Lecture 20

Chapter 30

Magnetic Fields Due to Currents

Review

- Force on a charged particle due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- Charged particles moving with *v* $\perp$ to a *B* field move in a circular path with radius, *r*

$$r = \frac{mv}{qB}$$

- Force on a current carrying wire due to a magnetic field is

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

Review

- Define the magnetic dipole moment to be

$$\mu = NiA$$

- Torque is

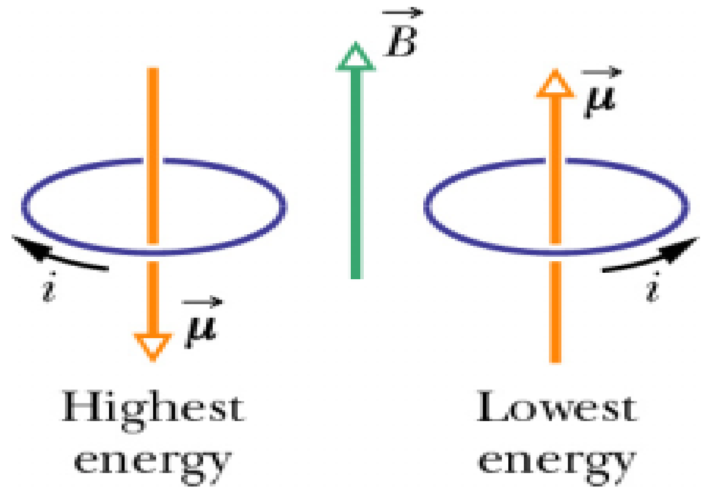
$$\tau = \vec{\mu} \times \vec{B}$$

- A magnetic dipole in a magnetic field has a magnetic potential energy, U

$$U = -\vec{\mu} \bullet \vec{B}$$

Review

- Magnetic dipole has lowest (highest) energy when μ is lined up with (directed opposite) the *B* field



$$U = -\vec{\mu} \bullet \vec{B} = -\mu B \cos\theta$$

- Work done on dipole by *B* field is

$$W = -\Delta U = -(U_f - U_i)$$

- Applied work done by an external agent is

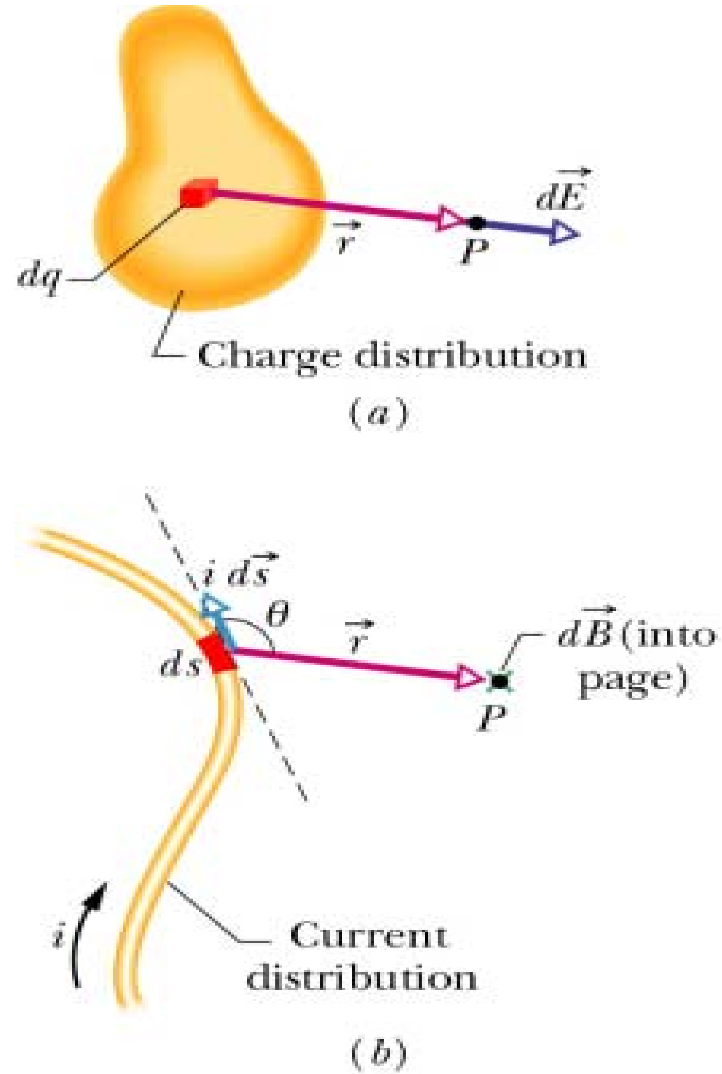
$$W_a = -W = U_f - U_i$$

B Fields from Currents (1)

- Calculate *B* field produced by distribution of currents
- Similar to finding *E* from distribution of charges

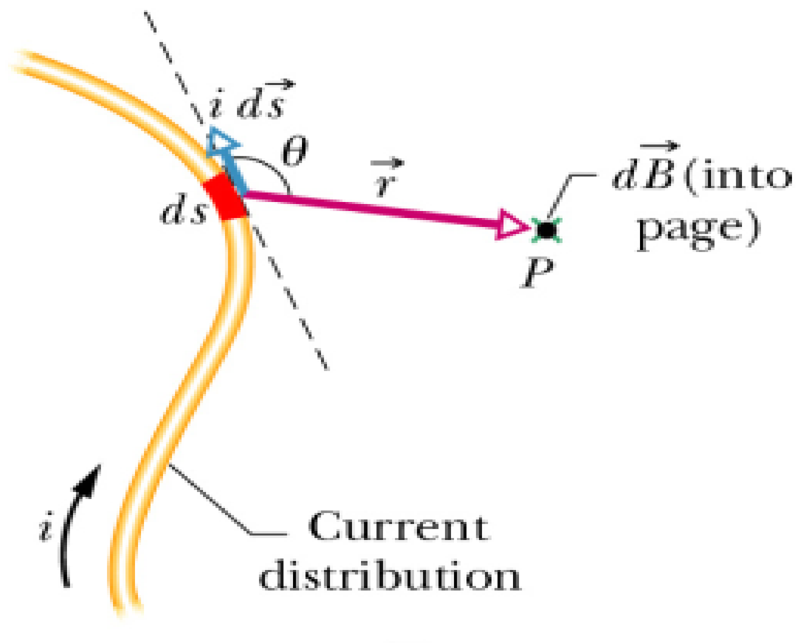
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

- *B* fields, like *E* fields, can be superimposed to find net field



B Fields from Currents (2)

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$



- Current-length element, $i\,d\vec{s}$, is product of a scalar and a vector
- Find net B field by integrating
- BUT remember it is a vector sum

- Permeability constant, μ_0

$$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m / A$$

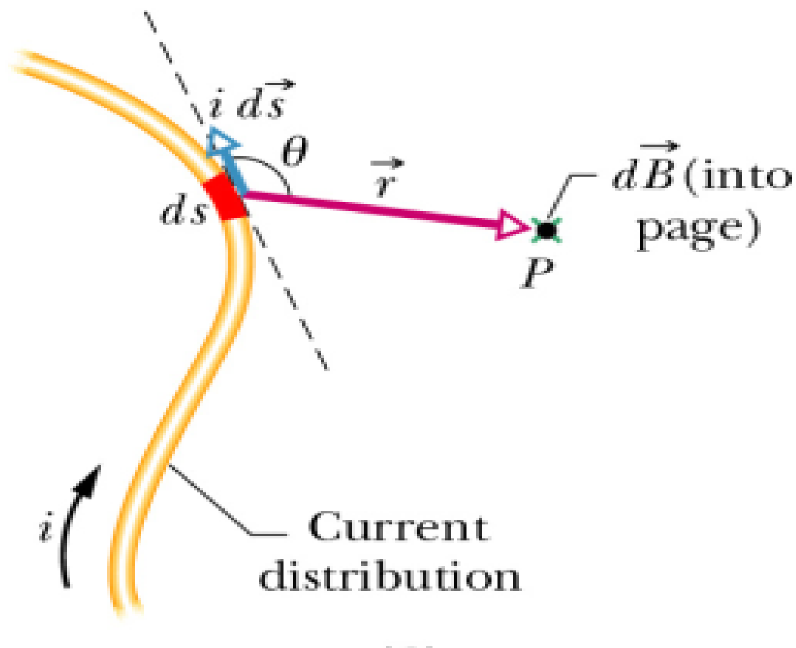
B Fields from Currents (3)

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin\theta}{r^2}$$

- Rewrite in vector form

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$$

- Known as Biot-Savart Law

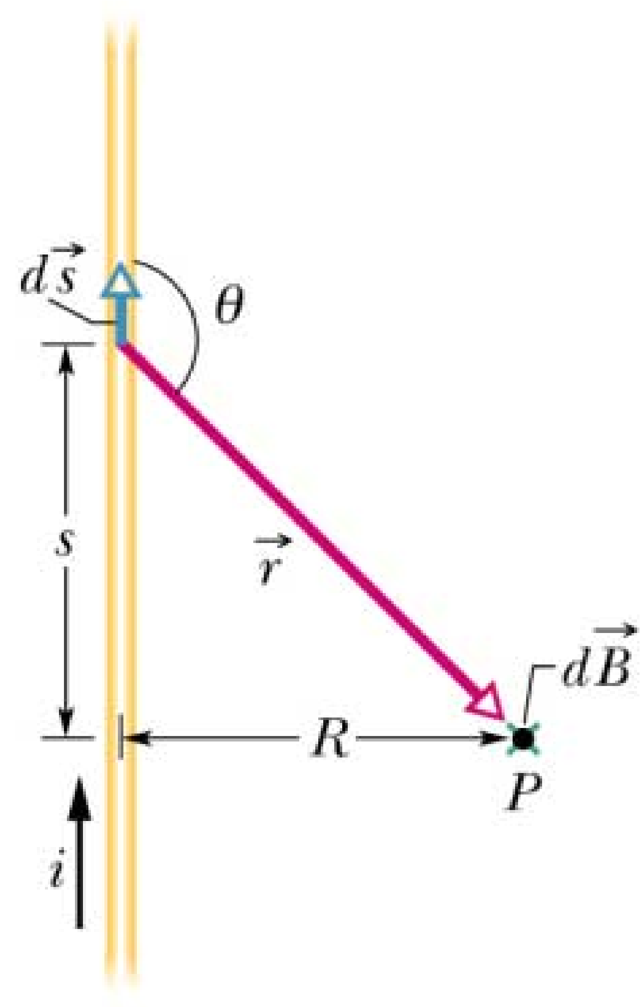


B Fields from Currents (4)

- Use Biot-Savart Law to calculate *B* field produced by an infinitely long straight wire with current, *i*

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

- Direction of *dB* at point *P* is into the page for all *ds* from -∞ to +∞



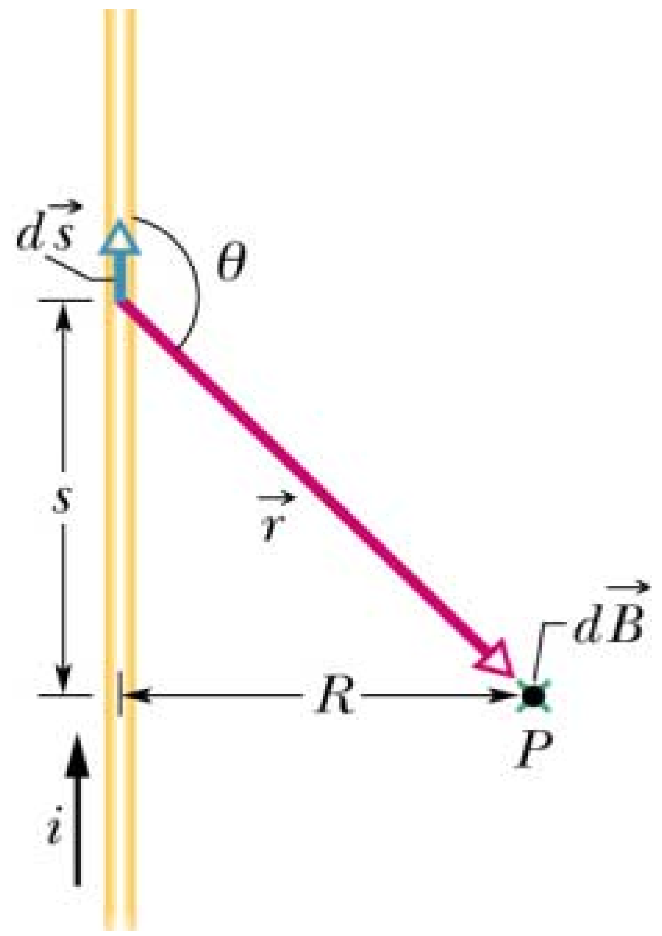
B Fields from Currents (5)

- Find total *B* field by integrating from 0 to +∞ and multiplying by 2

$$B = 2\int_0^{\infty} dB$$

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{\sin\theta}{r^2}ds$$



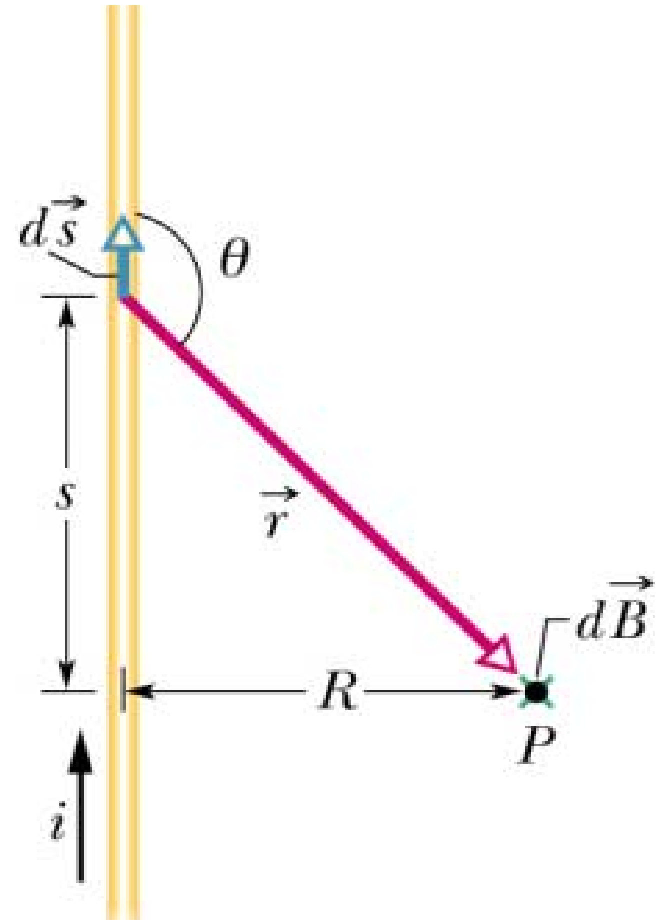
B Fields from Currents (6)

- Variables *r*, *s* and θ are related by

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r}$$

$$\sin\theta = \frac{R}{\sqrt{s^2 + R^2}}$$



B Fields from Currents (7)

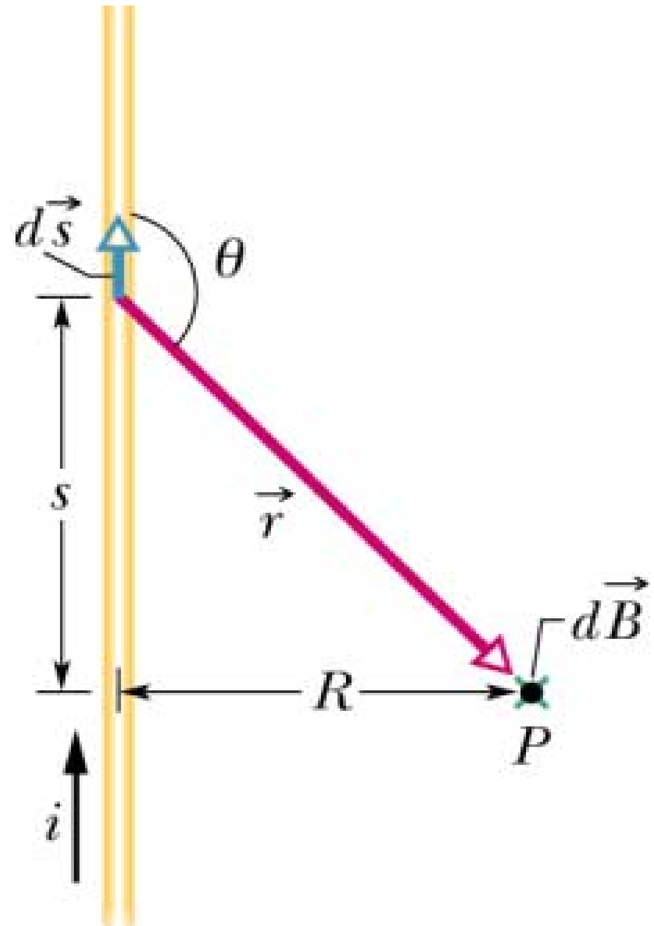
- Substituting

$$\sin\theta = \frac{R}{\sqrt{s^2+R^2}}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{\sin\theta}{r^2}ds = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{R\,ds}{\left(s^2+R^2\right)^{3/2}}$$

- Using the integral relation

$$\int\frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{x}{a^2\left(x^2+a^2\right)^{1/2}}$$



B Fields from Currents (8)

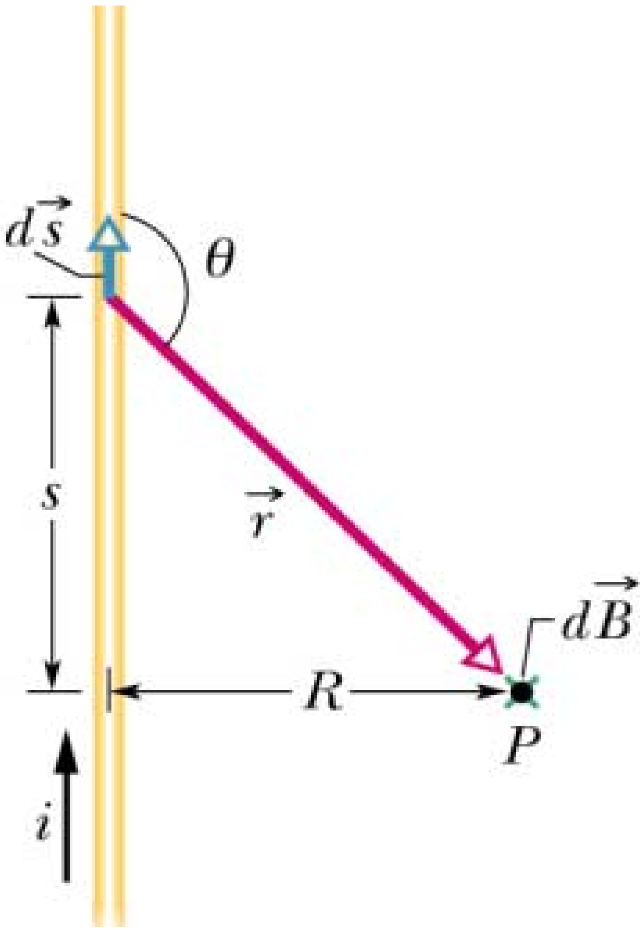
- Evaluating the integral gives

$$B = \frac{\mu_0 i R}{2\pi}\int_0^{\infty}\frac{ds}{\left(s^2+R^2\right)^{3/2}}$$

$$B = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{\sqrt{s^2+R^2}}\right]_0^{\infty}$$

- For a distance *R* from a long straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$

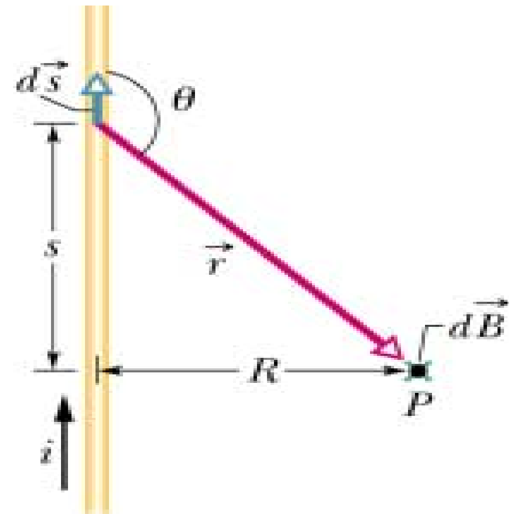


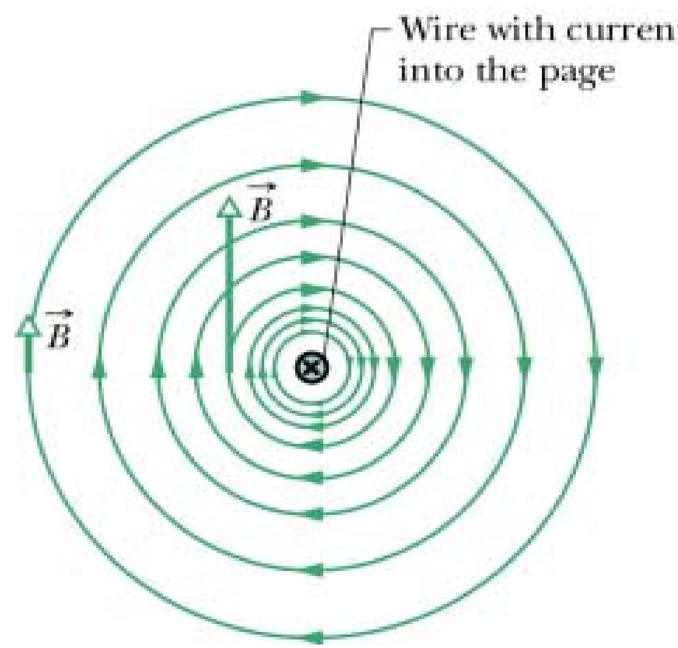
B Fields from Currents (9)

- Notice B field only depends on current, i, and $\perp$ distance R from wire

$$B = \frac{\mu_0 i}{2\pi R}$$

- B field forms concentric rings
- Magnitude of B decreases with distance as $1/R$

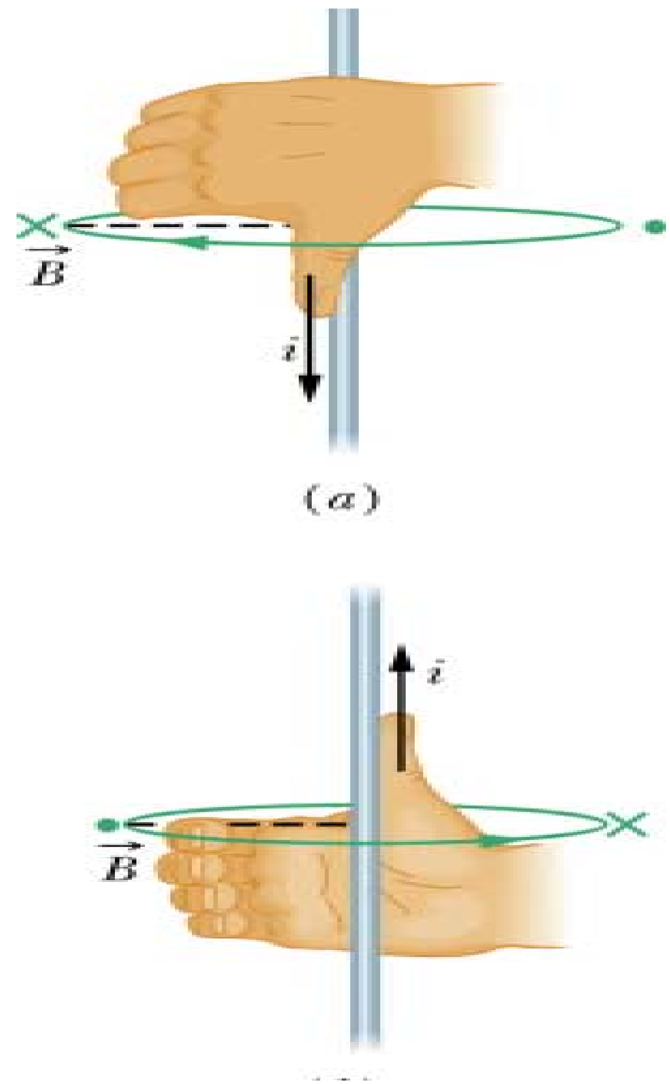




B Fields from Currents (10)

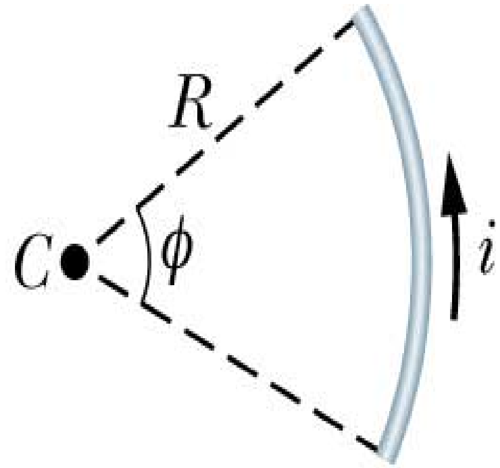
- New use for right-hand rule

- Point thumb in direction of current flow
- Fingers will curl in the direction of the magnetic field lines due to current
- *B* field is tangent to magnetic field line

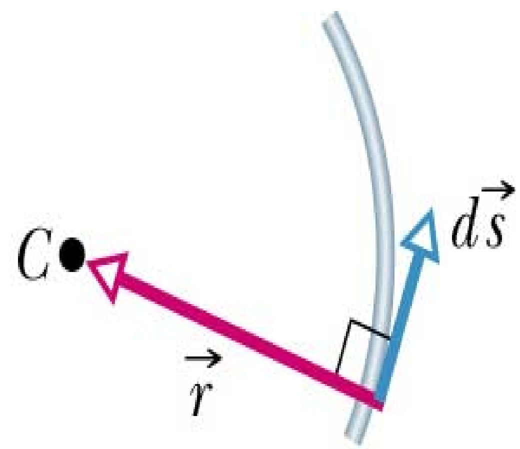


B Fields from Currents (11)

- What is B field due to circular arc of wire?
- Simplify problem by finding B at center of arc, point C
- Using Biot-Savart and the fact that r and ds are $\perp$ (θ=90)



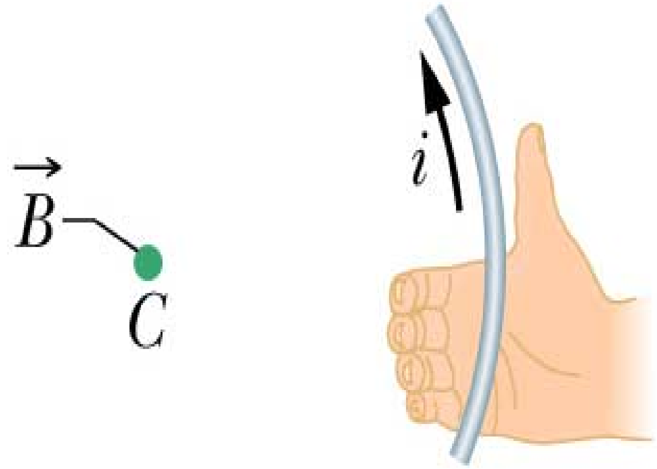
$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{ids}{R^2}$$

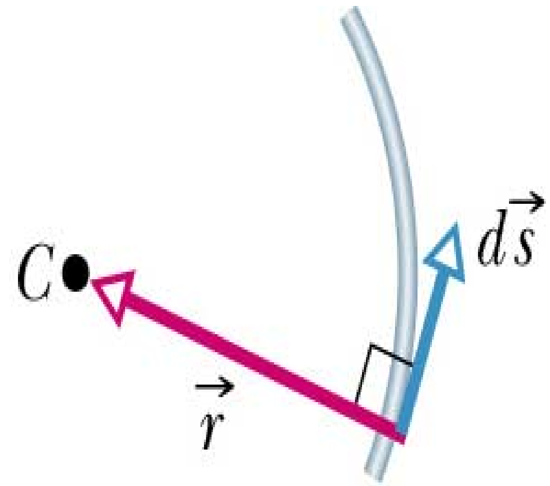


B Fields from Currents (12)

- Use right-hand rule to find direction of *B* field at C
- Every *ds* gives *dB* directed out of page so get net B by integrating over whole arc

$$B = \int dB = \frac{\mu_0}{4\pi}\int \frac{i\,ds}{R^2}$$





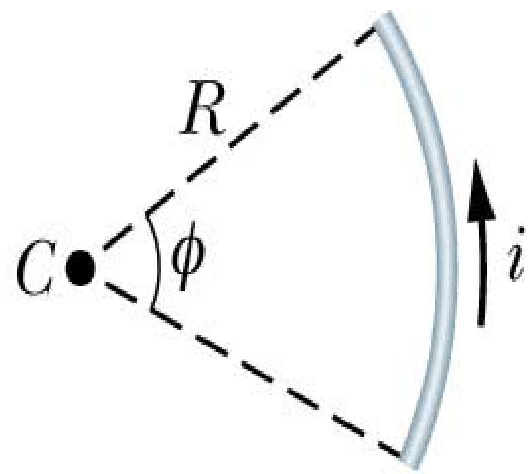
B Fields from Currents (13)

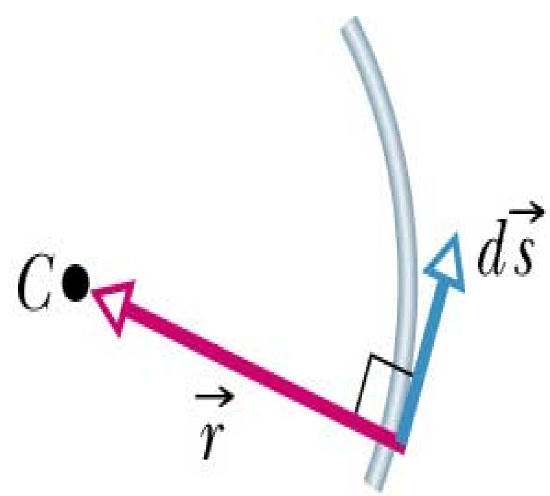
- Use identity to change integration variable

$$ds = Rd\phi$$

$$B = \frac{\mu_0 i}{4\pi}\int \frac{ds}{R^2} = \frac{\mu_0 i}{4\pi}\int_0^{\phi} \frac{Rd\phi}{R^2}$$

$$B = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi$$

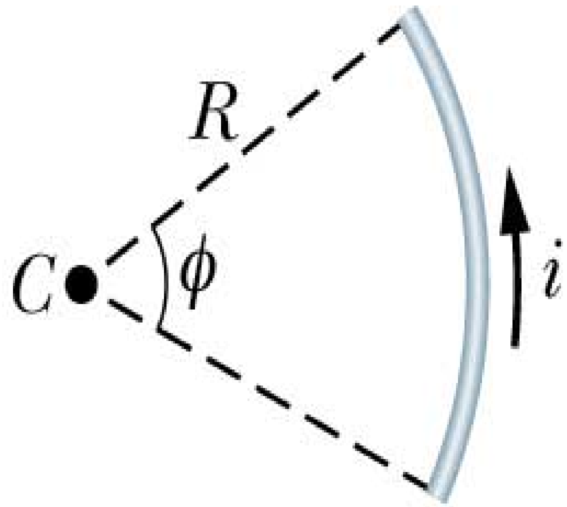




B Fields from Currents (14)

- *B* field at the center of an arc is

$$B = \frac{\mu_0 i \phi}{4\pi R}$$



- Express ϕ in radians not in degrees
- For a complete loop ($\phi = 2\pi$) then *B* is

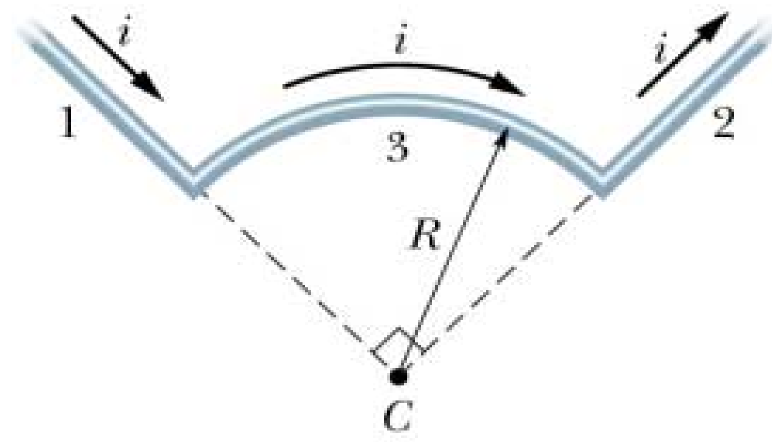
$$B = \frac{\mu_0 i}{2R}$$

B Fields from Currents (15)

- Calculate the *B* field at point C
- Use Biot-Savart law

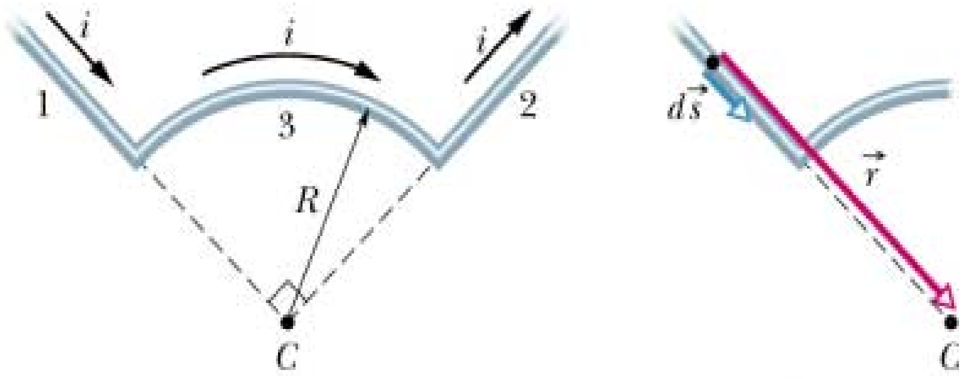
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

- Simplify problem by separating into 3 parts – sides 1, 2 & 3



- Side 1 – straight section on the left
- Side 2 – straight section on the right
- Side 3 – circular arc

B Fields from Currents (16)



- Side 1 – Angle, θ, between *ds* and *r* is zero so

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin 0}{r^2} = 0$$

$$B_1 = 0$$

- Side 2 – Angle, θ, between *ds* and *r* is 180 so

$$B_2 = 0$$

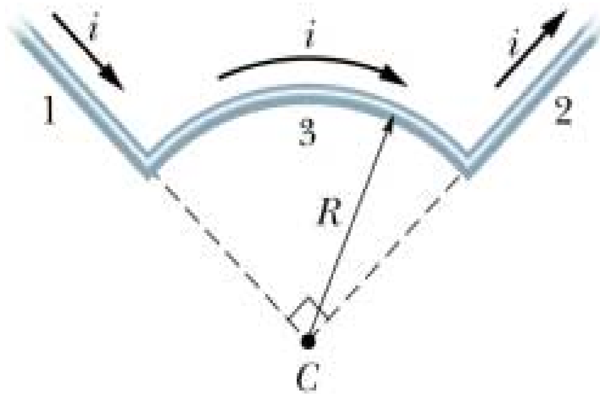
B Fields from Currents (17)

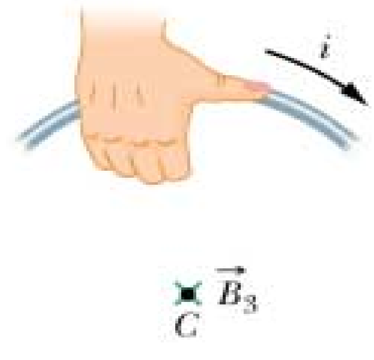
- Side 3 – circular arc
- Just derived *B* field at center of arc as

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

- Given that $\phi = \pi/2$ so

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}$$

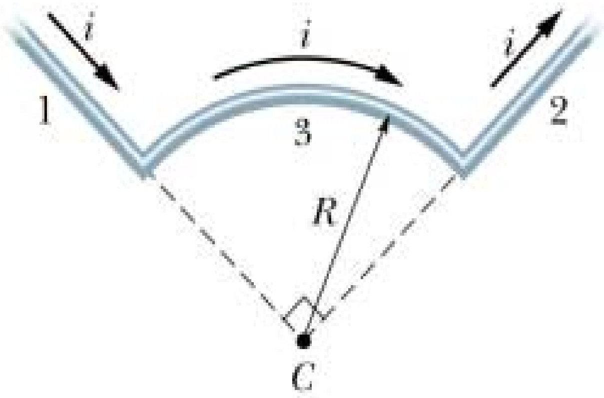




- Use right-hand rule to find that B_3 is directed into page

B Fields from Currents (18)

- Find net *B* field by combining the 3 fields
- Remember they combine as vectors!



$$B_1 = 0$$

$$B_2 = 0$$

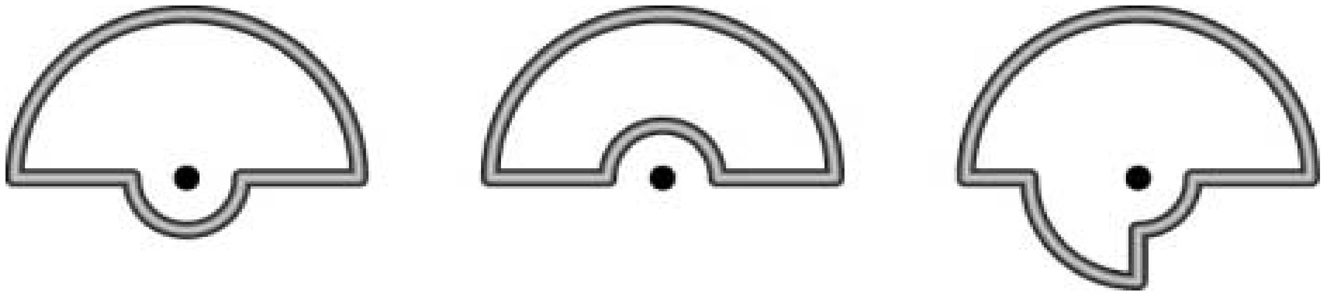
$$B_3 = \frac{\mu_0 i}{8R}$$

- Total *B* field is into the page and has magnitude

$$B = \frac{\mu_0 i}{8R}$$

B Fields from Currents (19)

- Checkpoint #1 – Three circuits with same *i* and various circular arcs of half (π) or quarter circles (π/2) and radii *r*, *2r* and *3r*. Rank magnitude of *B* field produced at the center (the dot), greatest first.



(a) (b) (c)

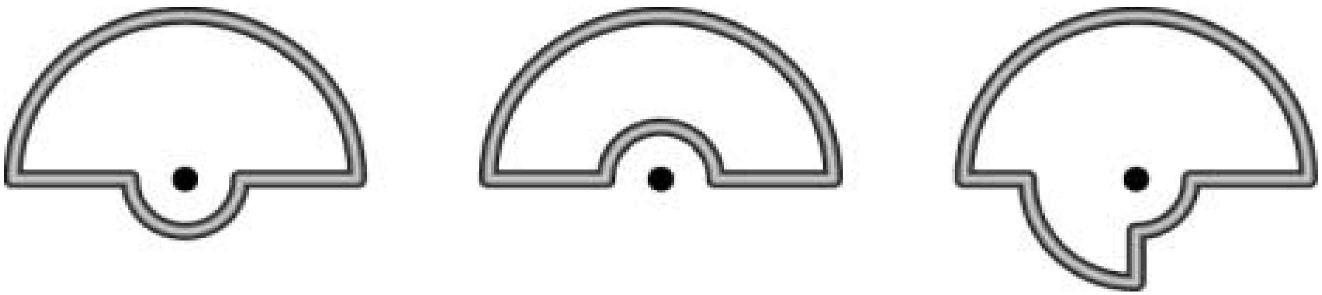
- For all straight sections θ = 0 or θ = 180 so

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2} = 0$$

B Fields from Currents (20)

- Recall *B* field at center of circular arc
- Find *B* field for each arc and then add them as vectors

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

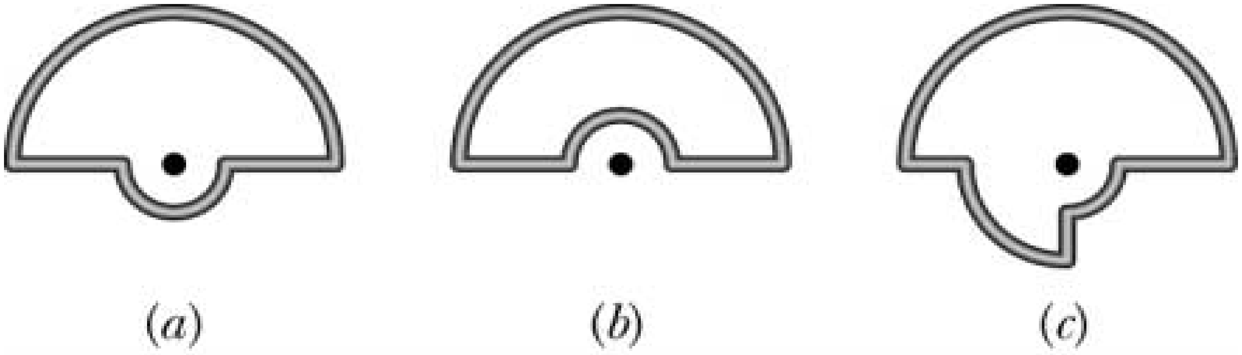


(a) (b) (c)

- All circuits have large upper arc with *B* field

$$B_{upper} = \frac{\mu_0 i \phi}{4\pi R} = \frac{\mu_0 i \pi}{4\pi(3r)} = \frac{\mu_0 i}{12r}$$

B Fields from Currents (21)



$$B = \frac{\mu_0 i \phi}{4\pi R}$$

- Circuits a & b each have small half arc

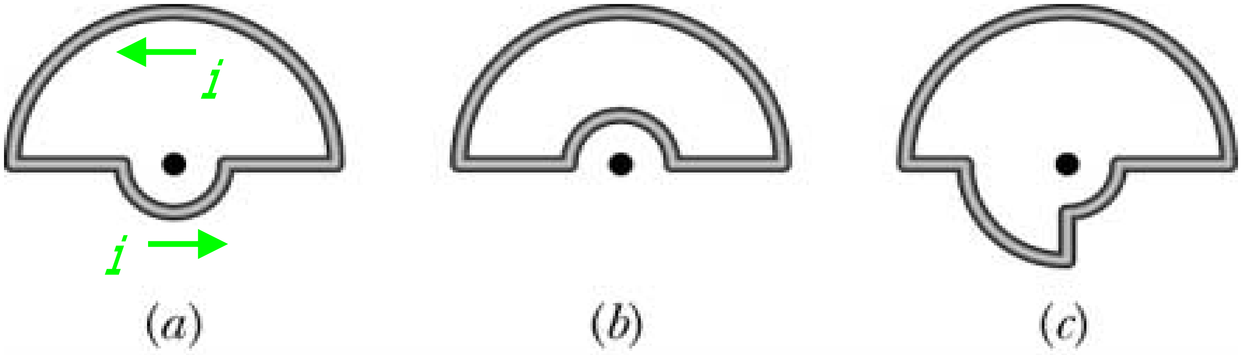
$$B_{small-half} = \frac{\mu_0 i \pi}{4\pi r} = \frac{\mu_0 i}{4r}$$

- Circuit c has a small and medium quarter arc

$$B_{small-quart} = \frac{\mu_0 i(\pi/2)}{4\pi r} = \frac{\mu_0 i}{8r}$$

$$B_{med-quart} = \frac{\mu_0 i(\pi/2)}{4\pi(2r)} = \frac{\mu_0 i}{16r}$$

B Fields from Currents (22)

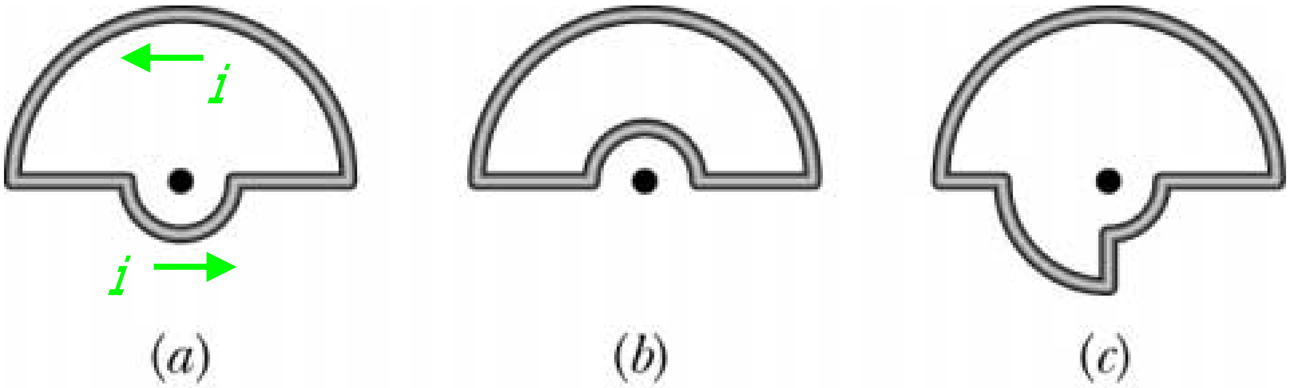


$$B = \frac{\mu_0 i \phi}{4\pi R}$$

- Assume *i* is flowing counterclockwise
- Use right-hand rule to find direction of *B*
- For all large upper arcs *B* field is out of page
- For circuit a
  - Small arc – B field is also out of page so

$$B_a = \frac{\mu_0 i}{12r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{3r}$$

B Fields from Currents (23)

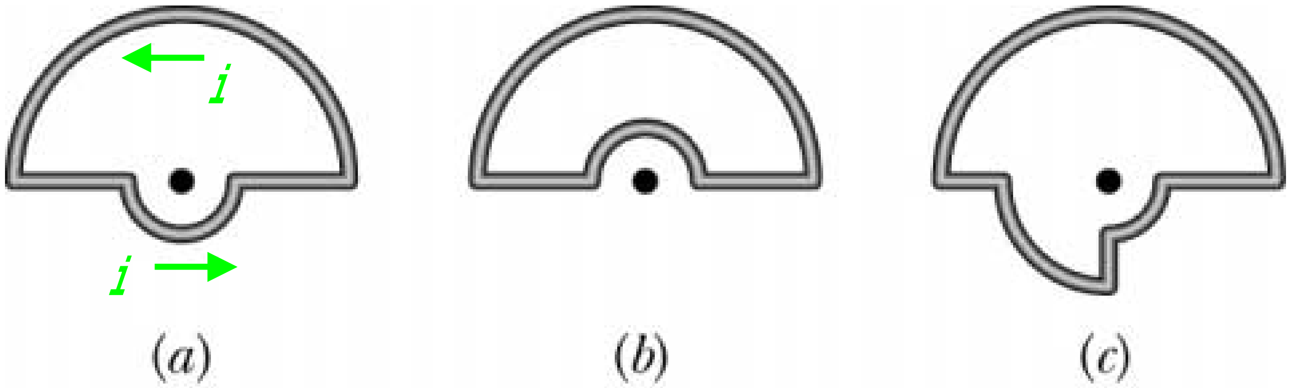


- For circuit b
  - Small arc – B field is into page so

$$B_b = \frac{\mu_0 i}{12r} - \frac{\mu_0 i}{4r} = -\frac{\mu_0 i}{6r}$$

- Negative sign means net *B* field points into page

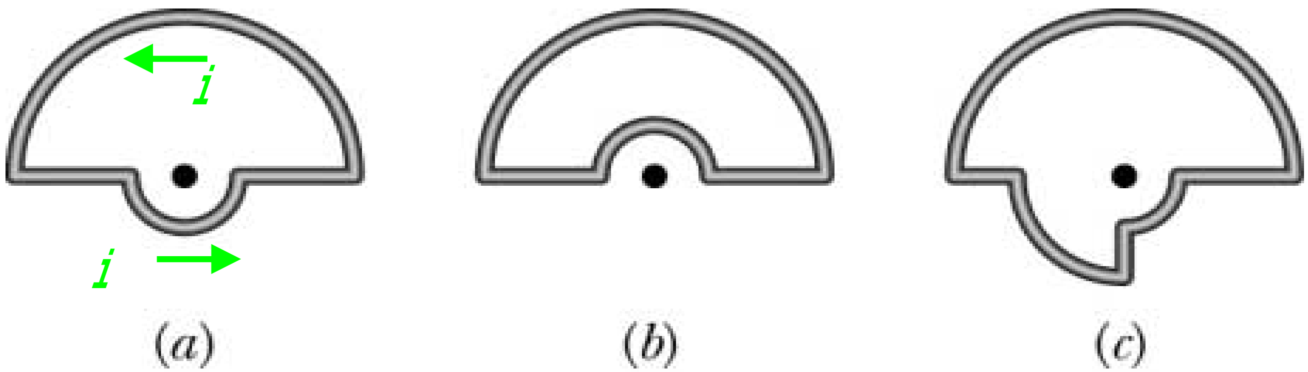
B Fields from Currents (24)



- For circuit c
  - For all arcs – *B* field is out page so

$$B_c = \frac{\mu_0 i}{12r} + \frac{\mu_0 i}{16r} + \frac{\mu_0 i}{8r} = \frac{13\mu_0 i}{48r}$$

B Fields from Currents (25)



- Net *B* field for each circuit is

$$B_a = \frac{\mu_0 i}{3r} \qquad B_b = -\frac{\mu_0 i}{6r} \qquad B_c = \frac{13\mu_0 i}{48r}$$

- Rank magnitude of *B* field, greatest first

a, c, b