Lecture 19

Chapter 29

Magnetic Fields

Review

- Force due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- Charged particles moving with $v \perp$ to a B field move in a circular path with radius, r

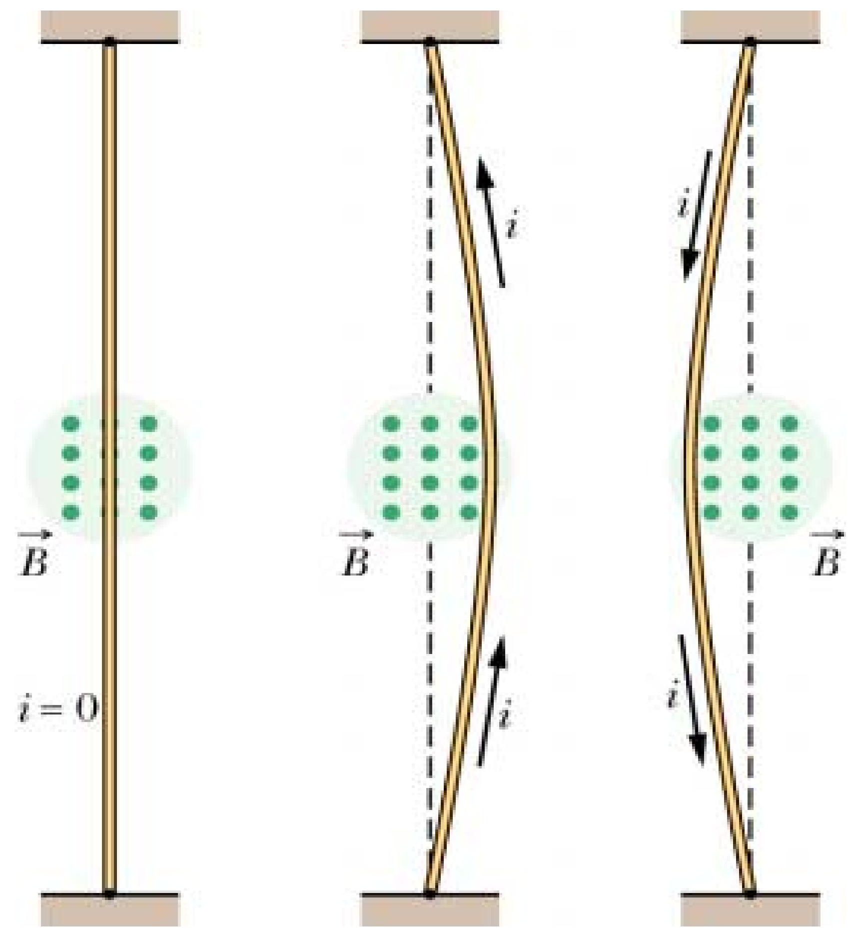
$$r = \frac{mv}{qB}$$

- Cyclotrons and synchrotrons

Review

- Demonstrated that a wire carrying current in a *B* field will feel a force
  - Wire jumped into (out of) horseshoe magnet when current was applied

- Change either direction of current or *B* field, reverses force on wire



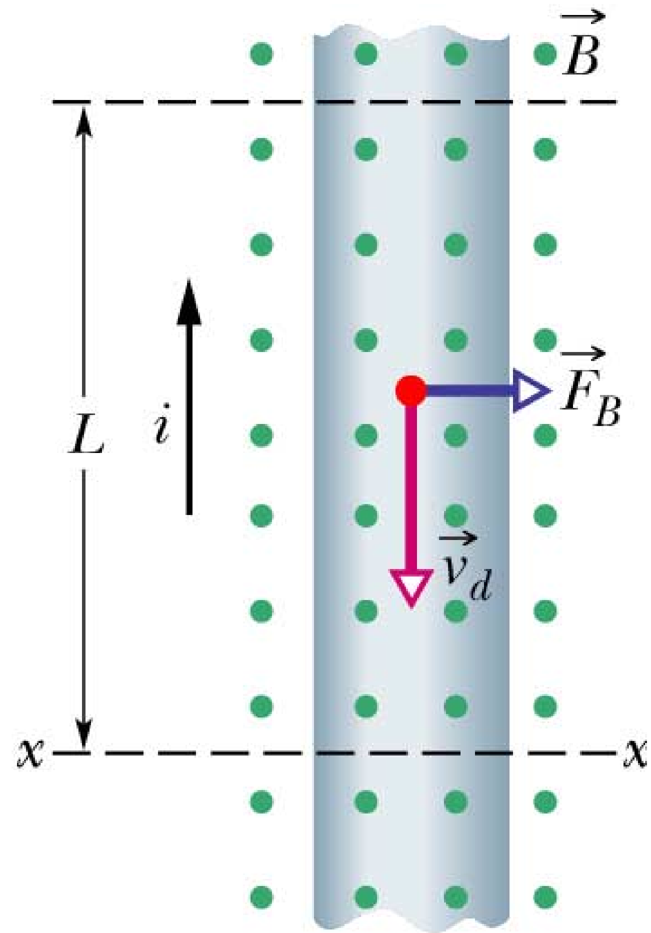
Magnetic Fields (47)

- Consider wire of length, *L*
- Conduction electrons drift past plane *xx* in time, *t*

$$t = \frac{L}{v_d}$$

- Amount of charge, *q*, passing through plane *xx* is then

$$q = it = i\frac{L}{v_d}$$



Magnetic Fields (48)

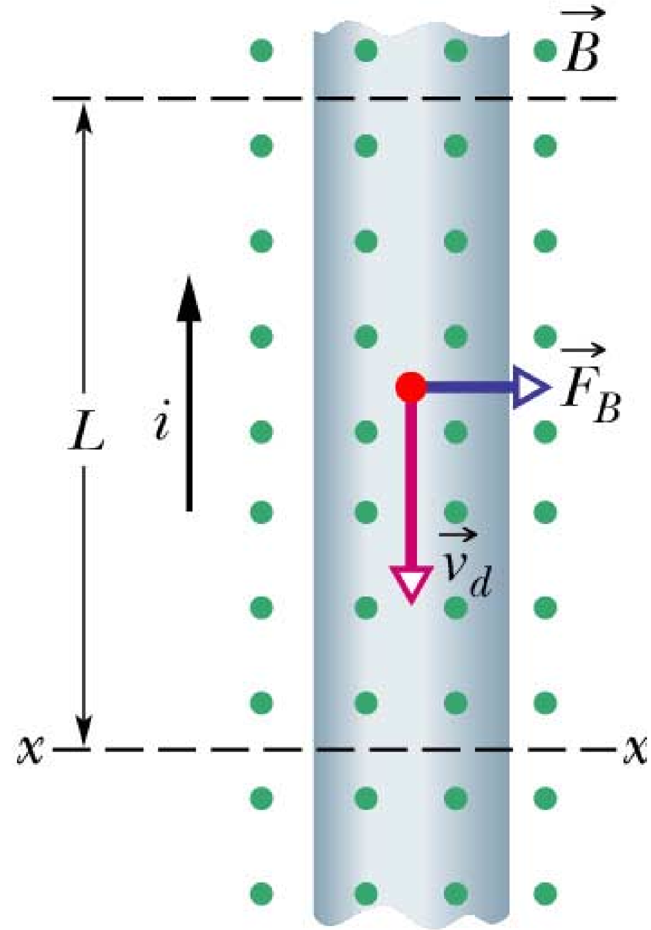
$$q = it = i\frac{L}{v_d}$$

- Substitute this for q in

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- Velocity is drift velocity, v_d

$$\vec{F}_B = qv_d B\sin\phi = \frac{iLv_d}{v_d} B\sin\phi$$



Magnetic Fields (49)

$$\vec{F}_B = iLB\sin\phi$$

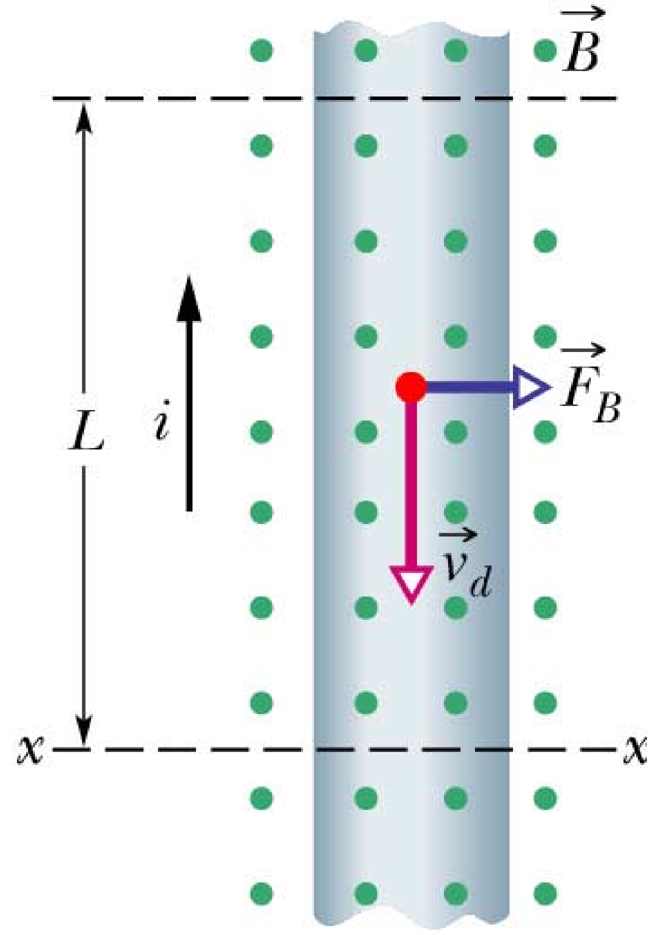
- Force on a current is

$$\vec{F}_B = i\vec{L}\times\vec{B}$$

- Vector *L* points along wire in the direction of the current

- Force on a single charge is

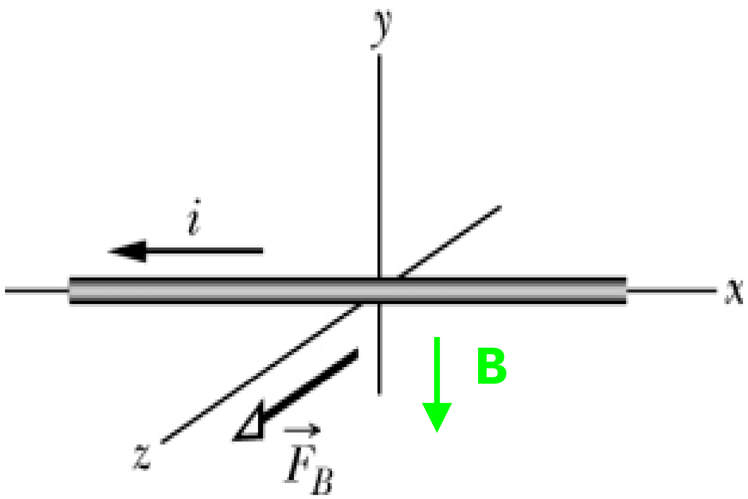
$$\vec{F}_B = q\vec{v}\times\vec{B}$$



Magnetic Fields (50)

- Checkpoint #5 – What is the direction of the *B* field so F_B is maximum?

$$\vec{F}_B = i\vec{L} \times \vec{B} = iLB\sin\phi$$



- Where's the maximum?

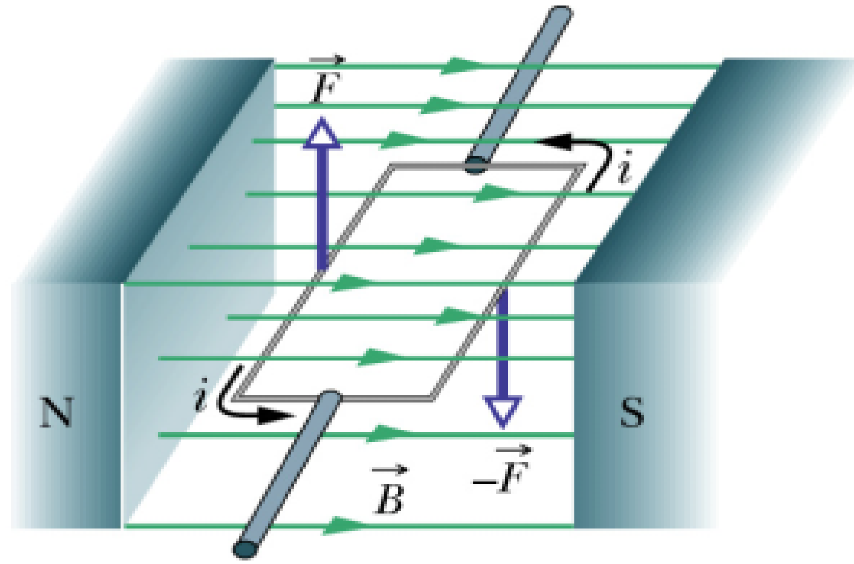
$$\sin\phi = 1, \phi = 90$$

- What's the direction of *B* ? Use right-hand rule

B points in -y

Magnetic Fields (51)

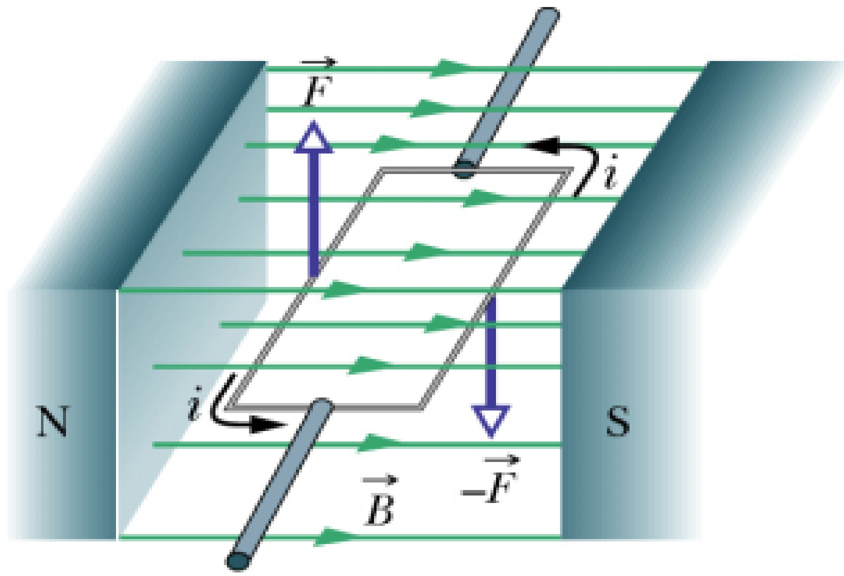
- What happens if we put a loop of wire carrying a current in a *B* field ?
- F_B on opposite sides of the loop produce a torque on the loop causing it to rotate



Magnetic Fields (52)

- Calculate the net force on the loop by vector sum of forces acting on each side
- For short sides of loop, i is || to B so $F_B = 0$
- For long sides of loop, i is ⊥ B so $F_B = iLB$, let length of long side $L = a$ so

$$\vec{F}_B = iaB$$

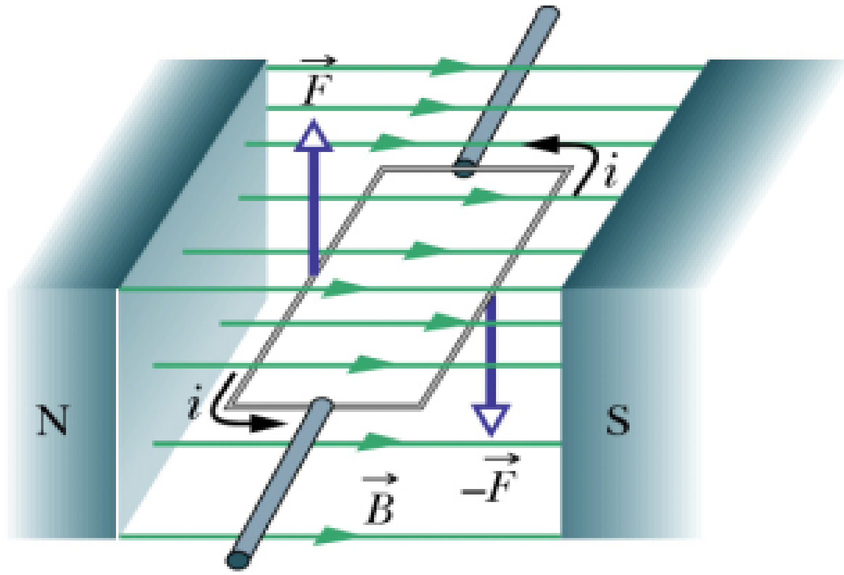


$$\vec{F}_B = i\vec{L} \times \vec{B} = iLB\sin\phi$$

Magnetic Fields (53)

- Force is in opposite directions for long sides of the loop
- Forces don't cancel because they don't share a common line of action
- Instead produce have a net torque and the loop rotates

$$\vec{F}_B = iaB$$

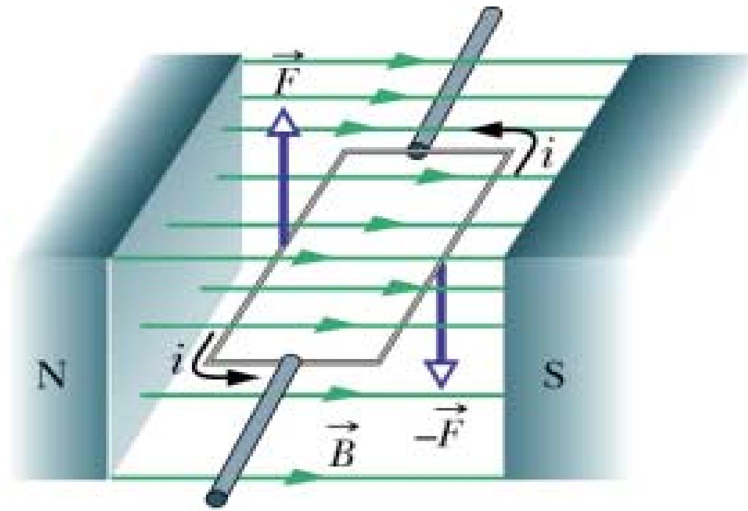


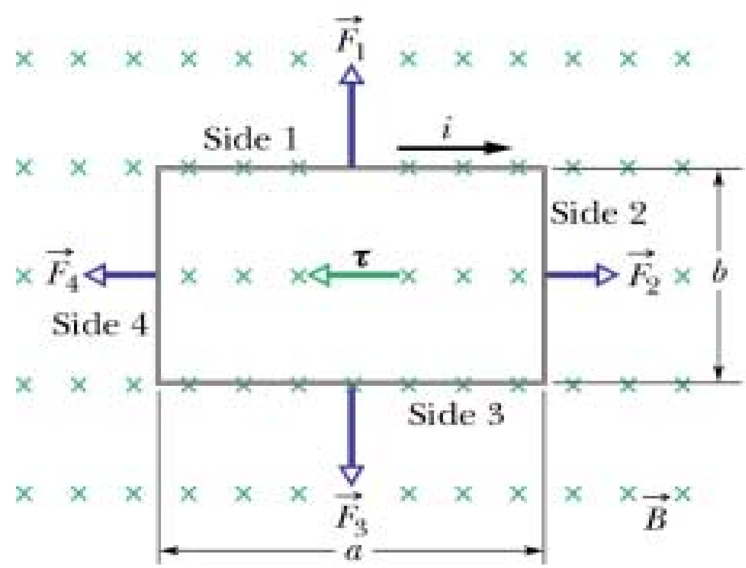
Magnetic Fields (54)

- Now rotate loop slightly so short sides are no longer || to B
- Short sides:
  - $F_B \neq 0$ instead

$$\vec{F}_B = i\vec{L}\times\vec{B} = iLB\sin\phi$$

  - Equal but opposite F_B s
  - Cancel each other since common line of action through center of loop





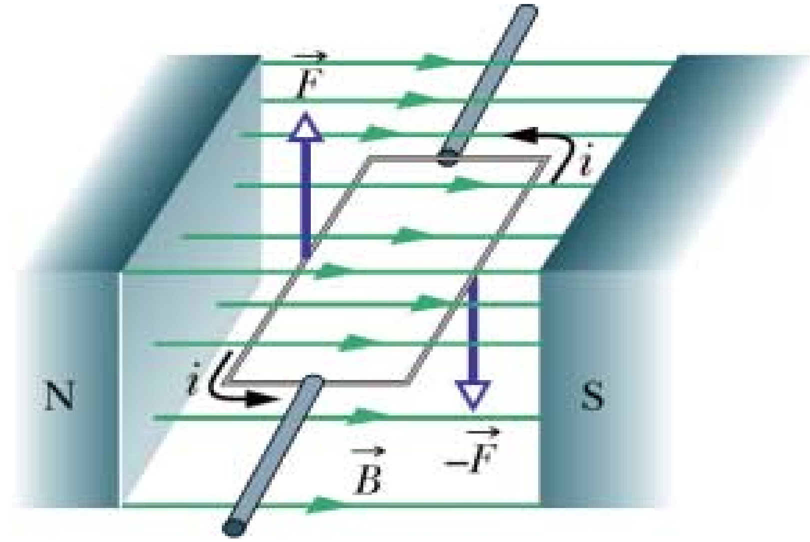
(a)

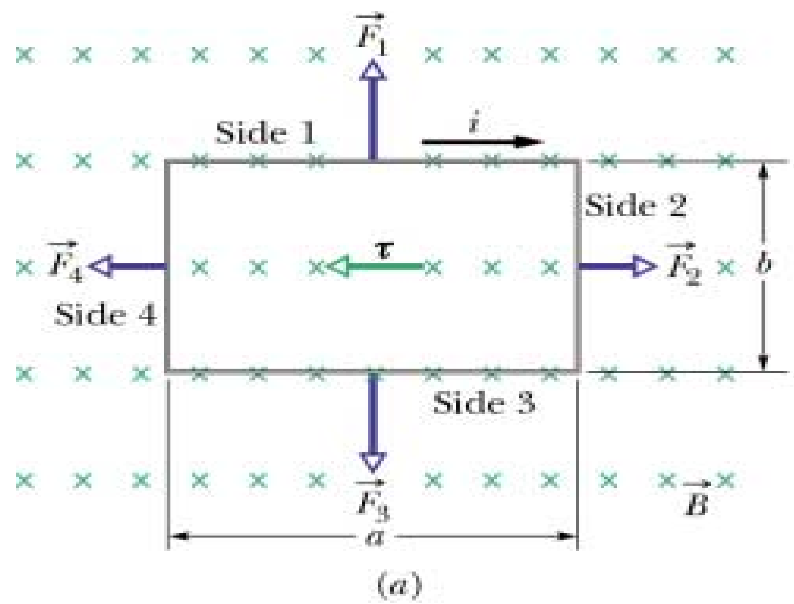
Magnetic Fields (55)

- Now rotate loop slightly so short sides are no longer || to B
- Long sides:
  - *i* is still ⊥ B so

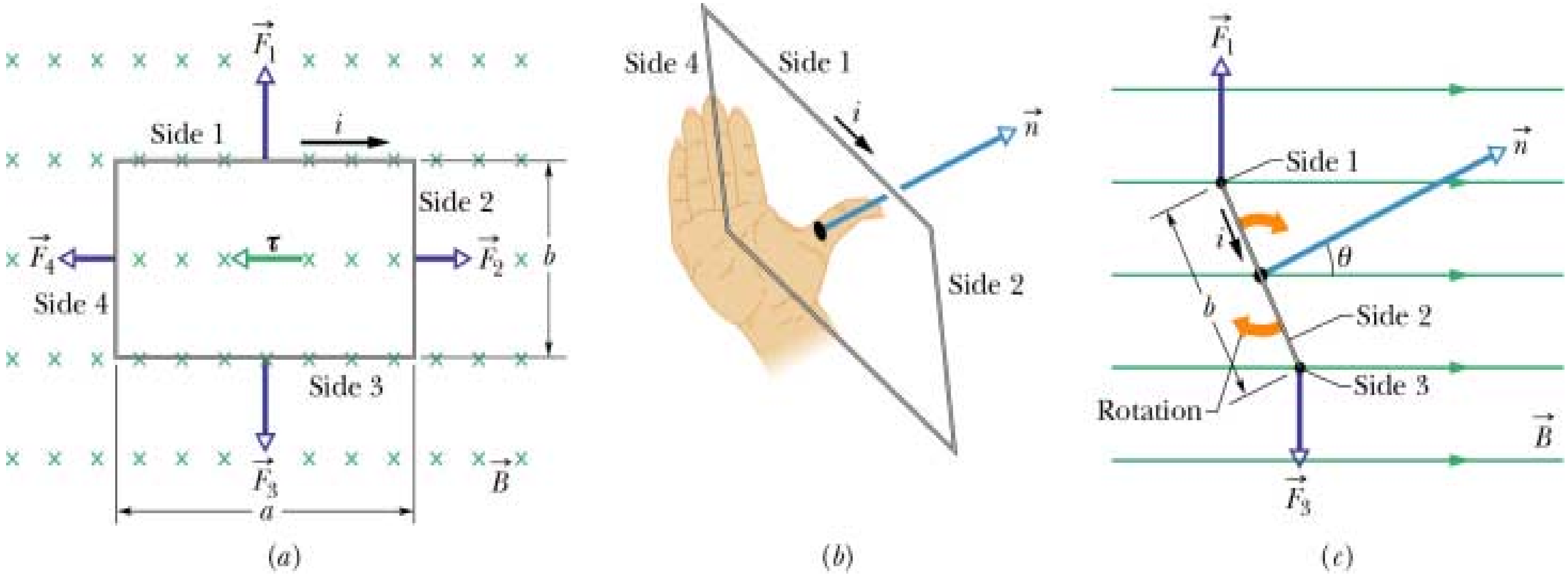
$$\vec{F}_B = i\vec{L} \times \vec{B} = iLB$$

$$\vec{F}_B = iaB$$





Magnetic Fields (56)



- Define normal vector, *n* ⊥ to loop
- Use right-hand rule to find direction of *n*
  - Fingers curl in direction of *i*, thumb points in direction of *n*

Magnetic Fields (57)

- Torque is defined as

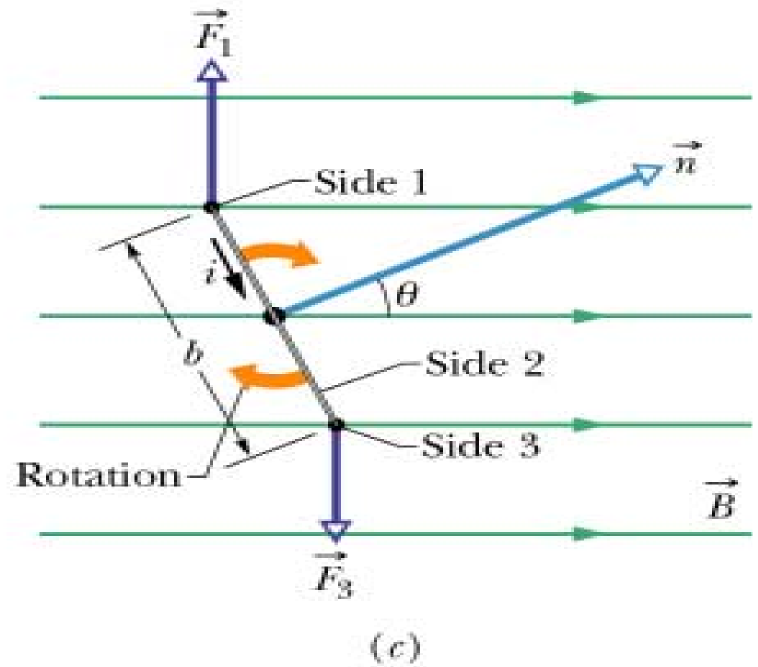
$$\tau = r_{\perp} F$$

- To find torque need to know the moment arm, $r_{\perp}$

$$r_{\perp} = \frac{b}{2}\sin\theta$$

- Torque for one side is

$$\tau = iaB\frac{b}{2}\sin\theta$$

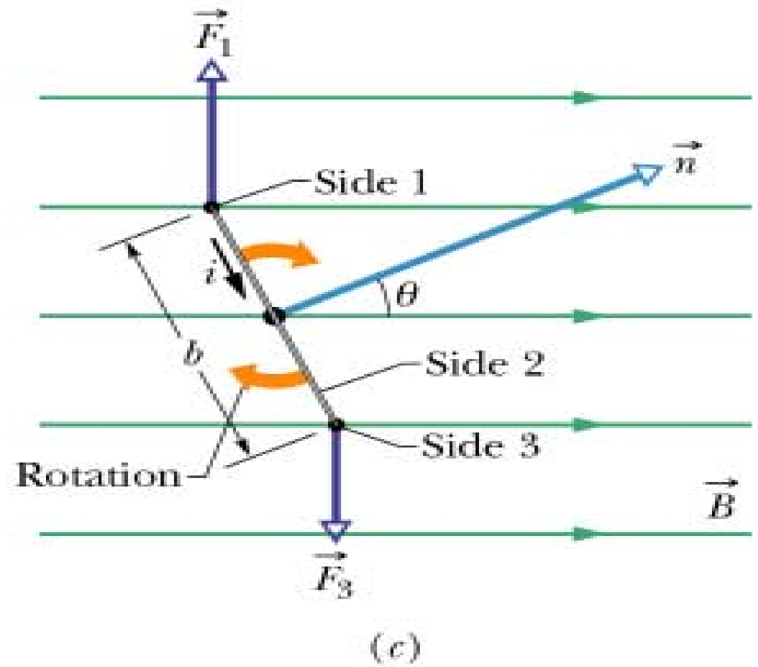


Magnetic Fields (58)

- Total torque is sum of torques for each long side

$$\tau = iaB\frac{b}{2}\sin\theta + iaB\frac{b}{2}\sin\theta$$

$$\tau = iabB\sin\theta$$



- Area of loop is

$$A = ab$$

so

$$\tau = iAB\sin\theta$$

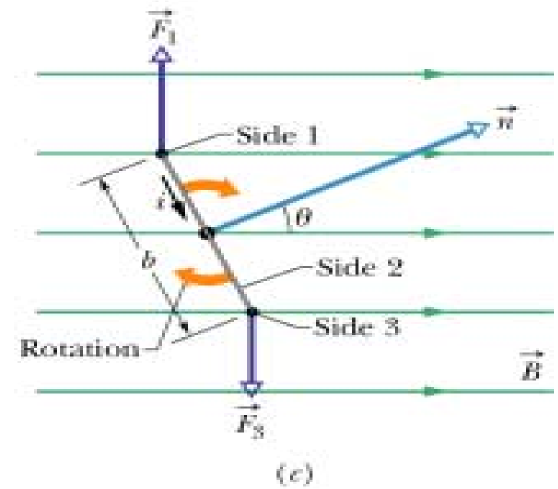
Magnetic Fields (59)

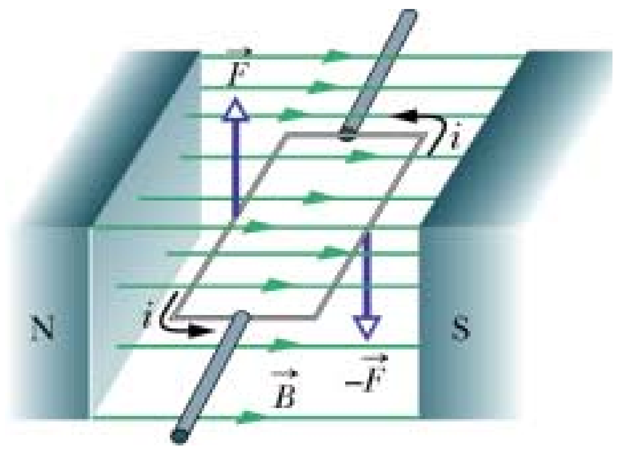
- Torque tends to rotate loop so to align *n* with *B* field
- Electric motor – oscillate polarity of *B* field to keep loop spinning
- Torque for single loop

$$\tau = iAB\sin\theta$$

- Replace single loop with coil of *N* loops or turns

$$\tau' = N\tau = (NiA)B\sin\theta$$





Magnetic Fields (60)

- Define the magnetic dipole moment to be

$$\mu = NiA$$

- Torque becomes

$$\tau = \mu B \sin\theta = \vec{\mu} \times \vec{B}$$

- Analogous to

$$\tau = \vec{p} \times \vec{E}$$

- A magnetic dipole in a magnetic field has a magnetic potential energy, U

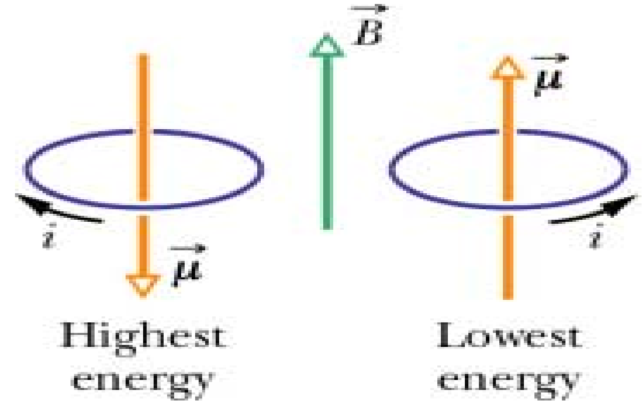
$$U = -\vec{\mu} \bullet \vec{B}$$

- Analogous to

$$U = -\vec{p} \bullet \vec{E}$$

Magnetic Fields (61)

- Magnetic dipole has lowest (highest) energy when μ is lined up with (directed opposite) the *B* field



$$U = -\vec{\mu} \bullet \vec{B} = -\mu B \cos\theta$$

$$\tau = \mu B \sin\theta = \vec{\mu} \times \vec{B}$$

- Checkpoint #6 – Rank, greatest first
- A) Magnitude of torque on dipole

All same

- B) Potential energy of dipole

1 & 4 tie, then 2 & 3 tie