Lecture 18

Chapter 29
Magnetic Fields
Review

• Force due to a magnetic field is

\[ \vec{F}_B = q\vec{v} \times \vec{B} \]

• \( \vec{F}_B \) is always \( \perp \) to \( \vec{v} \) and \( \vec{B} \)

• \( \vec{F}_B \) does not change the speed (magnitude of \( \vec{v} \))
  or kinetic energy of particle

• \( \vec{F}_B \) only changes direction of \( \vec{v} \)
Magnetic Fields (24)

- $F_B$ continually deflects path charged particles

- If $\mathbf{v}$ and $\mathbf{B}$ are $\perp$, $F_B$ causes charged particles to move in a circular path

- If looking in direction of $\mathbf{B}$, $+$ particles move counterclockwise and $-$ particles move clockwise

\[
\vec{F}_B = q\vec{v} \times \vec{B}
\]
Magnetic Fields (25)

- Derive radius of circular path for particle of charge, \( q \), and mass, \( m \), moving with velocity, \( v \), which is \( \perp \) to \( B \) field

\[
\vec{F}_B = q\vec{v} \times \vec{B} = qvB \sin \phi = qvB
\]

- Newton’s second law for circular motion is

\[
\vec{F} = m\vec{a} = m\frac{v^2}{r}
\]
Setting the forces equal and solving for \( r \):

\[
qvB = m \frac{v^2}{r}
\]

\[
r = \frac{mv}{qB}
\]

Checkpoint #4 – A proton and an electron travel at same \( v \) in a \( B \) field into the page

A) Which particle follows the smaller circle?

\( r \propto m \), \( m_p > m_e \) so the electron

B) What direction does it move in?

Clockwise
Magnetic Fields (27)

• Period, $T$, is the time for one full revolution

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\nu} \frac{mv}{qB} = \frac{2\pi m}{qB}$$

• Frequency, $f$, is # of revolutions per unit time

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

• Angular frequency, $\omega$, is

$$\omega = 2\pi f = \frac{qB}{m}$$
Magnetic Fields (28)

- Nuclear and high-energy physicists probe the structure of matter by
  - Circulating charged particles in a magnetic field and applying electrical kicks to accelerate the particles
  - Slam particle into solid target or collide it with another particle head-on

- 2 devices used to accelerate particles are:
  - Cyclotron – one right next door
  - Synchrotron – Fermi National Accelerator Laboratory (Fermilab) collides protons and anti-protons
Magnetic Fields (29)

- **Cyclotron**
  - Particles start at center
  - Circulate inside 2 hollow metal D shaped objects
  - Alternate the electric sign of the Dees so $V$ across gap alternates
  - Whole thing immersed in magnetic field $B \perp \nu$ ($B$ field out of page and approximately 1.5 T)
Magnetic Fields (30)

• **Cyclotron**
  – Proton starting in center will move toward negatively charged Dee
  – Inside Dee $E$ field = 0 (inside conductor) but $B$ field causes proton to move in circle with radius which depends on $v$

\[
r = \frac{mv}{qB}
\]
Magnetic Fields (31)

• **Cyclotron**
  - When proton enters gap between Dees E field is flipped so proton is again attracted to negatively charged Dee
  - Every time proton enters gap the polarity of the Dees is changed and the proton is given another kick (accelerated)
Magnetic Fields (32)

• Cyclotron
  - Key is that the frequency, $f$, of the proton does not depend on $v$ and must equal the $f_{osc}$ of the Dees

\[
f = f_{osc}
\]

\[
f = \frac{1}{T} = \frac{qB}{2\pi m}
\]

\[
qB = 2\pi mf_{osc}
\]
Magnetic Fields (33)

- **Cyclotron**
  - Has single $f_{osc}$
  - For proton, $q$ and $m$ are fixed
  - Tune cyclotron to get a beam of protons by varying $B$ field

\[ qB = 2\pi mf_{osc} \]
Magnetic Fields (34)

- Assumption that $f$ of charged particle is independent of its speed only works for speeds much smaller than the speed of light
- At higher $v$ the particle’s $f$ decreases as it is accelerated
- Cyclotron’s frequency, $f_{osc}$, gets out of step with the particle’s $f$
Magnetic Fields (35)

- To reach higher $v$ thus higher energies use a synchrotron

- The $B$ field and $f_{osc}$ vary with time

- Particles follow circular path instead of spiral
Magnetic Fields (36)

- **MSU cyclotron**
  - Fits in building next door
  - Can accelerate several different kinds of nucleons
  - Generates beams of particles with energies of 200 MeV/nucleon (1 MeV = $10^6$ eV)

- **FERMILAB**
  - Uses 6 synchrotrons – the largest with 4 mile circumference
  - Accelerates protons and anti-protons
  - Protons move at 99.9999% speed of light (Go around ring 50,000 times in second)
  - Beam energies of 1 TeV (1 TeV = $10^{12}$ eV)
Magnetic Fields (37)
Magnetic Fields (38)

- 1000 superconducting magnets in Tevatron
- Kept at 4.3 K (liquid helium)

\[ r = \frac{mv}{qB} \]

- Magnetic field changes with time
- Radius of circle remains constant
Magnetic Fields (39)

- So far assumed $v$ and $B$ were always $\perp$.
- If $v$ has a component $||$ to $B$ then particle will have helical path.
- FERMILAB protons move in helical path clockwise and anti-protons move in helical path counter-clockwise.
Magnetic Fields (40)

- What do we do with the particle beams?
- Can measure a particle’s mass using a mass spectrometer.
- Accelerate particle using potential difference, $V$
- Chamber with $B$ field causes particle to bend, striking photographic plate
Magnetic Fields (41)

- Conservation of energy

\[ \Delta K + \Delta U = 0 \]

\[ \frac{1}{2} mv^2 - qV = 0 \]

\[ v = \sqrt{\frac{2qV}{m}} \]

- Substituting \( v \) into relation for \( r \) gives

\[ r = \frac{mv}{qB} = \frac{m}{qB} \sqrt{\frac{2qV}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{q}} \]
Magnetic Fields (41)

- Rearranging for $m$

$$r = \frac{1}{B} \sqrt{\frac{2mV}{q}}$$

$$m = \frac{B^2 r^2 q}{2V}$$

- Distance $x = 2r$

$$m = \frac{B^2 x^2 q}{8V}$$
Magnetic Fields (42)

• Fermilab looks at collisions of protons and anti-protons
• Build 5,000 ton detectors around interaction point to observe what happens
• CDF (the Collider Detector at Fermilab) experiment
Magnetic Fields (43)

- Use giant solenoid to produce $B$ field of 1.4 T
- Put detectors which show particle’s path inside of $B$ field
Magnetic Fields (44)
Magnetic Fields (45)

- Direction of curvature tells us the sign of the particle.
- Amount of curvature, $r$, gives us the momentum.

\[ r = \frac{mv}{qB} \]

\[ \vec{p} = m\vec{v} = rqB \]

For more info on Fermilab see http://www.fnal.gov
Magnetic Fields (46)

- Hall effect - $B$ field exerts force on electrons moving in wire
- Electrons cannot escape wire so force is transmitted to wire itself
- Change either direction of current or $B$ field, reverses force on wire