Lecture 16

Chapter 28

Circuits

Circuits (39)

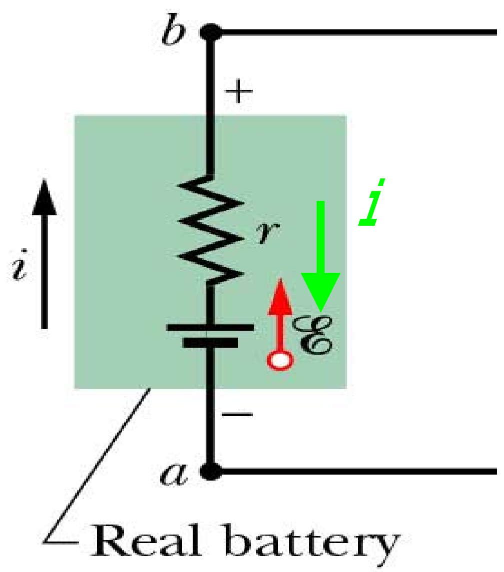
- Checkpoint #3 – A real battery has $\mathcal{E}$ =12V and r = 2Ω. Is the V across the terminals greater than, less than or equal to 12V if the current in the battery is
- A) from – to + terminal – LESS THAN

$$V_a + \mathcal{E} - ir = V_b \qquad V_b - V_a = \mathcal{E} - ir$$

- B) from + to – terminal - GREATER THAN

$$V_a + \mathcal{E} + ir = V_b \qquad V_b - V_a = \mathcal{E} + ir$$

- C) i = 0 - EQUAL TO 12V

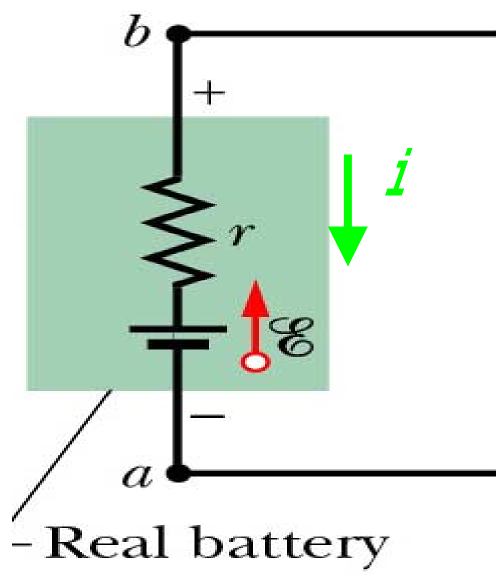


Circuits (40)

- Why is V greater when current is moving from + to – terminal?
- If start point a, go with emf arrow so $+\mathcal{E}$ but against current arrow so $+ir$ gives

$$V_a + \mathcal{E} + ir = V_b$$

$$V_b - V_a = \mathcal{E} + ir$$



- If start point b, go with current arrow so –ir but against emf arrow so –E gives

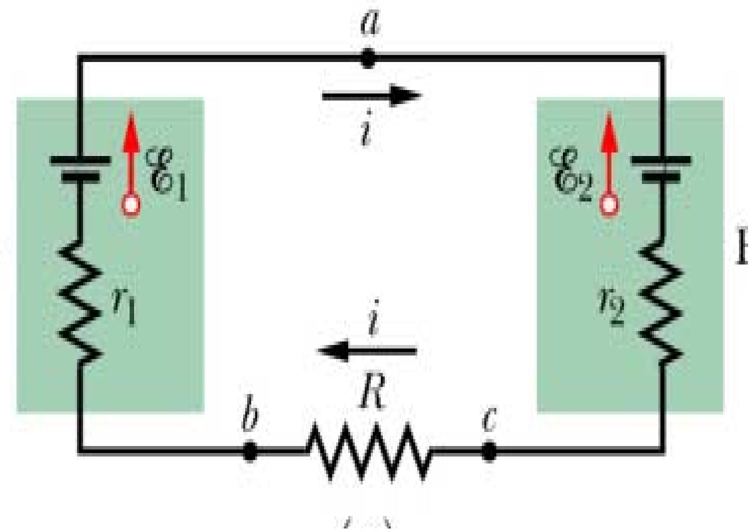
$$V_b - ir - \mathcal{E} = V_a$$

$$V_b - V_a = \mathcal{E} + ir$$

Circuits (41)

- What does it mean to have a *V* across the batteries terminals which is greater than its emf?

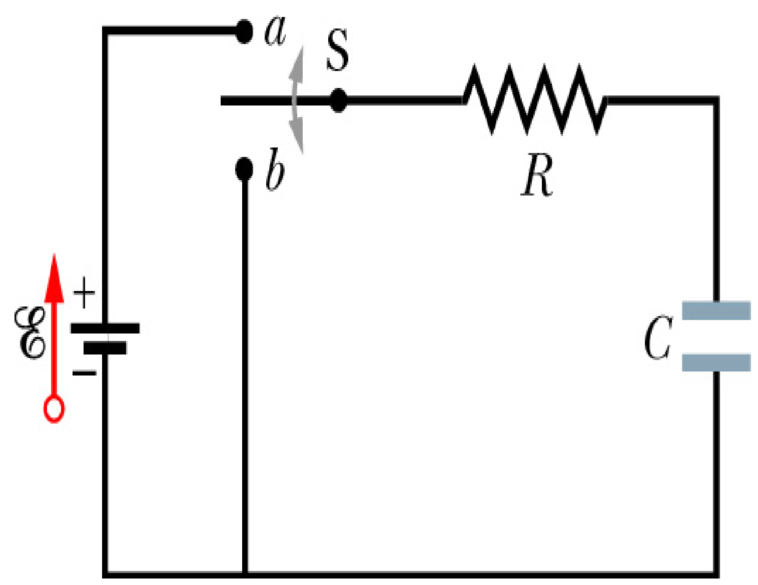
$$V_b - V_a = \mathcal{E} + ir$$



- How could you get a current flowing against the emf arrow of a battery?
- Battery of greater *V* connected in opposite direction would be charging the smaller battery

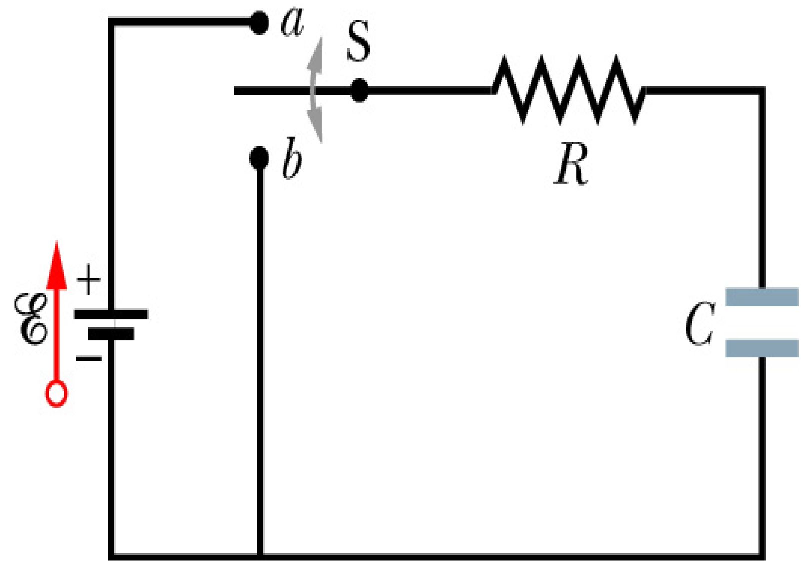
Circuits (42)

- Circuits where current varies with time
- RC series circuit – a resistor and capacitor are in series with a battery and a switch
- At t =0 switch is open and capacitor is uncharged so q =0



Circuits (43)

- Close the switch at point a
- Charge flows (current) from battery to capacitor, increasing *q* on plates and *V* across plates

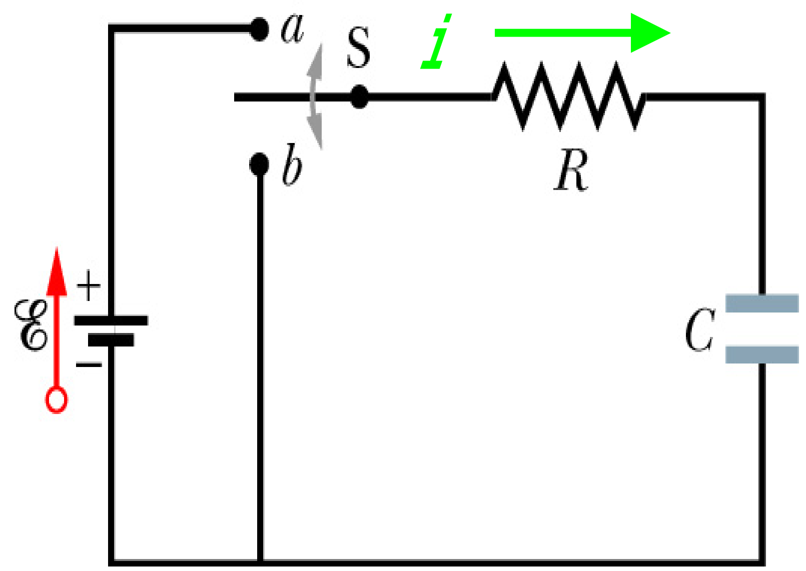


- When V_C equal $V_{battery}$ flow of charge stops (current is zero) and charge on capacitor is

$$q = CV = C\mathcal{E}$$

Circuits (44)

- Want to know how q and V of capacitor and i of the circuit change with time when charging the capacitor
- Apply loop rule, traversing clockwise from battery



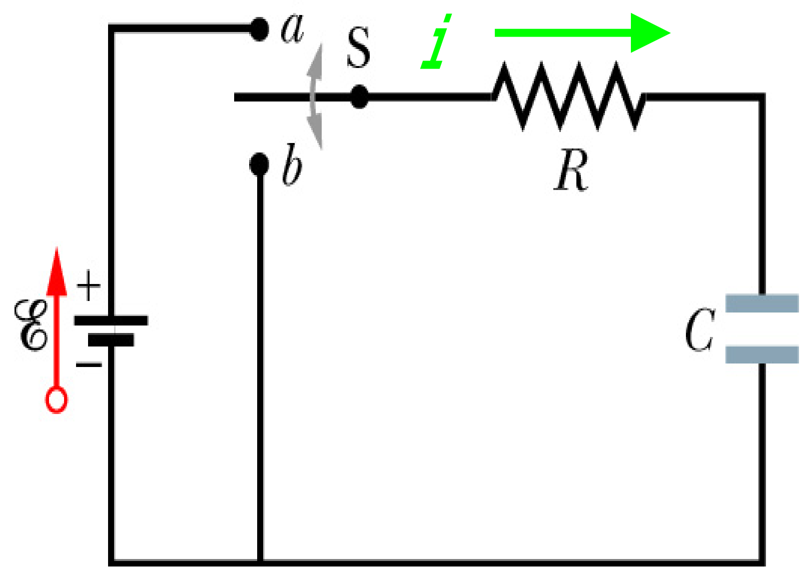
$$\mathcal{E} - iR - \frac{q}{C} = 0$$

Circuits (45)

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

- Contains 2 of the variables we want *i* and *q*
- Remember

$$i = \frac{dq}{dt}$$

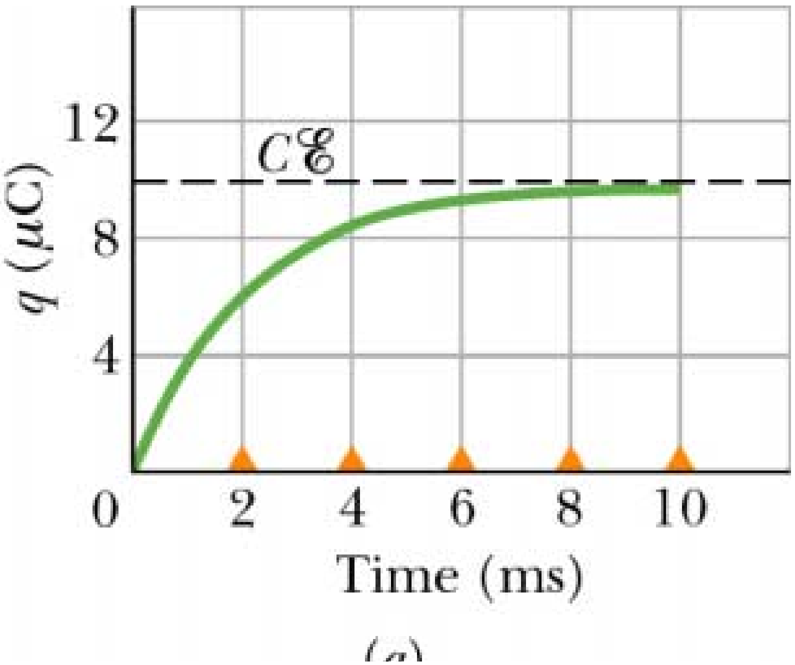


- Substituting gives

$$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$$

Circuits (46)

$$R\frac{dq}{dt}+\frac{q}{C}=\mathcal{E}$$

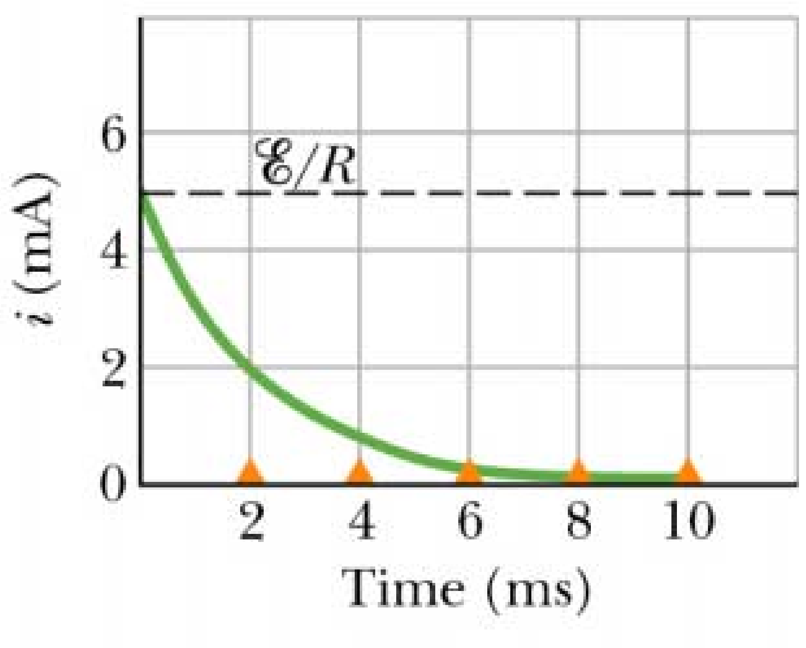


- Need a function which satisfies initial condition $q = 0$ at $t = 0$ and final condiction of $q = C\mathcal{E}$ at $t = \infty$
- For charging a capacitor

$$q=C\mathcal{E}\left(1-e^{-t/RC}\right)$$

Circuits (47)

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

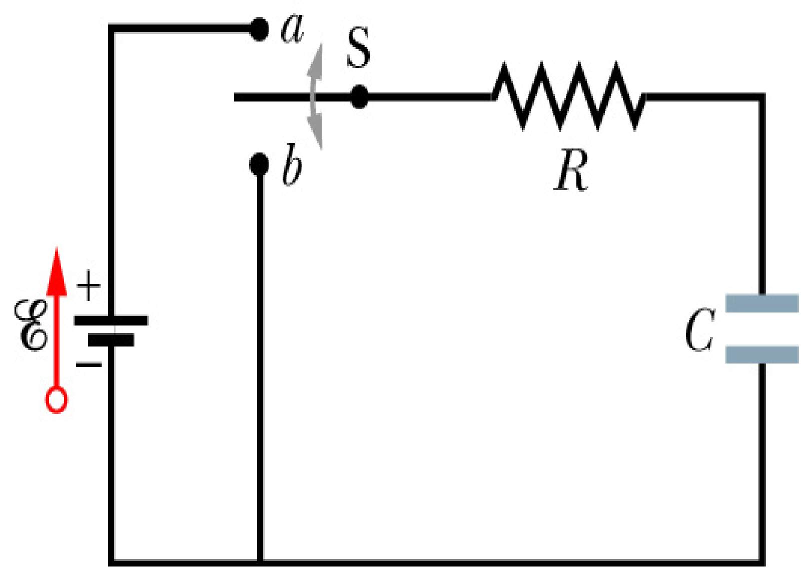


- Want current as a function of time
- For charging a capacitor

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC}$$

Circuits (48)

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

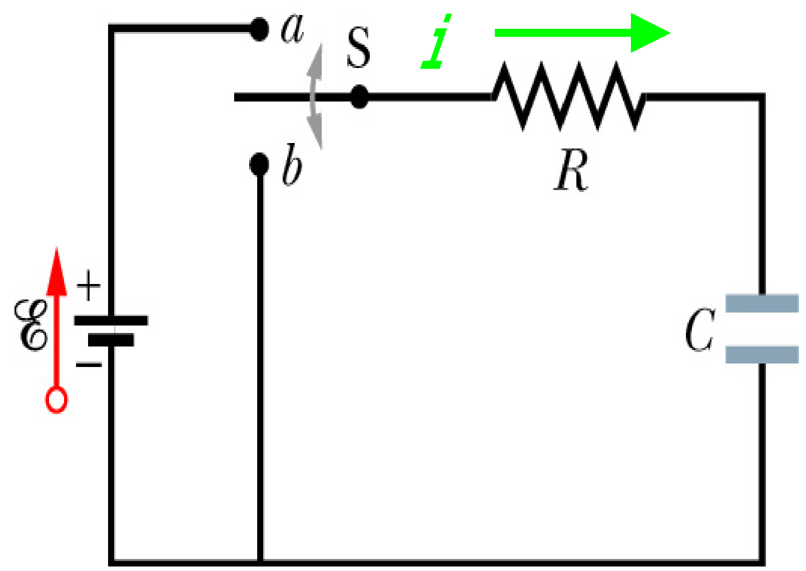


- Want *V* across the capacitor as function of time
- For charging a capacitor

$$V_C = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

Circuits (49)

- Want to know how *q* of capacitor and *i* of the circuit change with time when discharging the capacitor
- At new time $t = 0$, throw switch to point b and discharge capacitor through resistor *R*



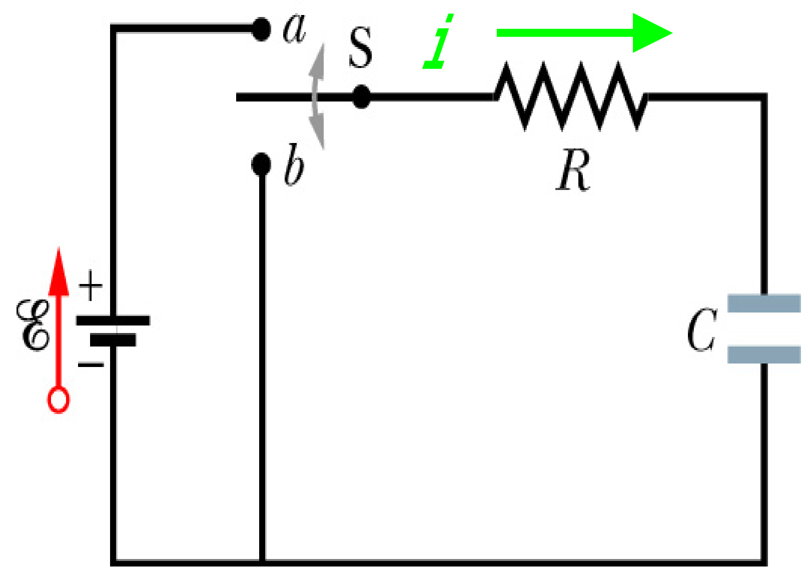
Circuits (50)

- Apply the loop rule again but this time no battery

$$-\ iR\ -\ \frac{q}{C} = 0$$

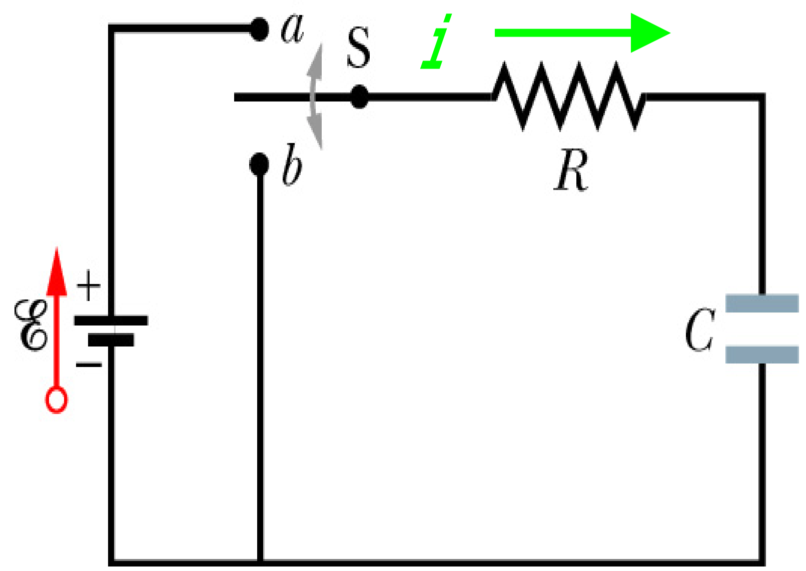
- Substituting for *i* again gives differential equation

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$



Circuits (51)

$$R\frac{dq}{dt}+\frac{q}{C}=0$$



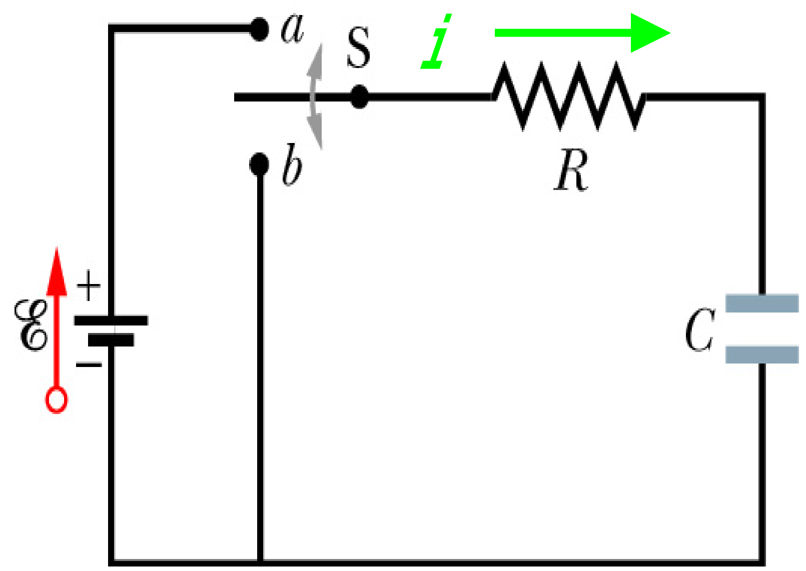
- Solution must satisfy initial condition that $q_0 = CV_0$
- For discharging a capacitor

$$q=q_0e^{-t/RC}$$

Circuits (52)

$$q = q_0 e^{-t/RC}$$

- Find i for discharging capacitor with initial condition at $i_0 = V_0/R = q_0/RC$ at $t = 0$



$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

Negative sign means charge is decreasing

Circuits (53)

- Charging capacitor

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$i = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC}$$

- Discharging capacitor

$$q = q_0 e^{-t/RC}$$

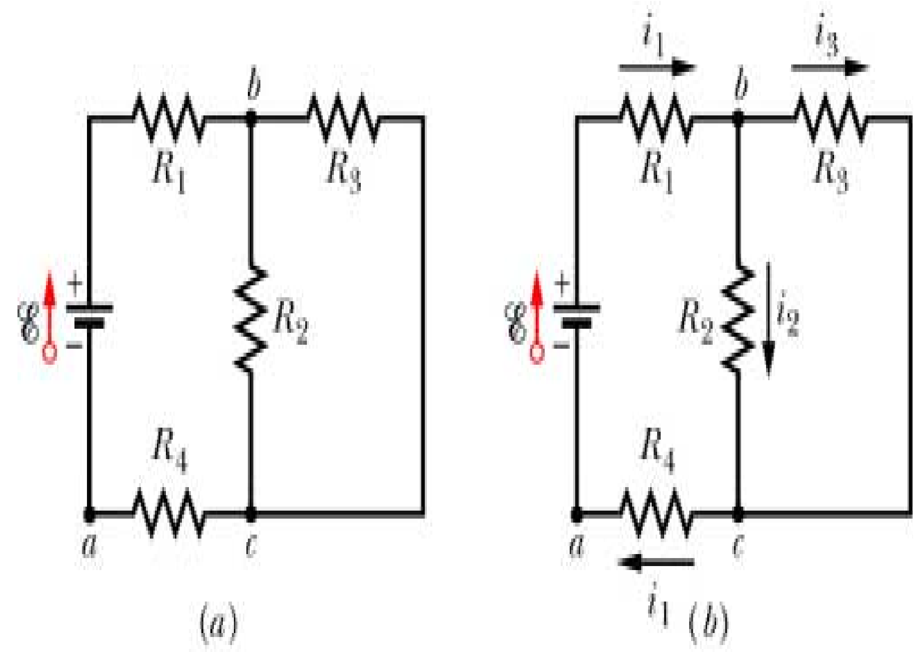
$$i = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

- Define capacitive time constant – greater τ, greater (dis)charging time

$$\tau = RC$$

Circuits (40)

- What is *i* through the battery?



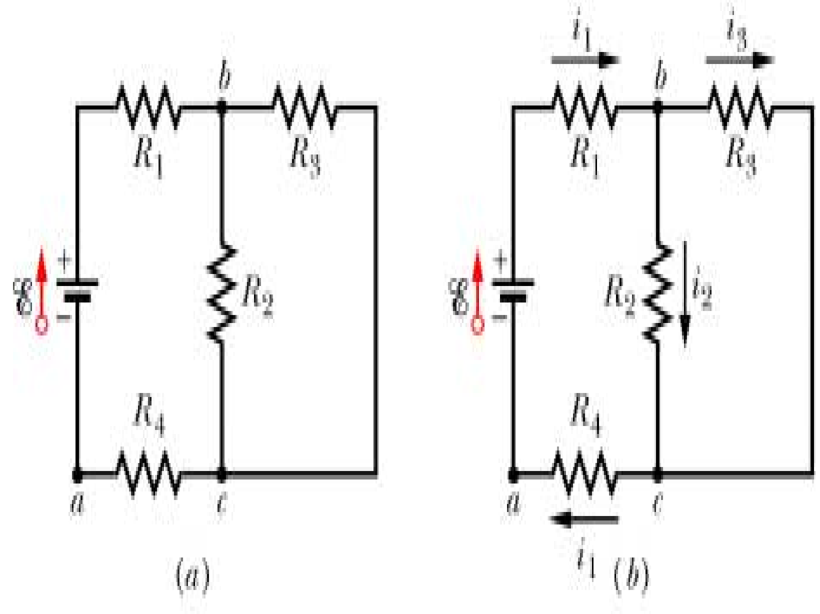
- Label currents in loops
- *i* through R_1 or R_4 is same as for battery

- Can use loop rule

$$\mathcal{E} - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$

Circuits (41)

$$\mathcal{E} - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$



- Equation has 2 unknowns so need to apply loop rule again

OR

- Realize R_2 and R_3 are in parallel and find R_{eq}

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

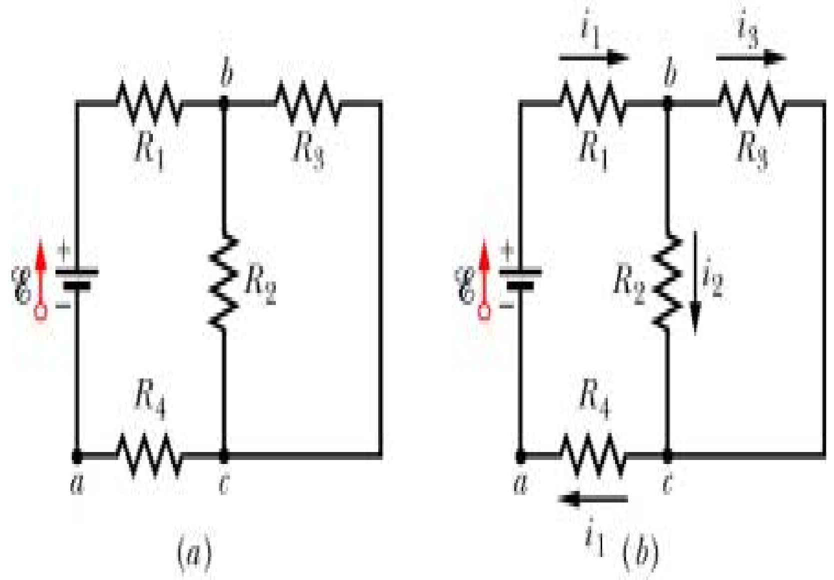
Circuits (42)

- Now i through R_{eq} is equal to i_1
- Apply loop rule

$$\mathcal{E} - i_1 R_1 - i_{23} R_{23} - i_1 R_4 = 0$$

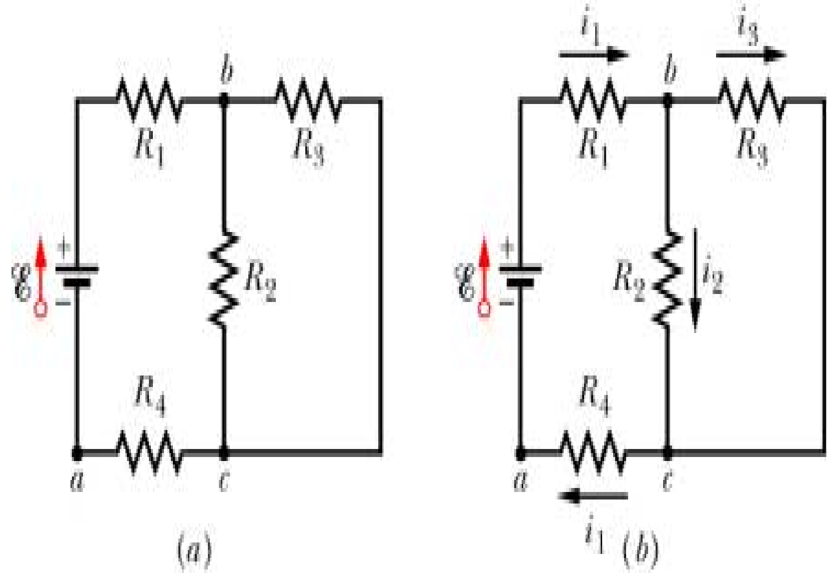
- Now solve for i_1

$$i_1 = \frac{E}{R_1 + R_2 + R_3}$$



Circuits (43)

- What is current i_2 through R_2?
- Work backwards from R_{eq} and realize V across R_{eq} is same for R_2 and R_3



$$V_{23} = i_1 R_{23}$$

$$V_{23} = V_2$$

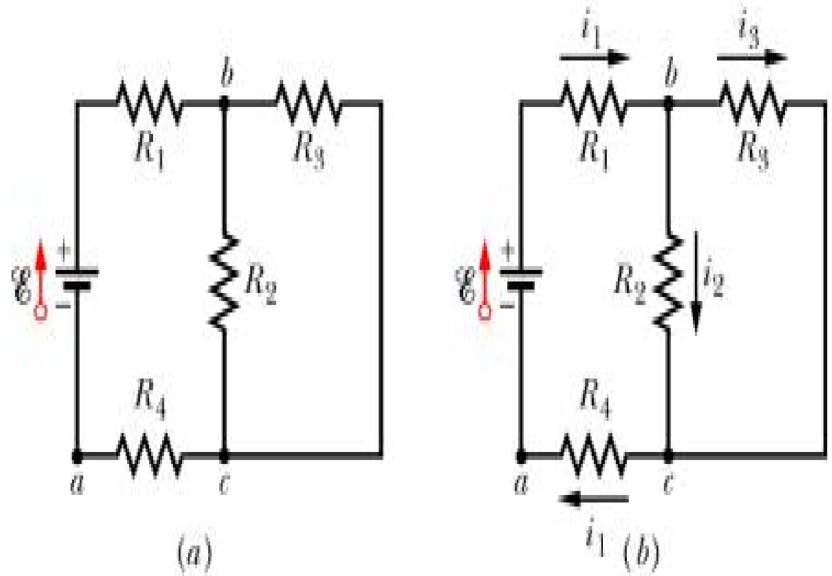
$$i_2 = \frac{V_2}{R_2}$$

Circuits (44)

- What is current i_3 through R_3?
- Can use same method of R_{eq} and V_{23}

OR

- Use junction rule



$$i_1 = i_2 + i_3$$

$$i_3 = i_1 - i_2$$