Lecture 18

Chapter 29

Magnetic Fields

Review

- Force due to a magnetic field is $\vec{F}_B = q\vec{v} \times \vec{B}$

- F_B is always $\perp$ to v and B

- F_B does not change the speed (magnitude of v) or kinetic energy of particle

- F_B only changes direction of v

Magnetic Fields (24)

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- F_B continually deflects path charged particles
- If v and B are $\perp$, F_B causes charged particles to move in a circular path
- If looking in direction of B , + particles move counterclockwise and – particles move clockwise

Magnetic Fields (25)

- Derive radius of circular path for particle of charge, *q*, and mass, *m*, moving with velocity, *v*, which is ⊥ to *B* field

$$\vec{F}_B = q\vec{v} \times \vec{B} = qvB\sin\phi = qvB$$

- Newton's second law for circular motion is

$$\vec{F} = m\vec{a} = m\frac{v^2}{r}$$

Magnetic Fields (26)

- Setting the forces equal and solving for *r*

$$qvB = m\frac{v^2}{r}$$

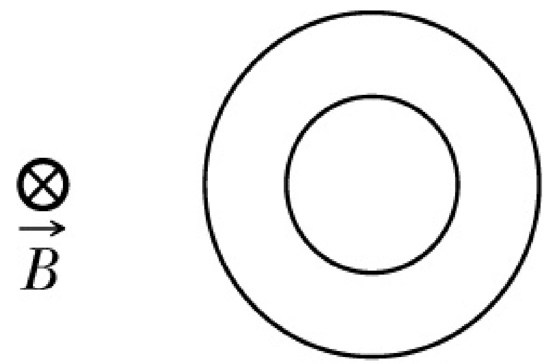
$$r = \frac{mv}{qB}$$

- Checkpoint #4 – A proton and an electron travel at same *v* in a *B* field into the page
- A) Which particle follows the smaller circle?

  $r \propto m$, $m_p > m_e$ so the electron

- B) What direction does it move in?

  Clockwise



Magnetic Fields (27)

- Period, *T*, is the time for one full revolution

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Frequency, *f*, is # of revolutions per unit time

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

- Angular frequency, *ω*, is

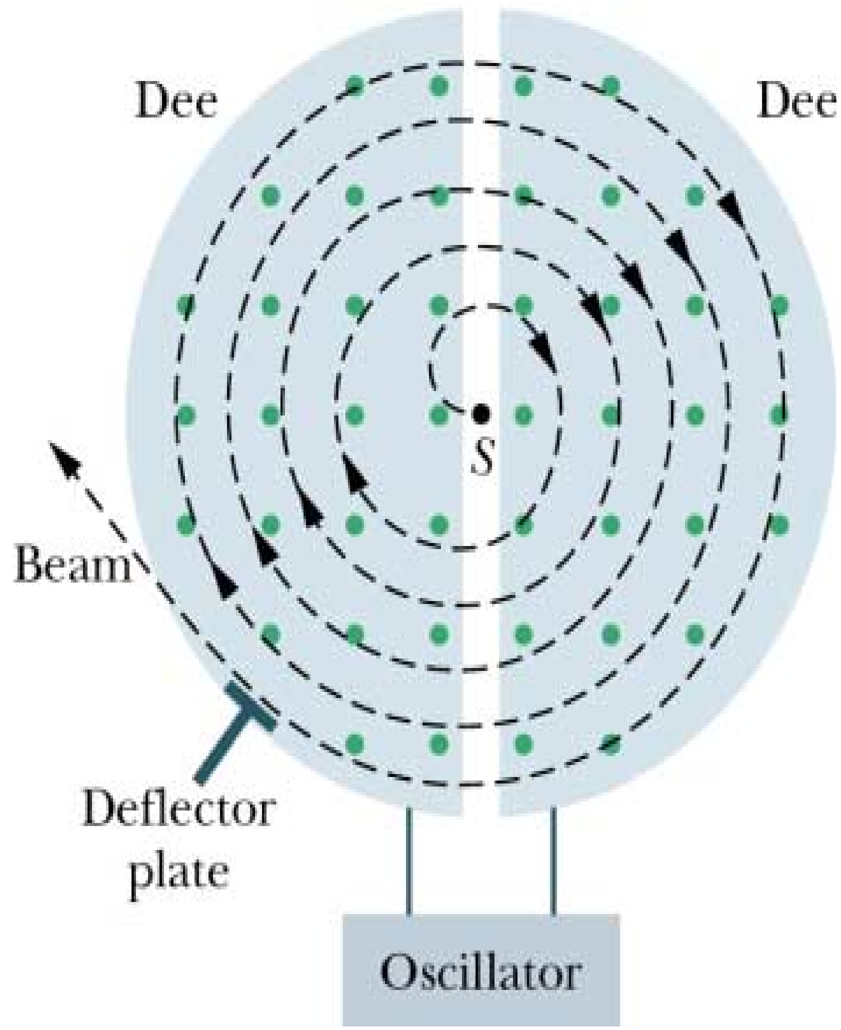
$$\omega = 2\pi f = \frac{qB}{m}$$

Magnetic Fields (28)

- Nuclear and high-energy physicists probe the structure of matter by
  - Circulating charged particles in a magnetic field and applying electrical kicks to accelerate the particles
  - Slam particle into solid target or collide it with another particle head-on
- 2 devices used to accelerate particles are:
  - Cyclotron – one right next door
  - Synchrotron – Fermi National Accelerator Laboratory (Fermilab) collides protons and anti-protons

Magnetic Fields (29)

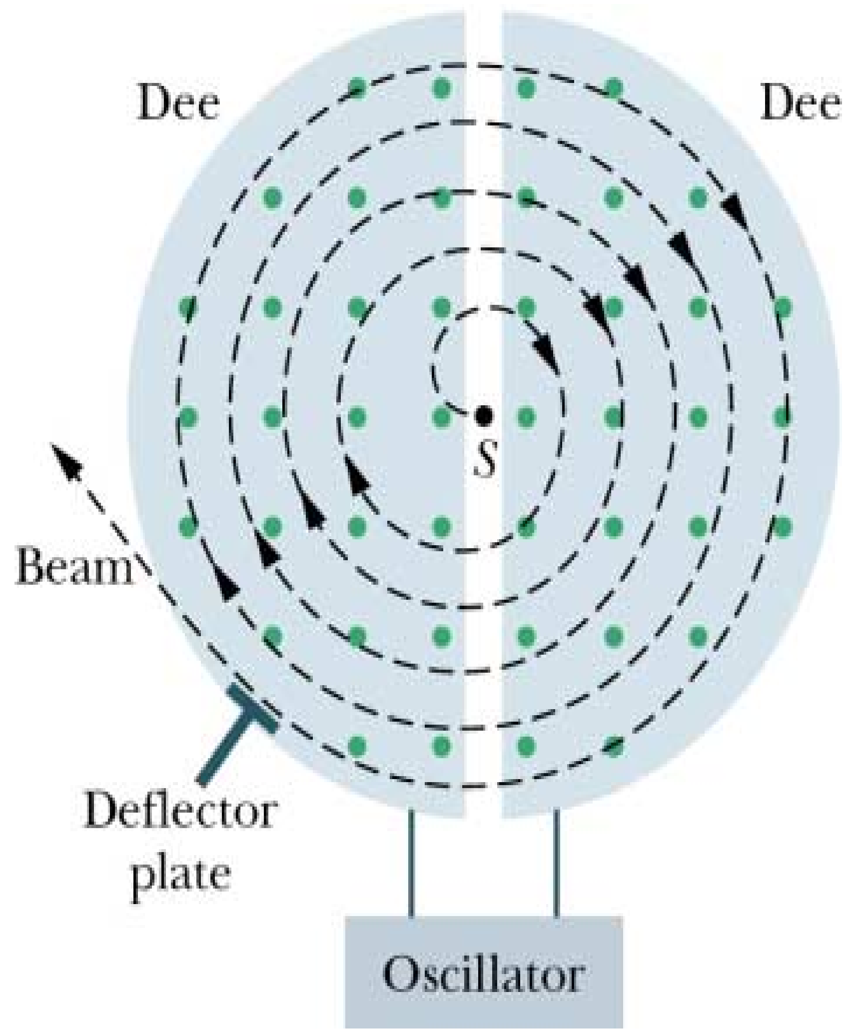
- Cyclotron
  - Particles starts at center
  - Circulate inside 2 hollow metal D shaped objects
  - Alternate the electric sign of the Dees so *V* across gap alternates
  - Whole thing immersed in magnetic field *B* $\perp$ to *v* (*B* field out of page and approximately 1.5 T)



Magnetic Fields (30)

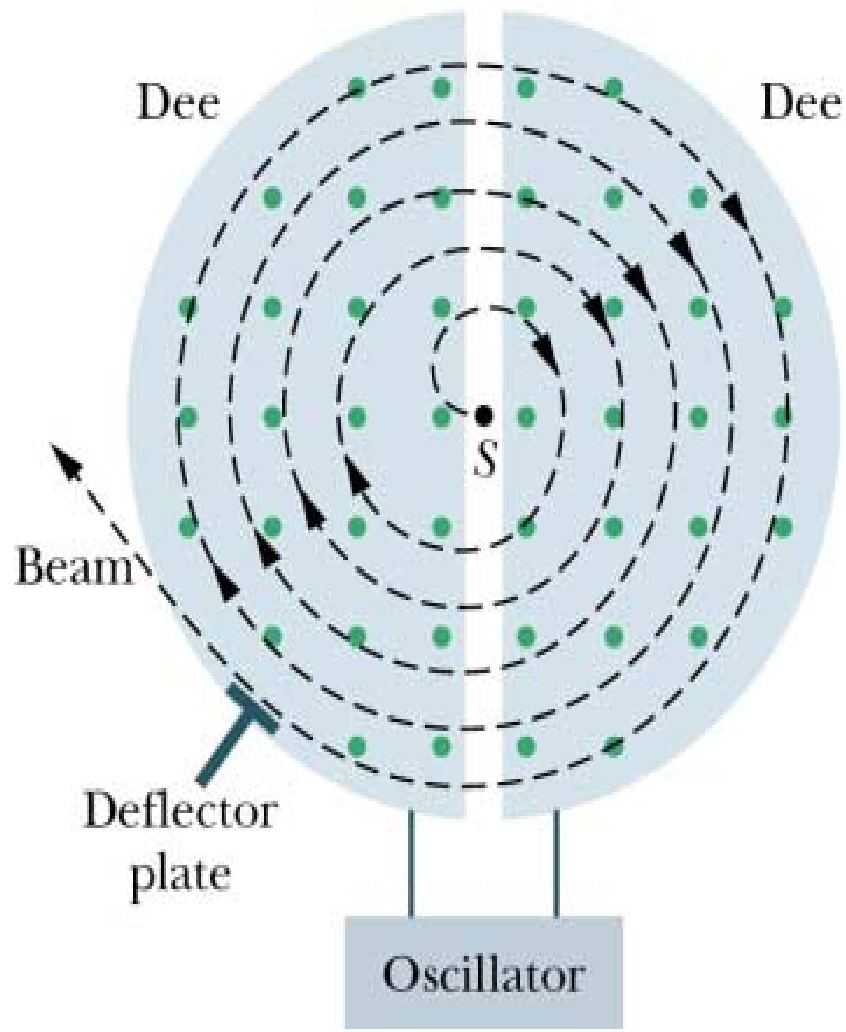
- Cyclotron
  - Proton starting in center will move toward negatively charged Dee
  - Inside Dee *E* field = 0 (inside conductor) but *B* field causes proton to move in circle with radius which depends on *v*

$$r = \frac{mv}{qB}$$



Magnetic Fields (31)

- Cyclotron
  - When proton enters gap between Dees E field is flipped so proton is again attracted to negatively charged Dee
  - Every time proton enters gap the polarity of the Dees is changed and the proton is given another kick (accelerated)



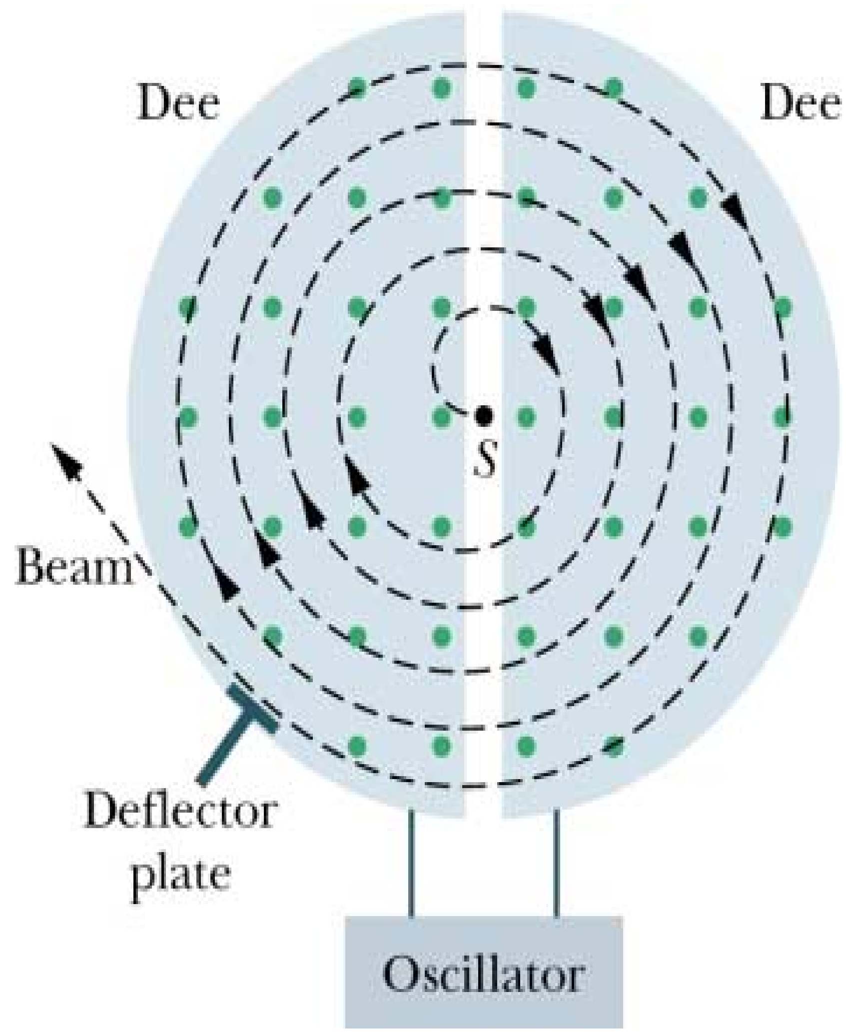
Magnetic Fields (32)

- Cyclotron
  - Key is that the frequency, *f*, of the proton does not depend on *v* and must equal the f_{osc} of the Dees

$$f = f_{osc}$$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

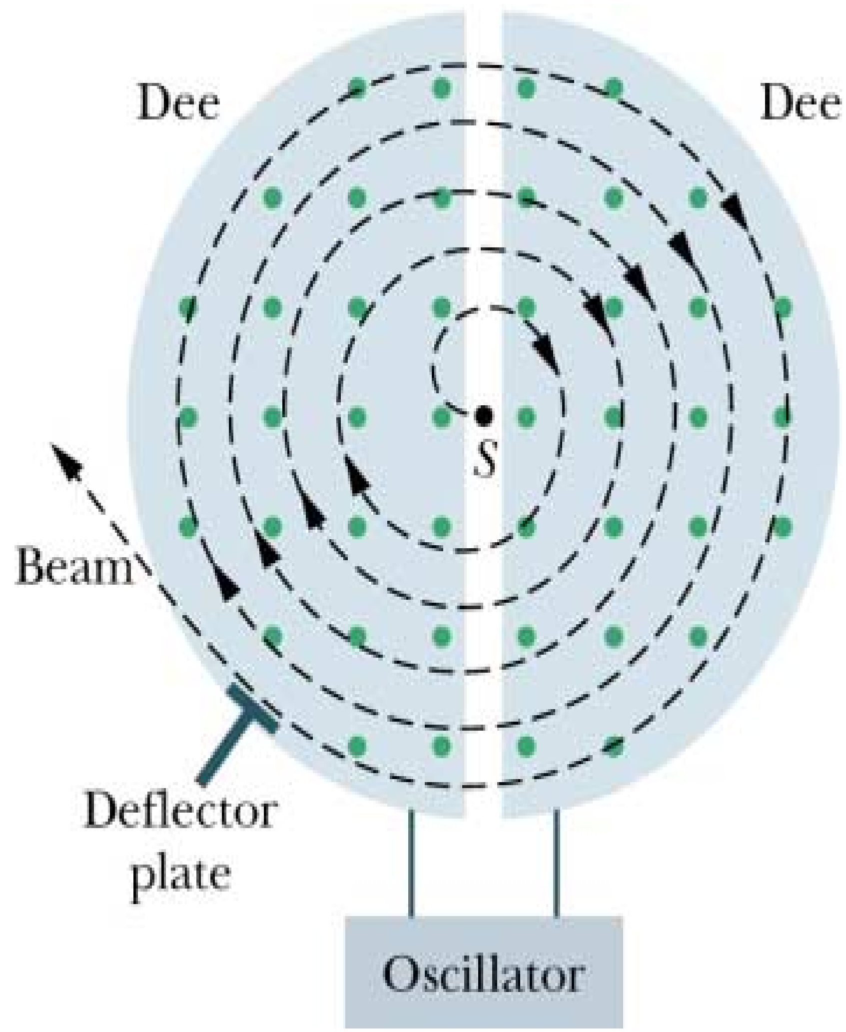
$$qB = 2\pi m f_{osc}$$



Magnetic Fields (33)

- Cyclotron
  - Has single f_{osc}
  - For proton, q and m are fixed
  - Tune cyclotron to get a beam of protons by varying B field

$$qB = 2\pi m f_{osc}$$



Magnetic Fields (34)

- Assumption that *f* of charged particle is independent of its speed only works for speeds much smaller than the speed of light
- At higher *v* the particle's *f* decreases as it is accelerated
- Cyclotron's frequency, f_{osc}, gets out of step with the particle's *f*

Magnetic Fields (35)

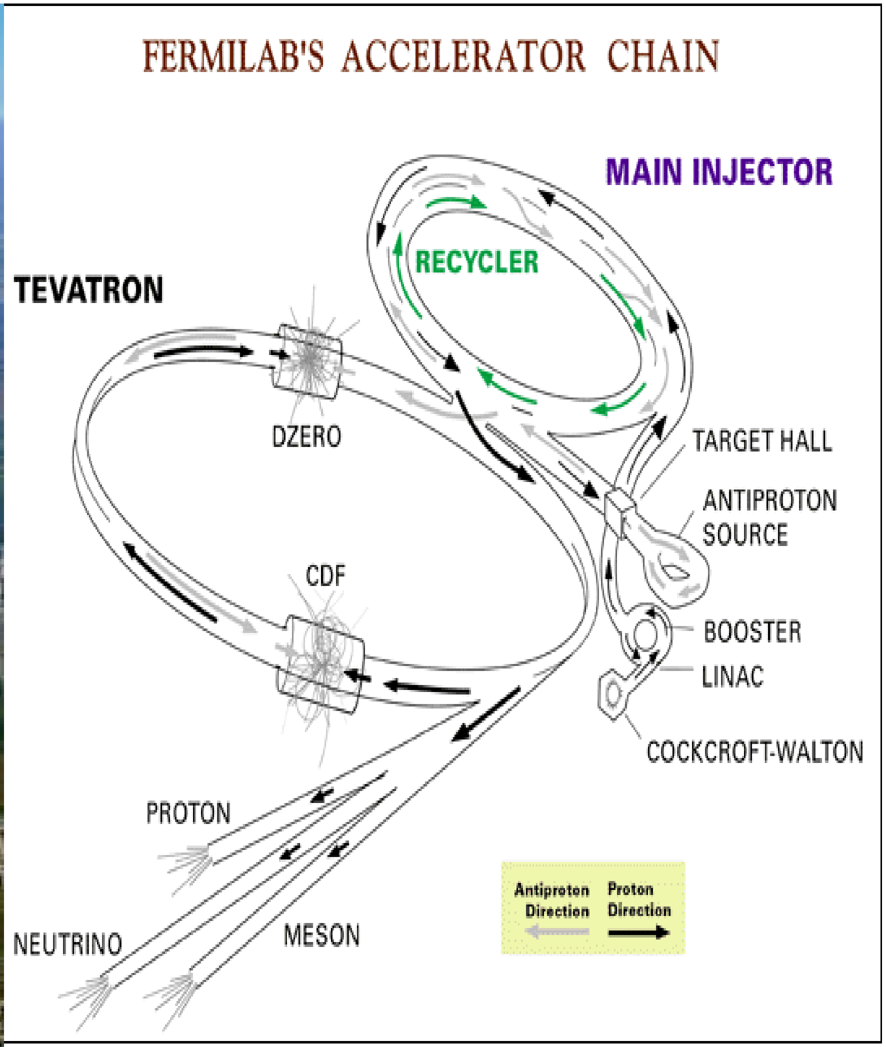
- To reach higher *v* thus higher energies use a synchroton
- The *B* field and f_{osc} vary with time
- Particles follow circular path instead of spiral

FERMILAB

Magnetic Fields (36)

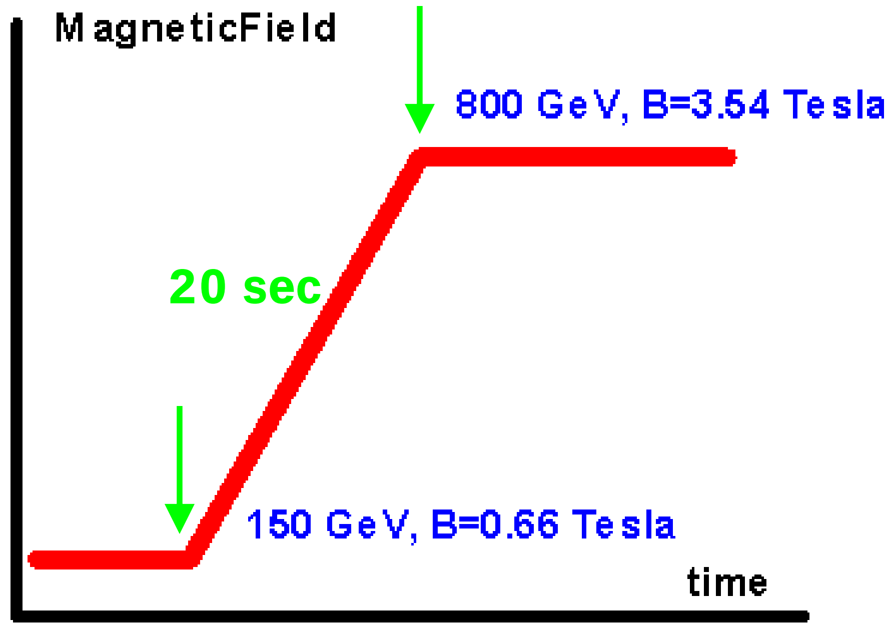
- MSU cyclotron
  - Fits in building next door
  - Can accelerate several different kinds of nucleons
  - Generates beams of particles with energies of 200 MeV/nucleon (1 MeV = 10^6 eV)
- FERMILAB
  - Uses 6 synchrotrons – the largest with 4 mile circumference
  - Accelerates protons and anti-protons
  - Protons move at 99.9999% speed of light (Go around ring 50,000 times in second)
  - Beam energies of 1 TeV (1 TeV = 10^{12} eV)

Magnetic Fields (37)



Magnetic Fields (38)

- 1000 superconducting magnets in Tevatron
- Kept at 4.3 K (liquid helium)



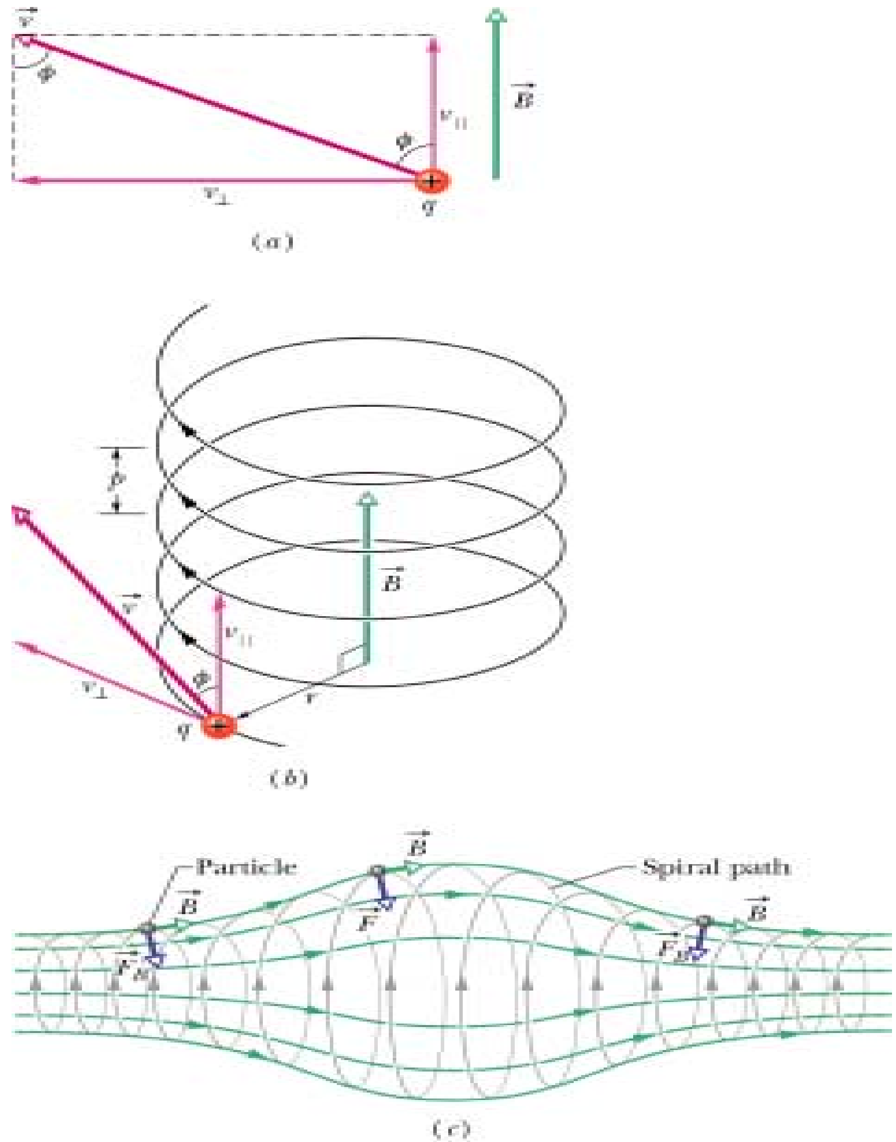
$$r = \frac{mv}{qB}$$

- Magnetic field changes with time
- Radius of circle remains constant

Magnetic Fields (39)

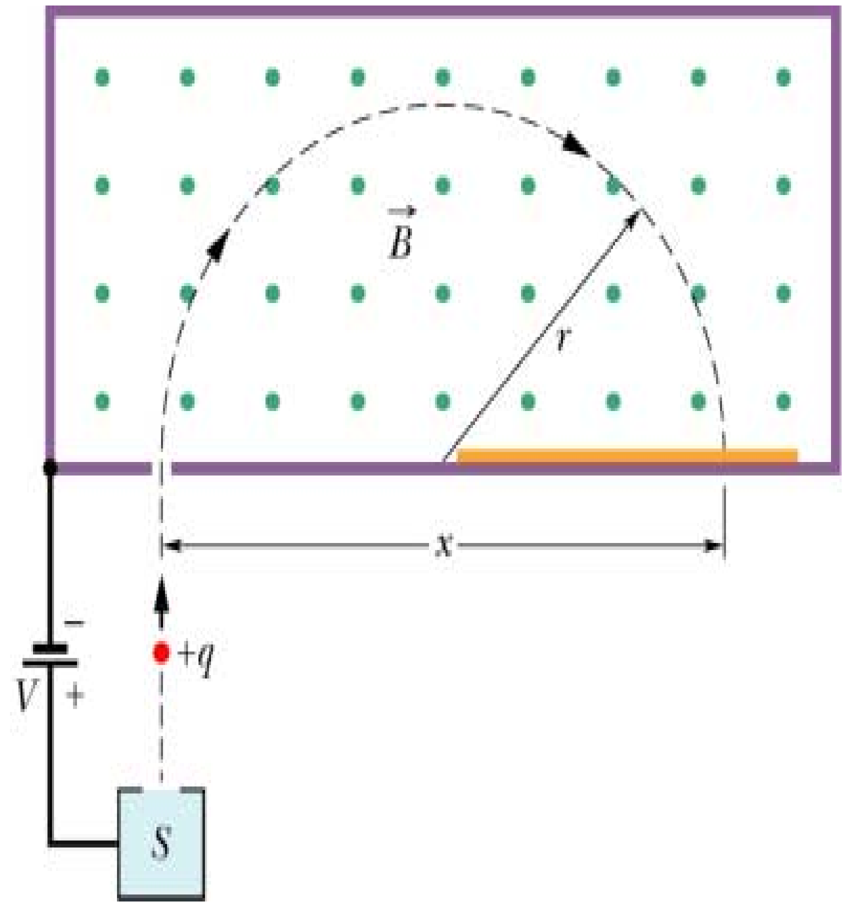
- So far assumed *v* and *B* were always ⊥
- If *v* has a component || to *B* then particle will have helical path

- FERMILAB protons move in helical path clockwise and anti-protons move in helical path counter-clockwise



Magnetic Fields (40)

- What do we do with the particle beams?
- Can measure a particle's mass using a mass spectrometer.
- Accelerate particle using potential difference, *V*
- Chamber with *B* field causes particle to bend, striking photographic plate



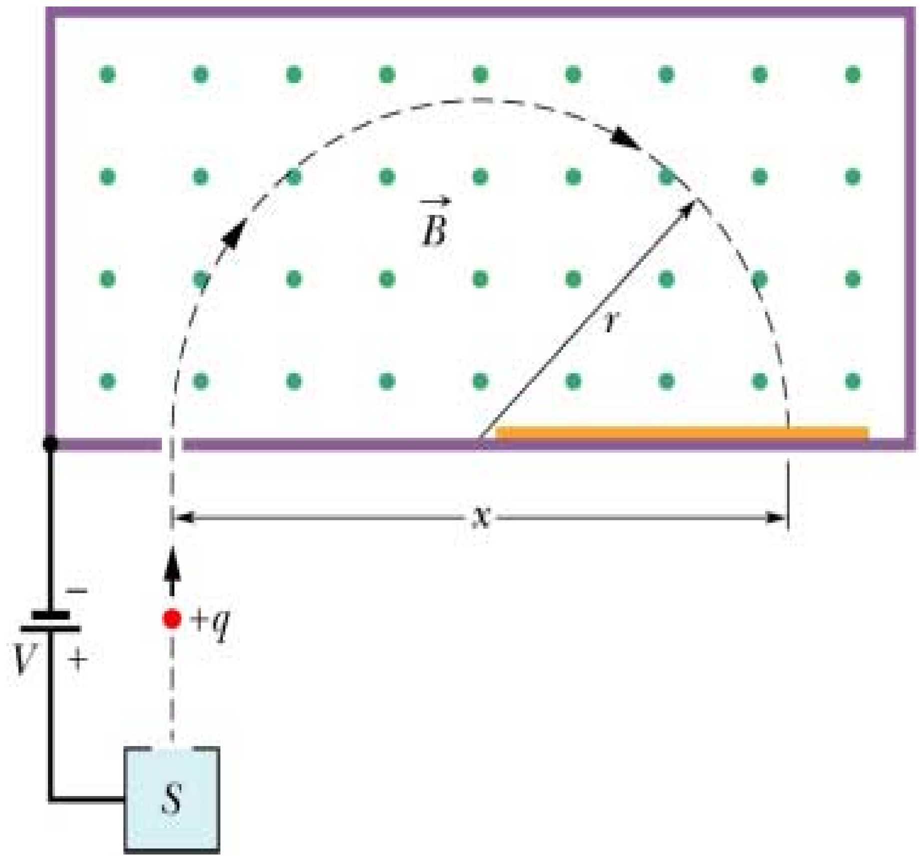
Magnetic Fields (41)

- Conservation of energy

$$\Delta K + \Delta U = 0$$

$$\tfrac{1}{2}mv^2 - qV = 0$$

$$v = \sqrt{\frac{2qV}{m}}$$



- Substituting *v* into relation for *r* gives

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

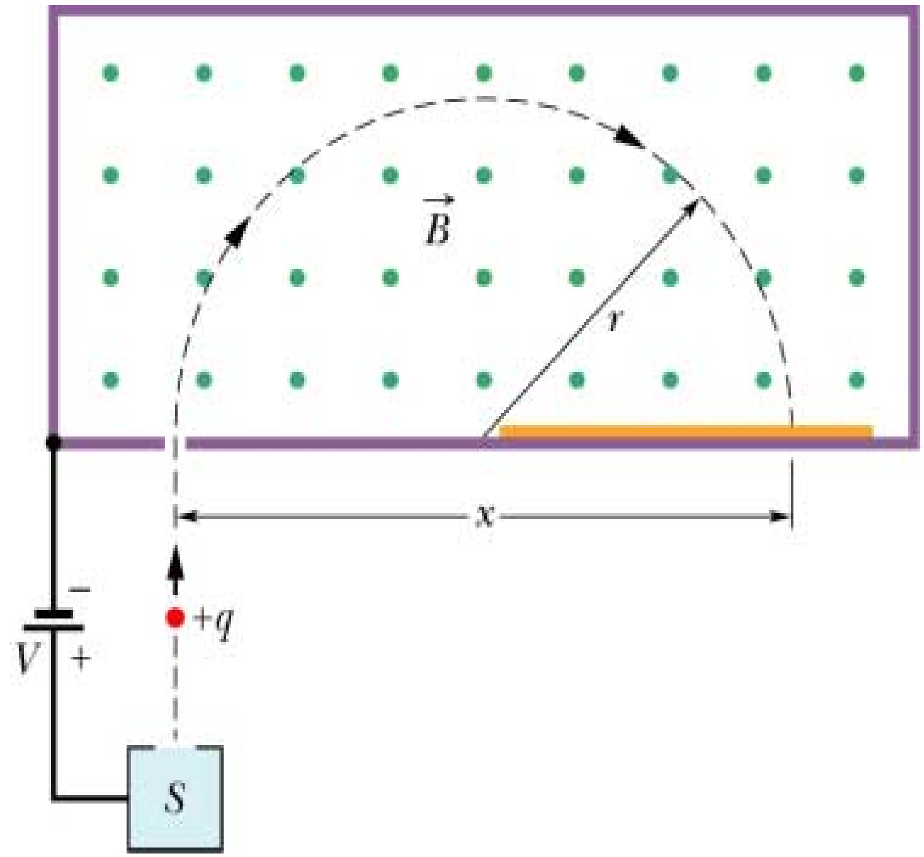
Magnetic Fields (41)

- Rearranging for *m*

$$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$m = \frac{B^2 r^2 q}{2V}$$

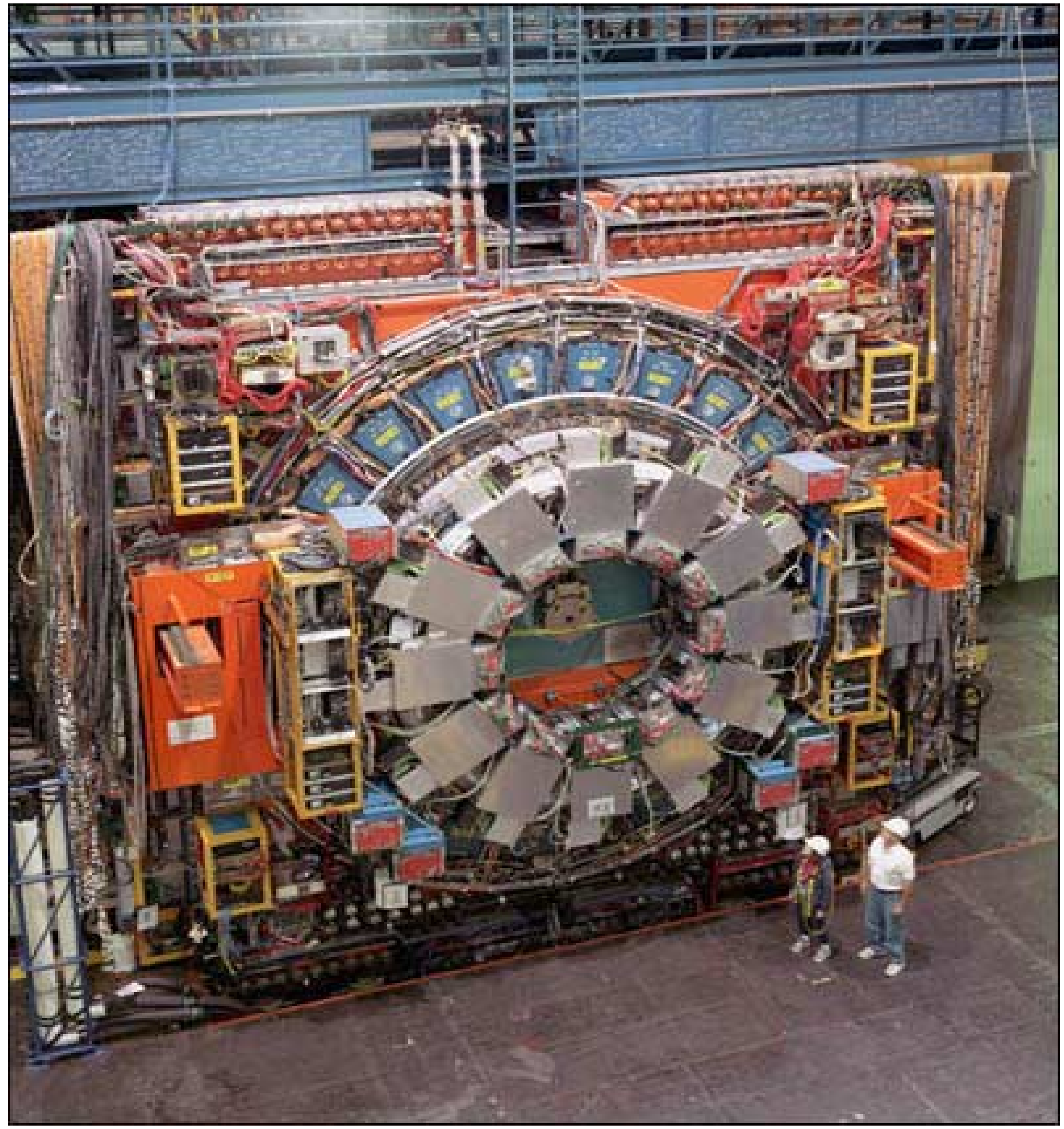
- Distance $x = 2r$



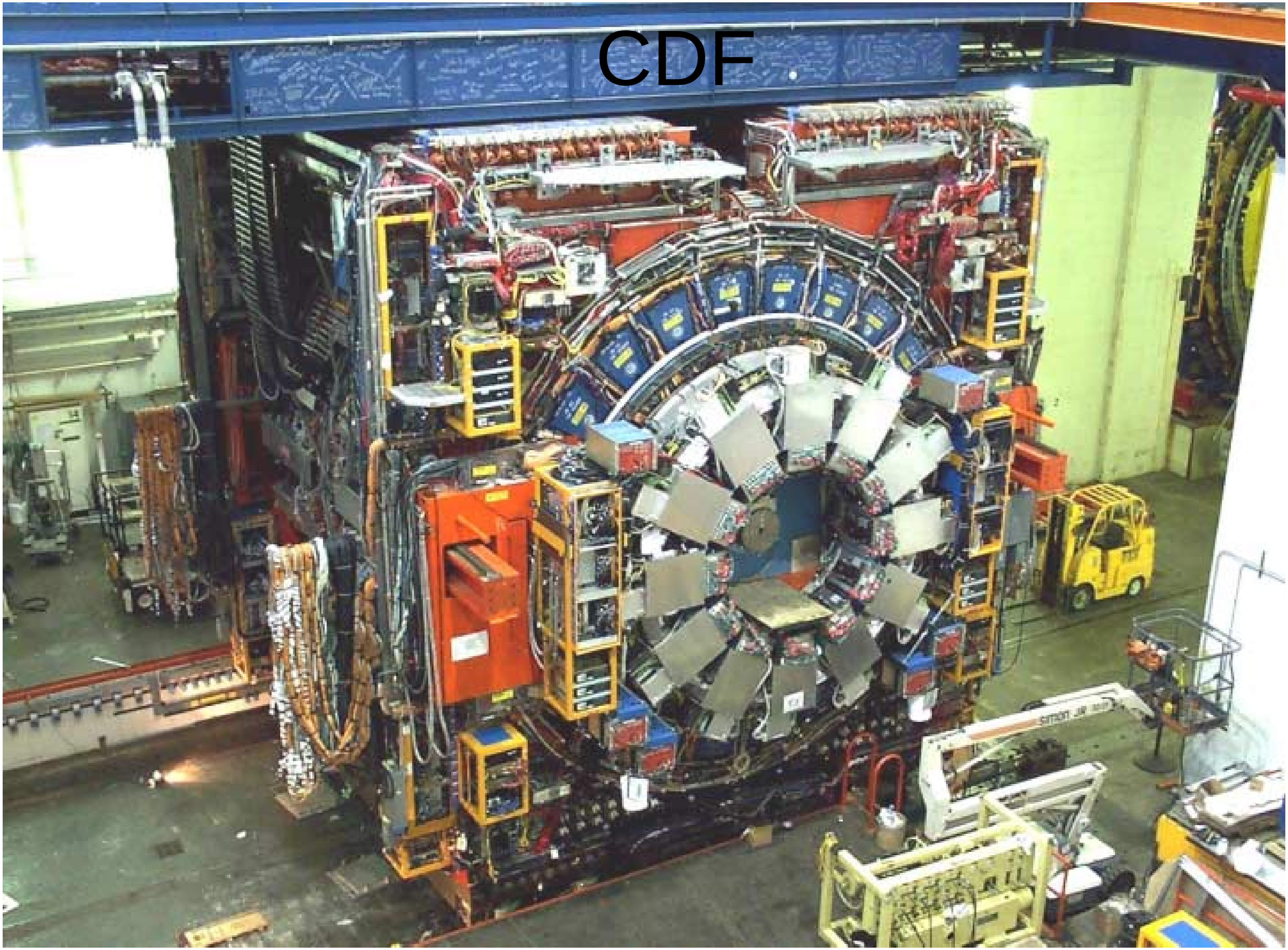
$$m = \frac{B^2 x^2 q}{8V}$$

Magnetic Fields (42)

- Fermilab looks at collisions of protons and anti-protons
- Build 5,000 ton detectors around interaction point to observe what happens
- CDF (the Collider Detector at Fermilab) experiment

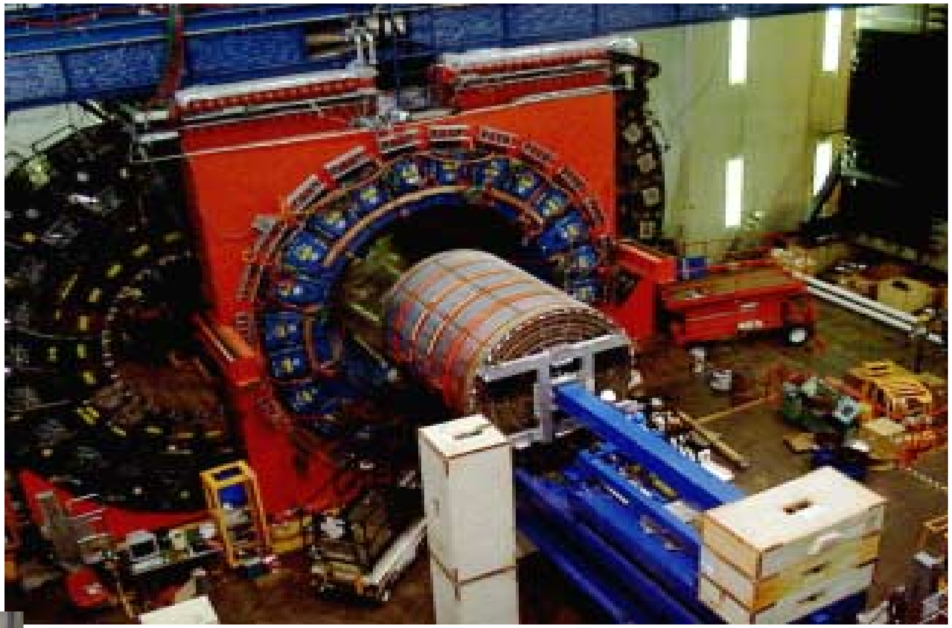


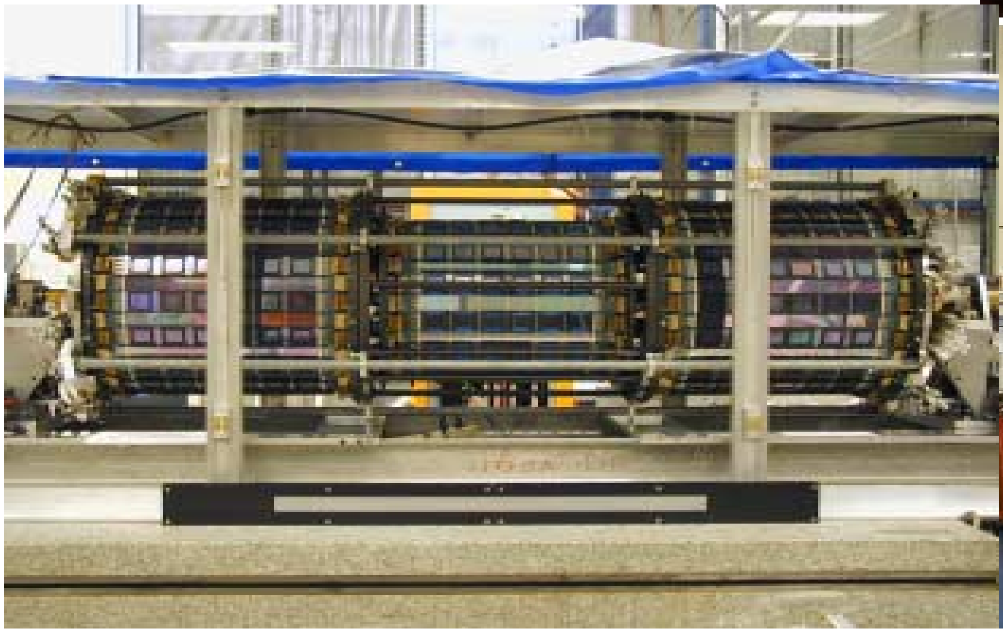
CDF



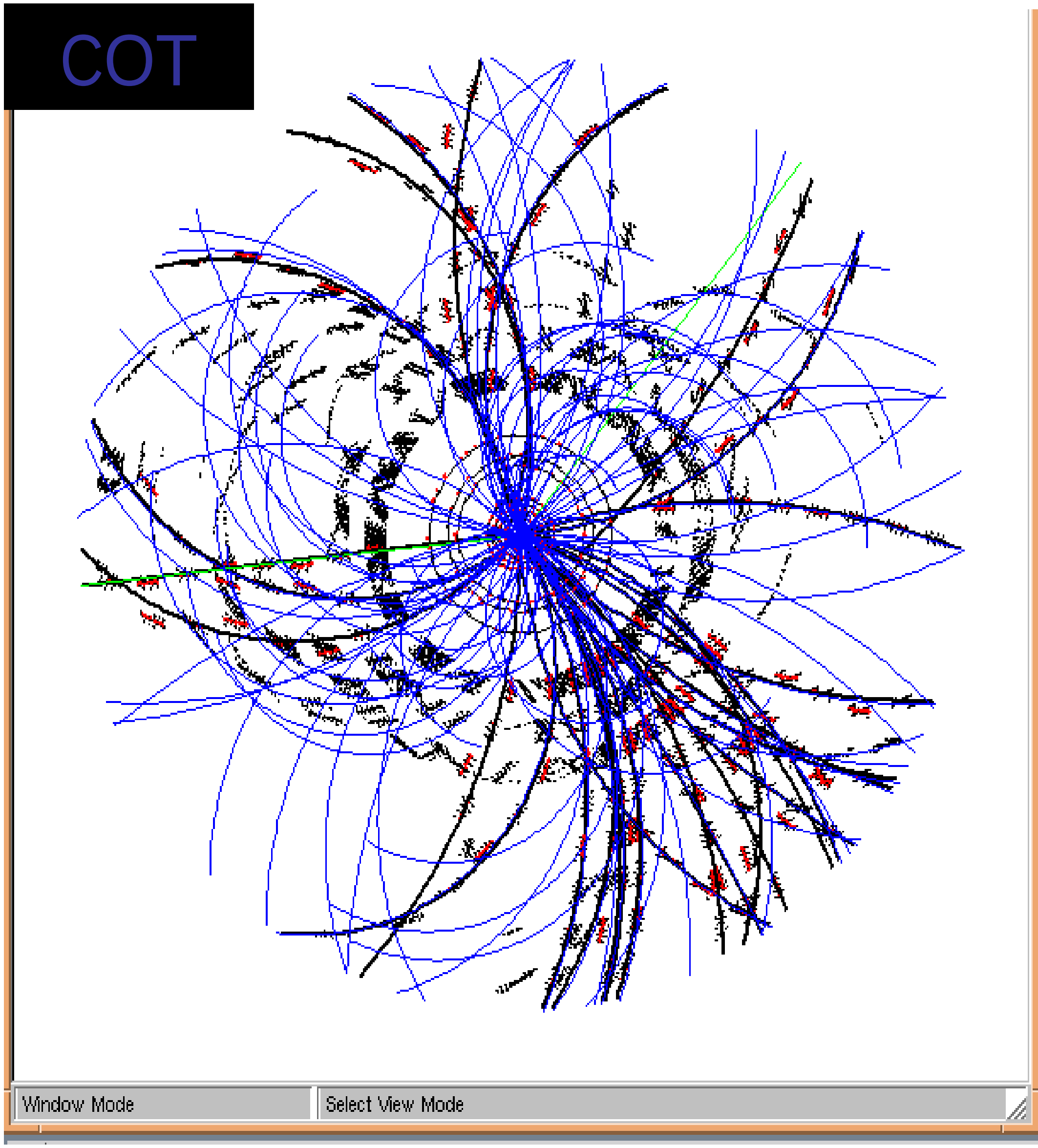
Magnetic Fields (43)

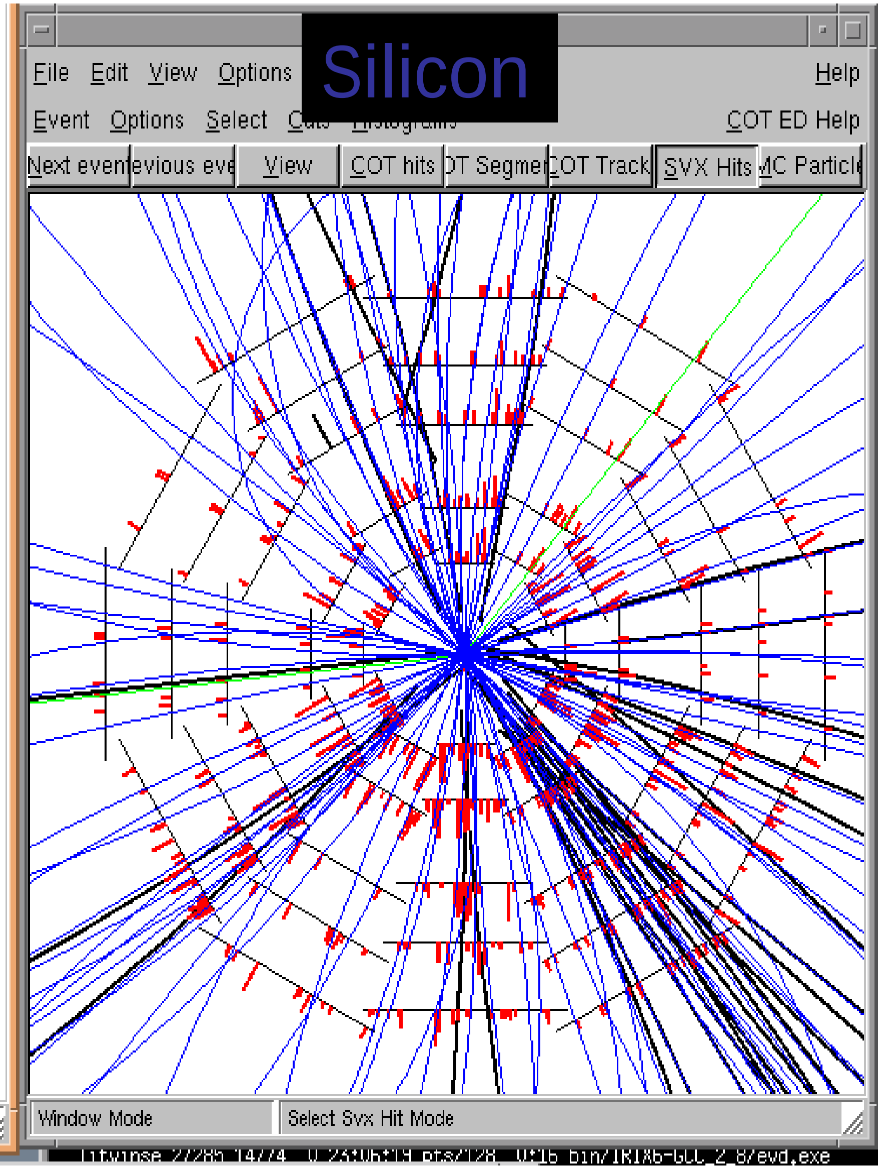
- Use giant solenoid to produce *B* field of 1.4 T
- Put detectors which show particle's path inside of *B* field





Magnetic Fields (44)



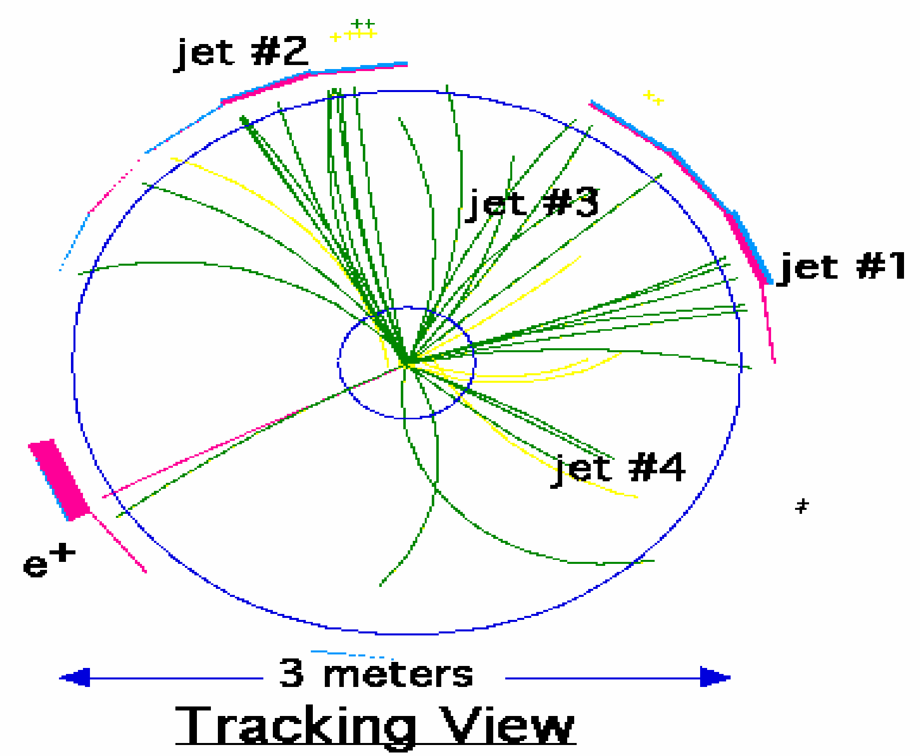


Magnetic Fields (45)

- Direction of curvature tells us the sign of the particle
- Amount of curvature, *r*, gives us the momentum

$$r = \frac{mv}{qB}$$

$$\vec{p} = m\vec{v} = rqB$$



For more info on Fermilab see http://www.fnal.gov

Magnetic Fields (46)

- Hall effect - *B* field exerts force on electrons moving in wire
- Electrons cannot escape wire so force is transmitted to wire itself
- Change either direction of current or *B* field, reverses force on wire