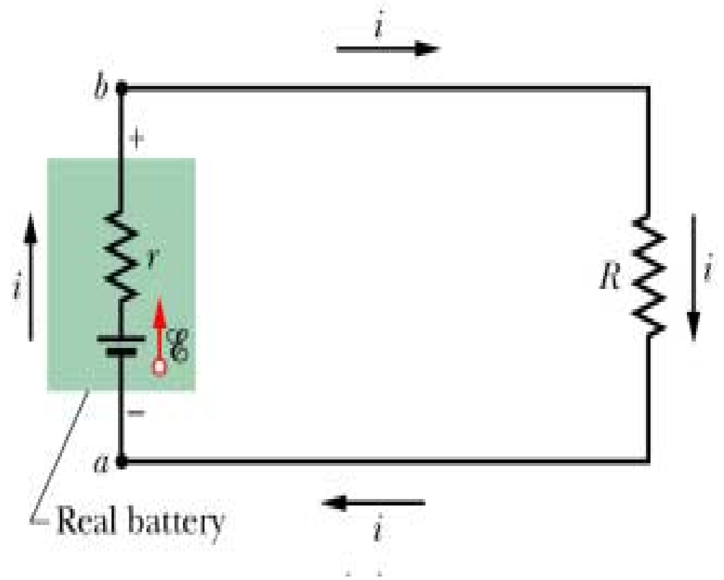


Review

- Put real battery in circuit
- Using Kirchhoff's loop rule and starting at point a gives

$$\mathcal{E} - ir - iR = 0$$

$$\mathcal{E} = i(r + R)$$



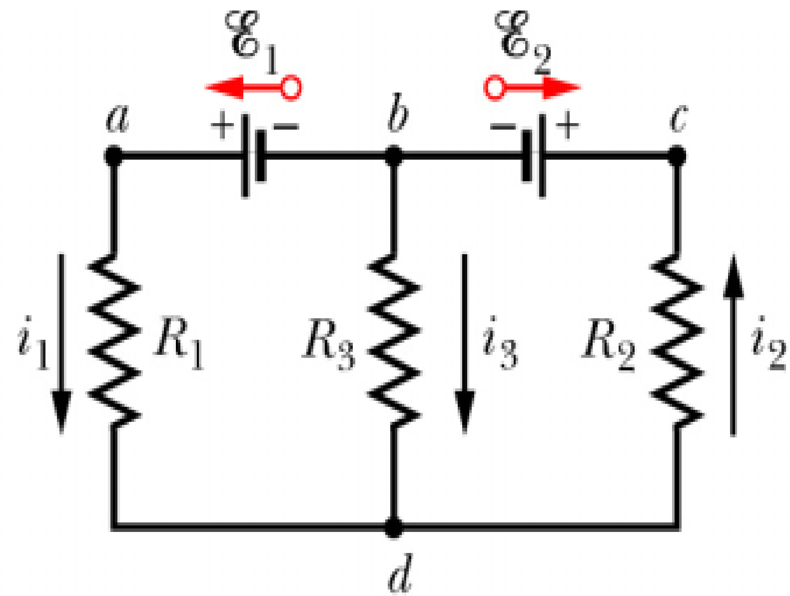
- For ideal battery, r = 0 and we get same as before

$$\mathcal{E} = iR$$

Circuits (25)

- Arbitrarily label currents, using different subscript for each branch
- Using conservation of charge can write

$$i_{in} = i_{out}$$



- At point d $$i_1 + i_3 = i_2$$
- At point b $$i_1 + i_3 = i_2$$
- At point a $$i_1 = i_1$$
- At point c $$i_2 = i_2$$

Circuits (26)

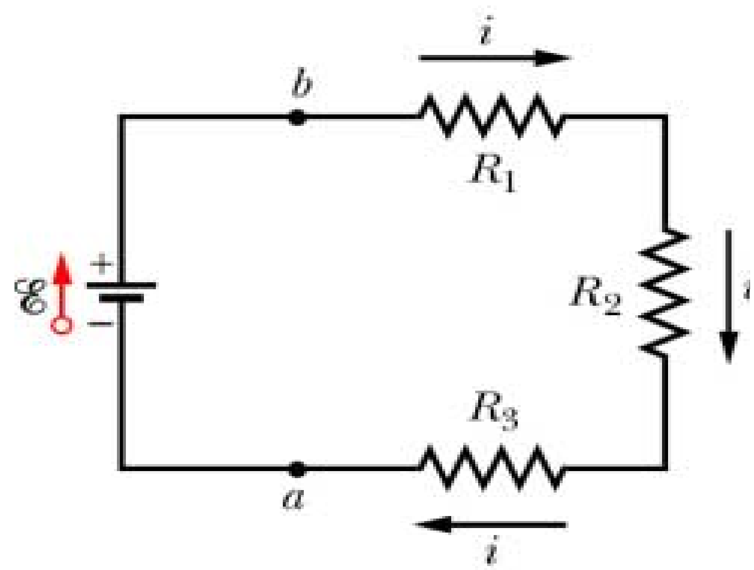
- Kirchhoff's junction rule (or current law) –
  - From conservation of charge
  - Sum of currents entering a junction is equal to sum of currents leaving that junction
- Kirchhoff's loop rule (or voltage law) –
  - From conservation of energy
  - Sum of changes in potential going around a complete circuit loop equals zero

Circuits (27)

- Resistors in series
- Have identical currents, *i*, through them
- Use Kirchhoff's loop rule

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$

$$i = \frac{E}{R_1 + R_2 + R_3}$$

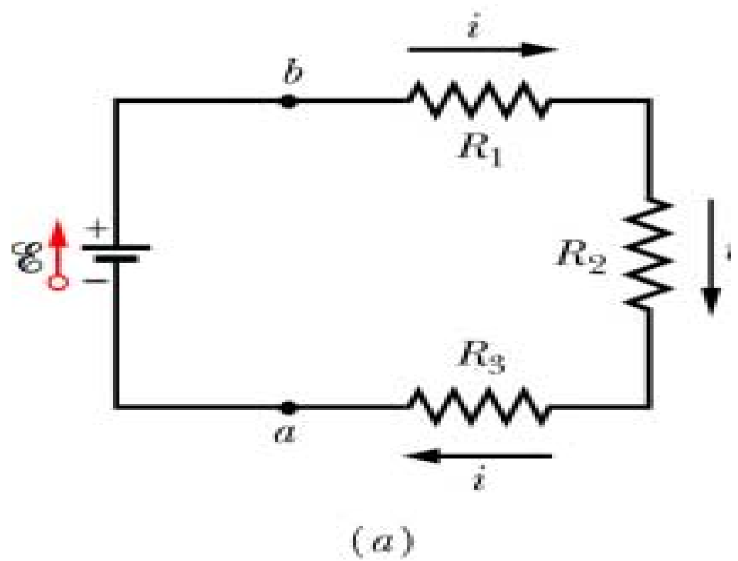


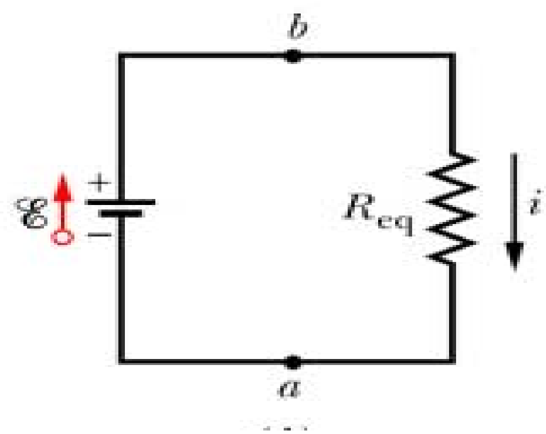
Circuits (28)

- Resistors in series
- Replace 3 resistors with equivalent resistor R_{eq}

$$i = \frac{E}{R_1 + R_2 + R_3}$$

$$R_{eq} = R_1 + R_2 + R_3$$

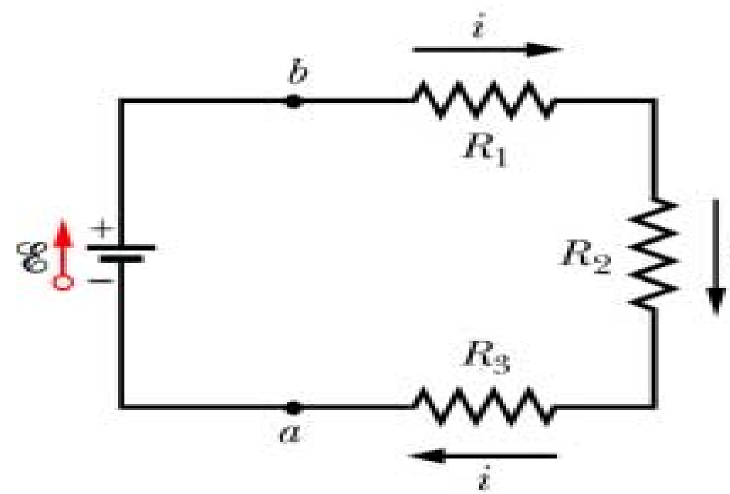




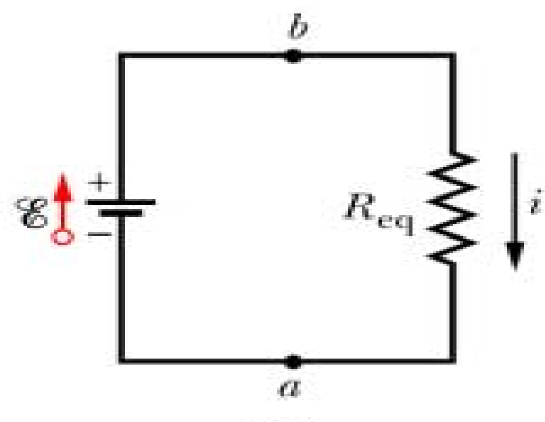
Circuits (29)

- Resistors in series –
- Resistors have identical currents, *i*
- Sum of *V* s across resistors = applied *V*
- R_{eq} is sum of all resistors

$$R_{eq} = \sum_{j=1}^{n} R_j$$



(a)



Circuits (30)

- Checkpoint #2 – If R1>R2>R3, rank greatest first

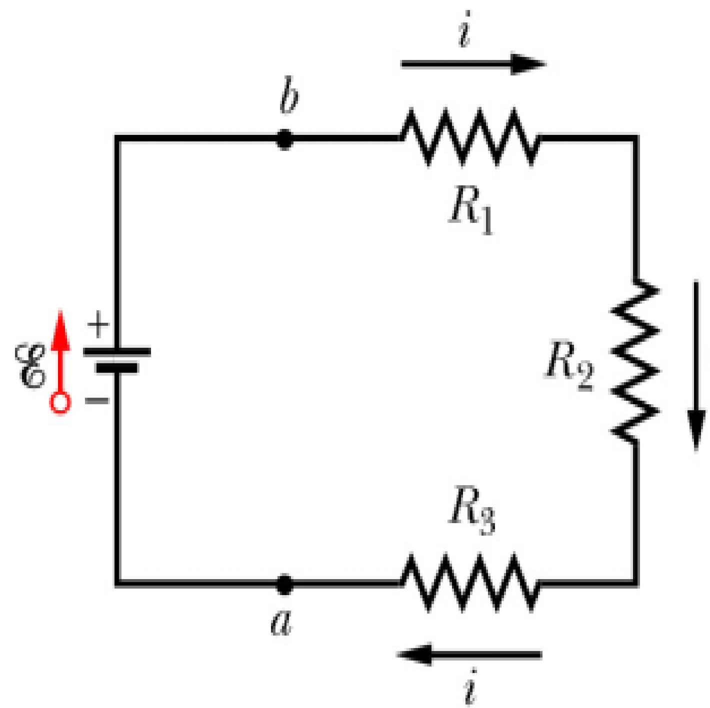
- A) current through resistors

  i is same for all, tie

- B) V across them

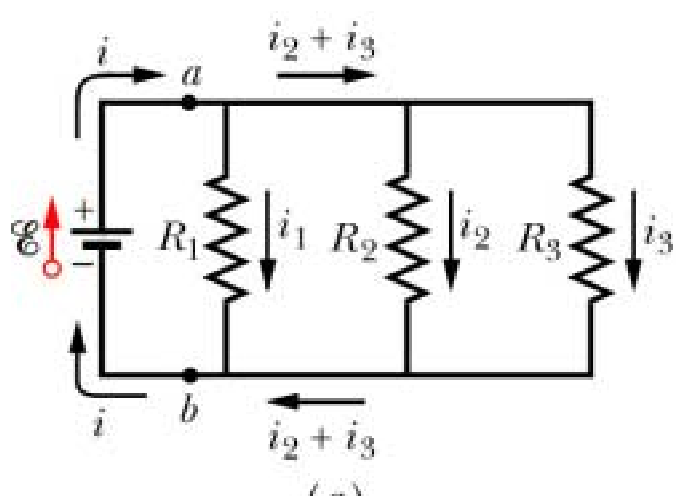
$$V = iR$$

R1, R2, R3



Circuits (31)

- Resistors in parallel
- Have same V across them
- Arbitrarily choose direction for currents in each branch
- Write down current relation for each resistor



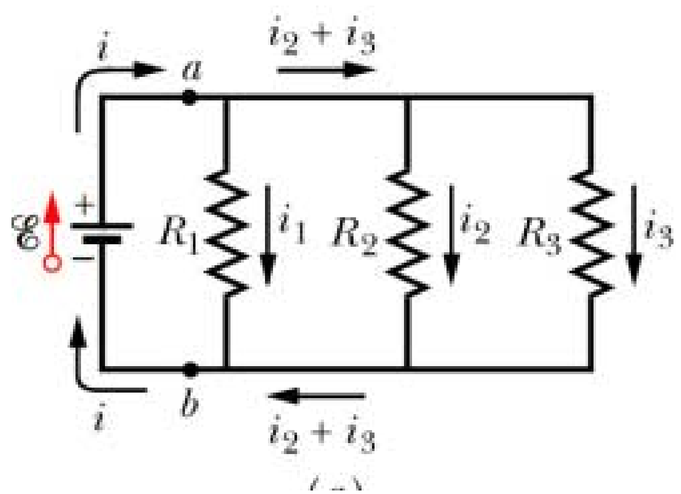
$$i_1 = \frac{V}{R_1}$$

$$i_2 = \frac{V}{R_2}$$

$$i_3 = \frac{V}{R_3}$$

Circuits (32)

- Resistors in parallel
- Apply Kirchhoff's junction rule at point a



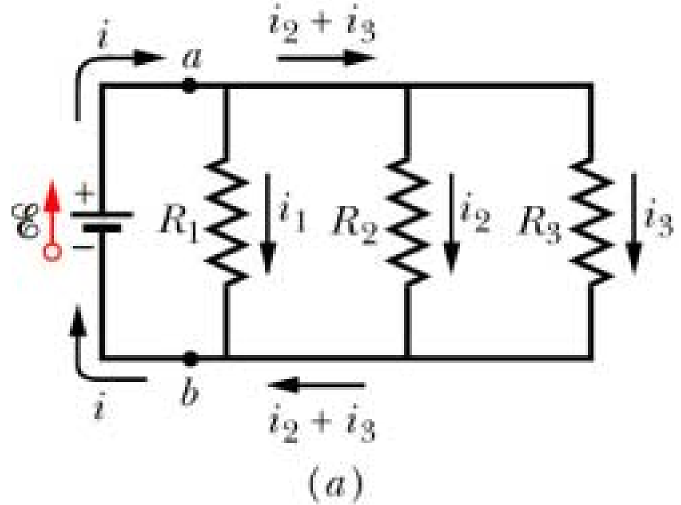
$$i = i_1 + i_2 + i_3$$

- Substitute current values

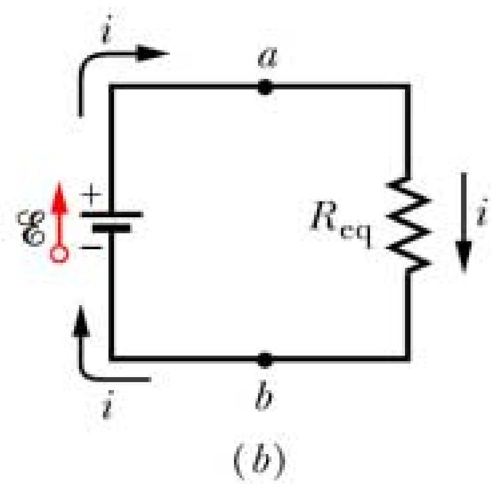
$$i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Circuits (33)

- Resistors in parallel
- Replace 3 resistors with equivalent resistor, R_{eq}



$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$



Circuits (34)

- Resistors
- Series

$$R_{eq} = \sum_{j=1}^{n} R_j$$

- Parallel

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

- Capacitors
- Series

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

- Parallel

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Circuits (35)

- Checkpoint #4 – Battery with potential V supplies current i to 2 identical resistors
- What is *V* across and *i* through either of the resistors if they are connected in
- A) Series – What is constant?

  i is same, $V_1 = V/2$

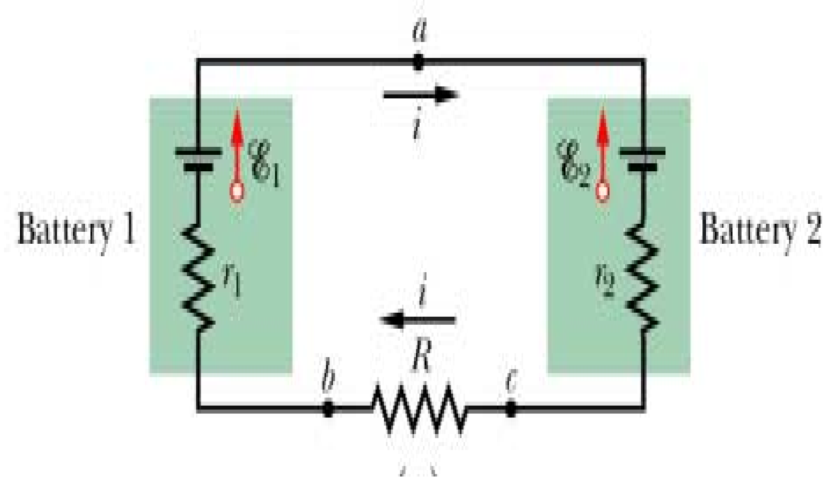
$$V = iR$$

- B) Parallel – What is constant?

  V is same, $i_1 = i/2$

Circuits (36)

- What is *i* of the circuit?
- Use Kirchhoff's loop rule



- Clockwise from point a gives

$$-\mathcal{E}_2 - ir_2 - iR - ir_1 + \mathcal{E}_1 = 0$$

- Counterclockwise from point a gives

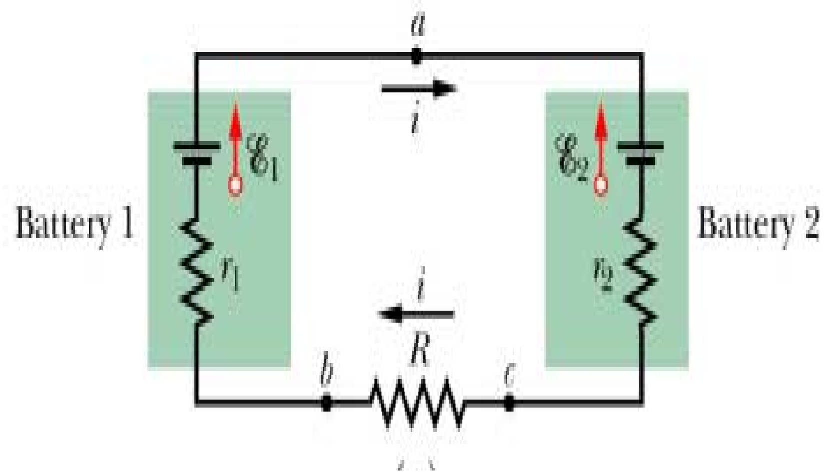
$$-\mathcal{E}_1 + ir_1 + iR + ir_2 + \mathcal{E}_2 = 0$$

Circuits (37)

- Solve for *i*

$$-\mathcal{E}_1 + ir_1 + iR + ir_2 + \mathcal{E}_2 = 0$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2}$$



$$\mathcal{E}_1 = 4.4V$$ $$\mathcal{E}_2 = 2.1V$$

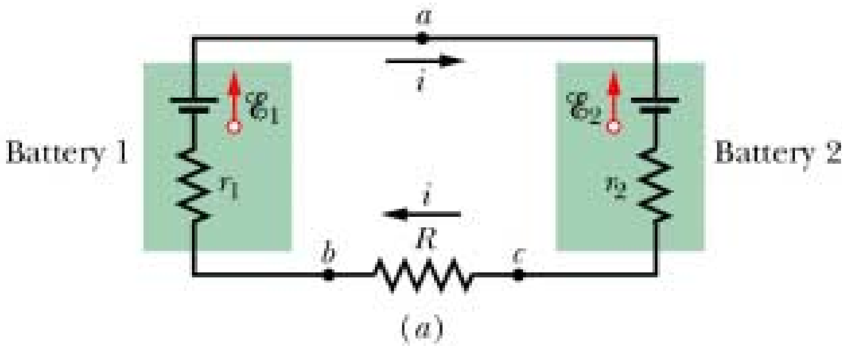
$$r_1 = 2.3\Omega$$ $$r_2 = 1.8\Omega$$

$$R = 5.5\Omega$$

$$i = 0.2396A \approx 240mA$$

Circuits (38)

- What is *V* across battery 1's teminals?
- Sum potential differences from point b to point a
  - Clockwise
  - Counterclockwise



$$V_a - V_b = \mathcal{E}_1 - ir_1$$

$$V_b - ir_1 + \mathcal{E}_1 = V_a$$

$$V_a - V_b = 3.84V$$

$$V_b + iR + ir_2 + \mathcal{E}_2 = V_a$$

$$V_a - V_b = i\left(R + r_2\right) + \mathcal{E}_2$$

Circuits (39)

- Checkpoint #3 – A real battery has $\mathcal{E}$ =12V and r = 2Ω. Is the V across the terminals greater than, less than or equal to 12V if the current in the battery is

- A) from – to + terminal
- LESS THAN

$$V_a + \mathcal{E} - ir = V_b$$

- B) from + to -
- GREATER THAN

$$V_a + \mathcal{E} + ir = V_b$$

- C) i = 0
- EQUAL TO 12V