Lecture 21

Chapter 30

Magnetic Fields Due to Currents

Review

- Used Biot-Savart law to calculate *B* field due to a distribution of currents

$$dB = \frac{\mu_0}{4\pi} \frac{ids \sin\theta}{r^2}$$

- Add *B* fields as vectors
- Direction of *B* from right-hand rule

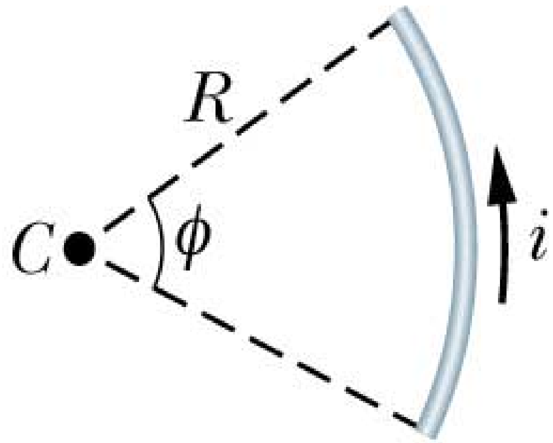
- *B* field for long straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$

B Fields from Currents (26)

- *B* field at the center of a circular arc (ϕ in radians)

$$B = \frac{\mu_0 i \phi}{4\pi R}$$



- For a complete loop (full circle) of wire

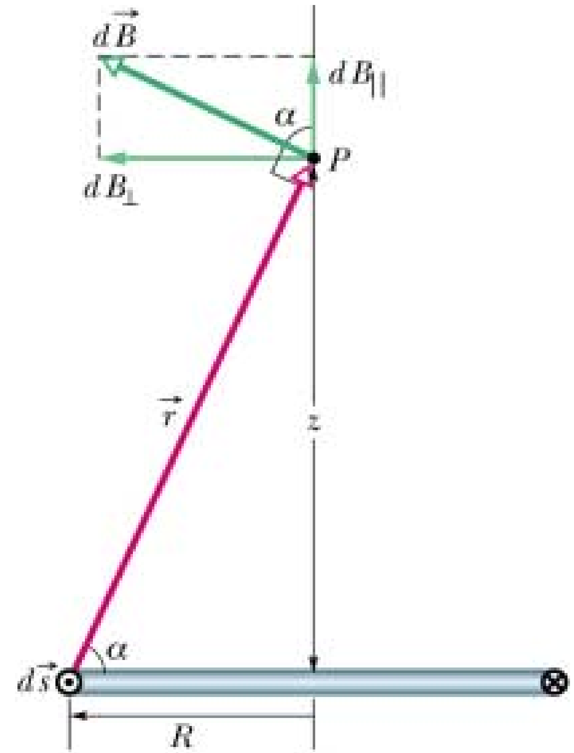
$$\phi = 2\pi$$

$$B = \frac{\mu_0 i 2\pi}{4\pi R} = \frac{\mu_0 i}{2R}$$

B Fields from Currents (27)

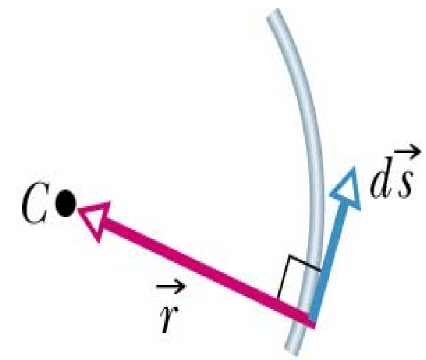
- What is the *B* field from a loop of wire at point P along its central axis?
- Use Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$



- *ds* $\perp$ *r* so sinθ = 1

$$dB = \frac{\mu_0}{4\pi}\frac{ids}{r^2}$$

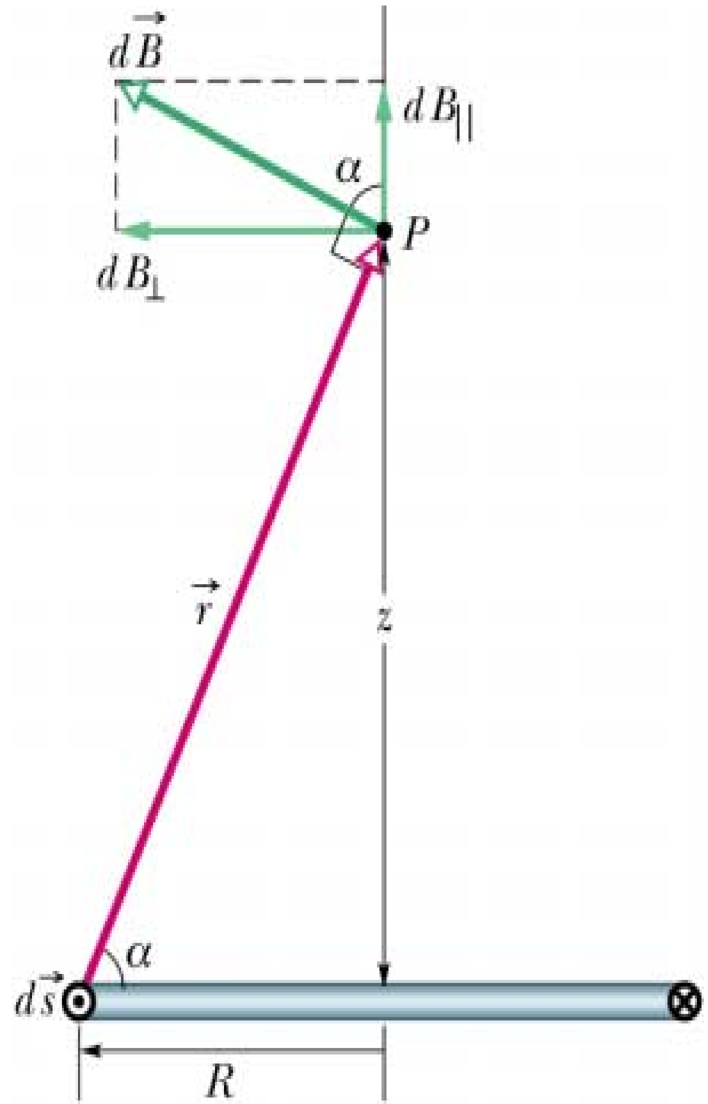


B Fields from Currents (28)

- From cross-product (or using right-hand rule) know that *dB* is ⊥ to plane of *ds* x *r*
- Resolve *dB* into components along the axis of the loop $dB_{||}$ and ⊥ to the loop $dB_{\perp}$

- From symmetry see that vector sum of all $dB_{\perp} = 0$

$$\vec{B}_{\perp} = \int dB_{\perp} = 0$$



B Fields from Currents (29)

- Only need to worry about || components of *B* field

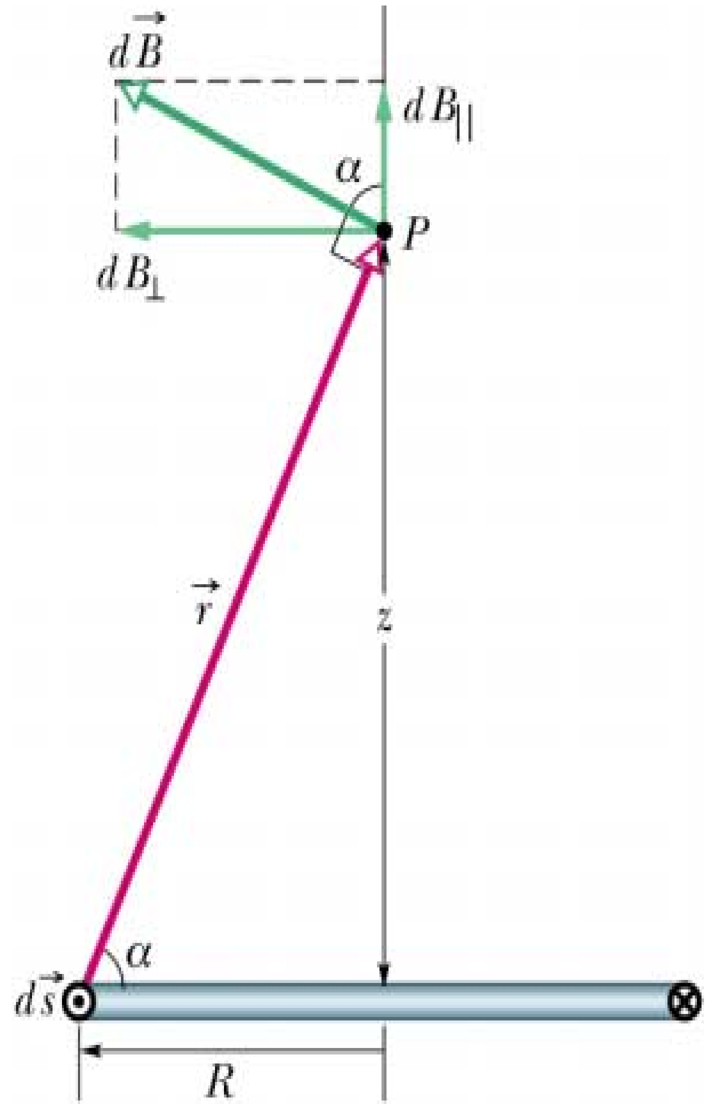
$$B = \int dB_{||}$$

- $dB_{||}$ is related to dB by

$$dB_{||} = dB\cos\alpha$$

- Already found that dB is

$$dB = \frac{\mu_0}{4\pi}\frac{ids}{r^2}$$



B Fields from Currents (30)

- Substituting find

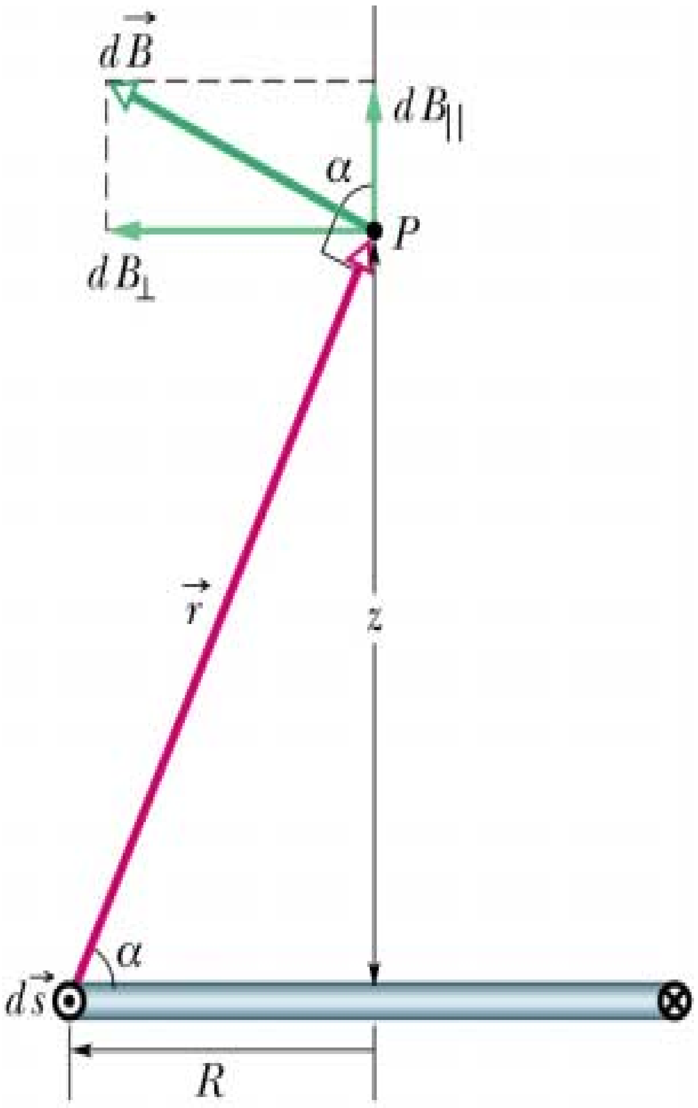
$$dB_{\parallel} = \frac{\mu_0 i ds}{4\pi r^2}\cos\alpha$$

- r and α are not independent but related by

$$r = \sqrt{R^2 + z^2}$$

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}$$

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}} ds$$



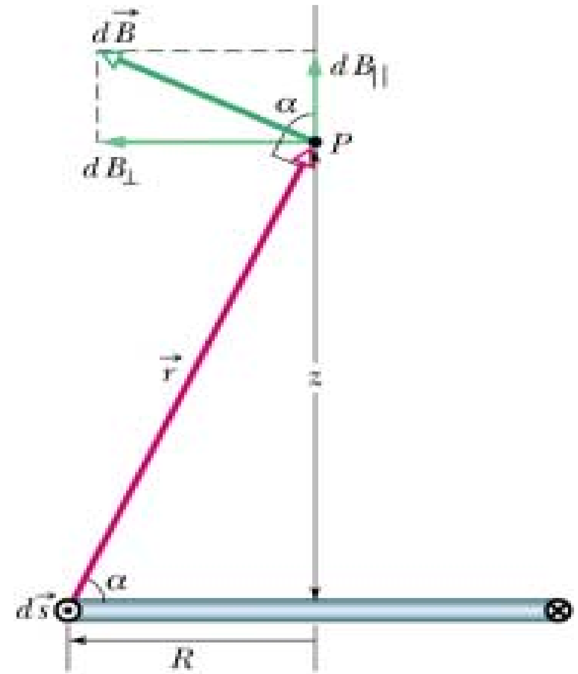
B Fields from Currents (31)

- *R*, *z* are constants so integral becomes

$$B = \int dB_{\|} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \int ds$$

- Integral is circumference of loop or $\int ds \; = \; 2\pi R$

- So $$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$



- At z=0 get same result as before

$$B = \frac{\mu_0 i}{2R}$$

B Fields from Currents (32)

- *B* field for point on central axis of a loop depends on radius of loop, *R*, and distance from loop, *z*

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

- If far away from loop than *z>>R*

$$B(z) = \frac{\mu_0 i R^2}{2z^3}$$

- If have N loops then

$$B(z) = N\frac{\mu_0 i R^2}{2z^3}$$

- Recalling that area of the loop is πR^2 rewrite *B(z)* as

$$B(z) = N\frac{\mu_0 i \pi R^2}{2\pi z^3} = \frac{\mu_0 N i A}{2\pi z^3}$$

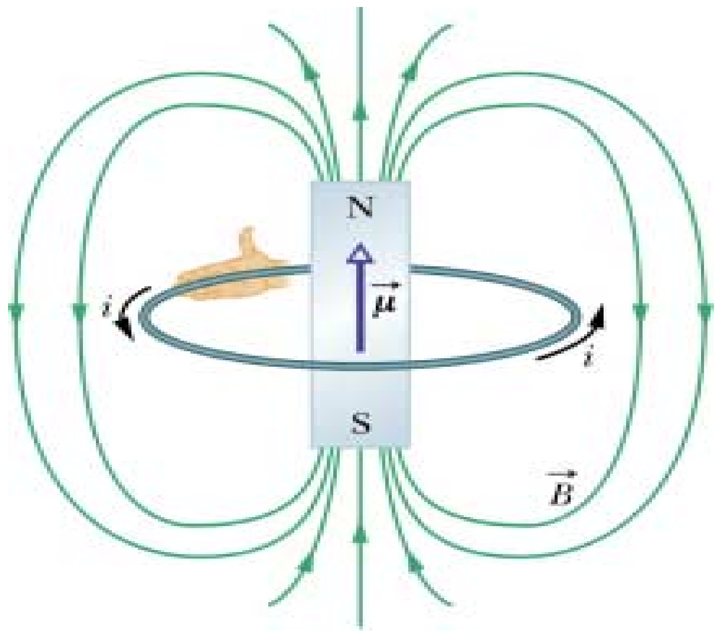
B Fields from Currents (33)

- Recall that magnetic dipole moment, μ, is

$$\boxed{\mu = NiA}$$

- Rewrite *B* field as

$$\boxed{\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}}$$

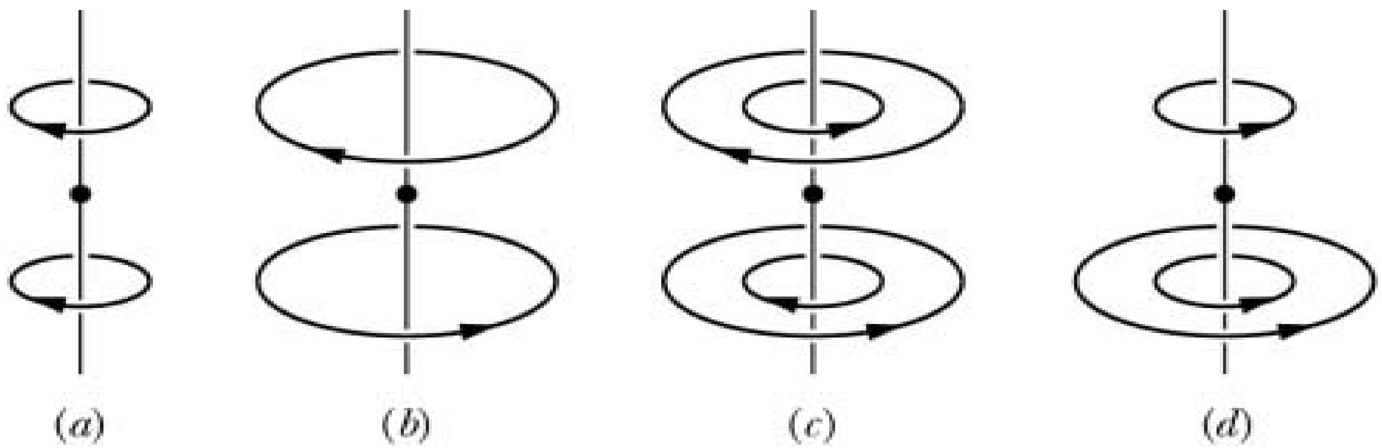


B and μ have same direction

- Current-carrying coil acts as a magnetic dipole
  - Experiences a torque in an external *B* field
  - Generates its own intrinsic *B* field

B Fields from Currents (34)

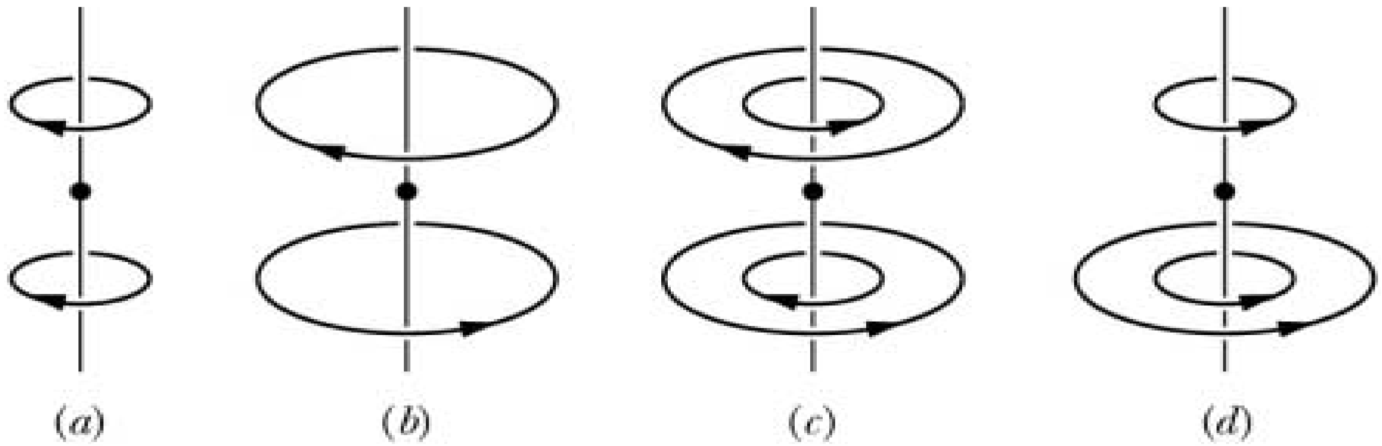
- Checkpoint #4 – 4 arrangements of circular loops of radius 2 and 2r, all carrying identical currents. Rank magnitude of net *B* field at midpoint (the dot), greatest first



  - What quantities are constants?

Current and distance, z

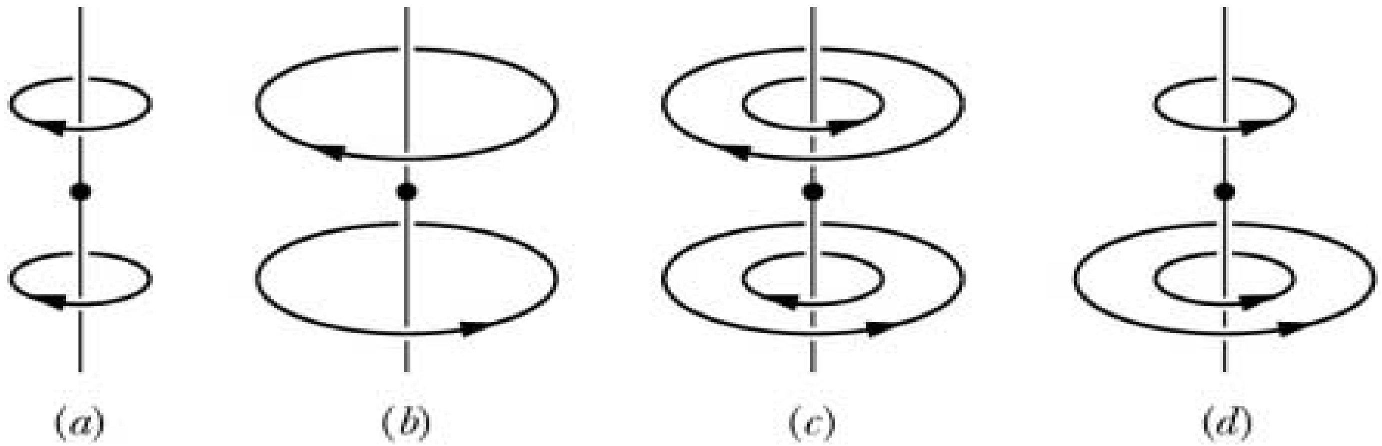
B Fields from Currents (35)



- Recall *B* field for loop $$B(z) = \frac{\mu_0 i R^2}{2z^3}$$
- So $$B(z) \propto R^2$$
- What else do we have to worry about?

Direction of *B* field for each loop

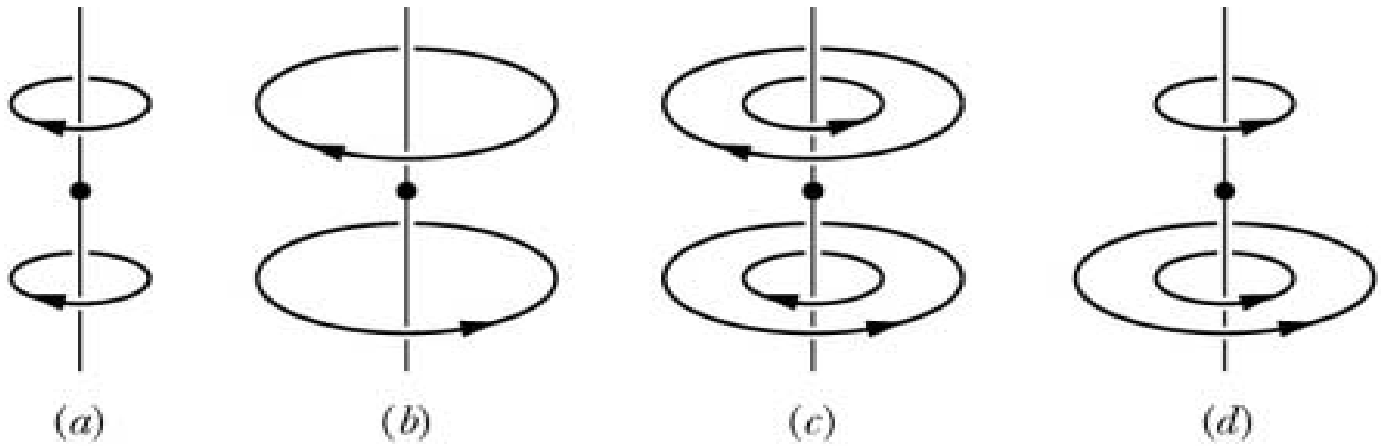
B Fields from Currents (35)



- What is the direction of *B* field for a?
- Use right-hand rule
  - Curl fingers in direction of current in loop
  - Thumb points in direction of *B* field
- *B* field points down for both
  in diagram a so *B* s add

$$B(z) \propto 2r^2$$

B Fields from Currents (36)



- What is the direction of *B* field for b?

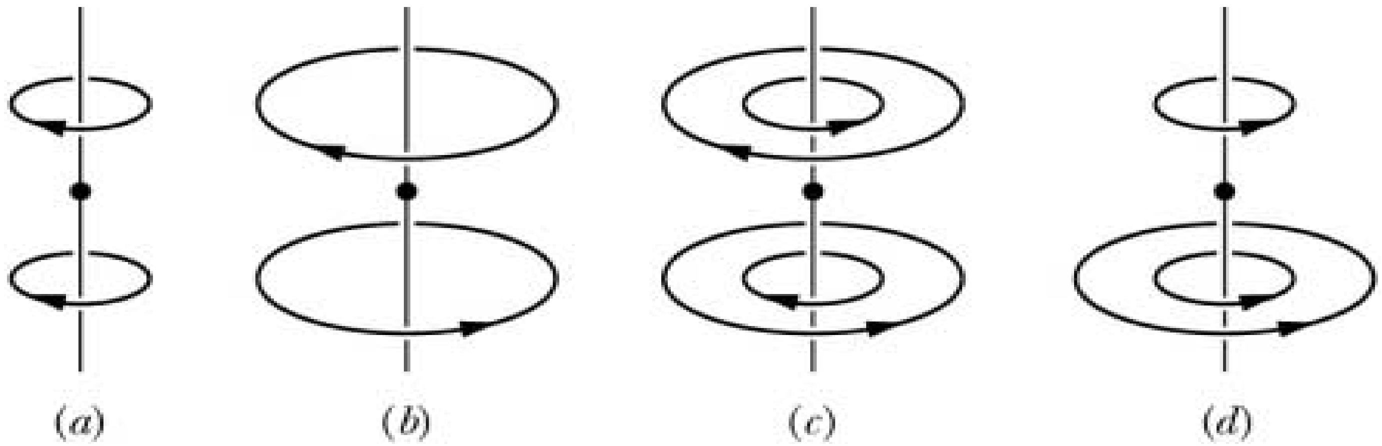
  Top loop gives B field down

  Bottom loop gives B field up

- Add *B* fields as vectors so

$$\boxed{B(z)=0}$$

B Fields from Currents (37)



- What are *B* fields for c and d?
- For c and b $B(z) = 0$

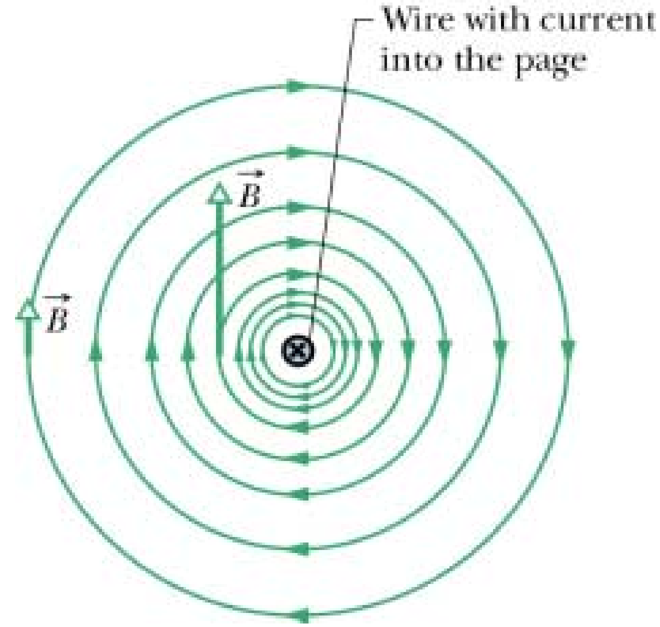
d, a, then b & c tie

- For d $B(z) \propto 2r^2 + (2r)^2 = 6r^2$
- Remember for a $B(z) \propto 2r^2$

B Fields from Currents (38)

- Shown wire with a current produces a *B* field

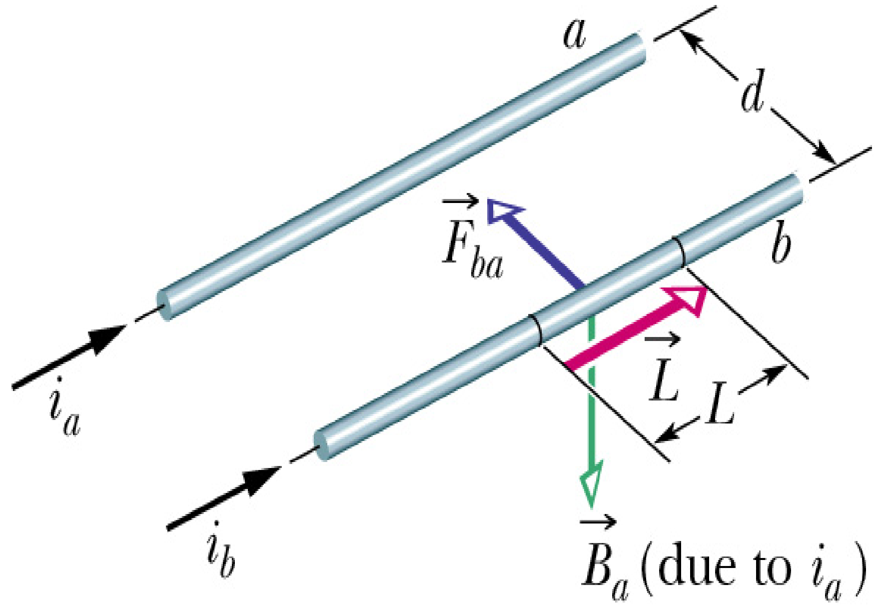
$$B = \frac{\mu_0 i}{2\pi R}$$



- What happens if we bring 2 wires, each carrying a current, near each other?
- Will it matter if the currents are in the same direction or opposite each other?

B Fields from Currents (39)

- What is the force, F_{ba}, on wire b due to the current in wire a?
- Wires are separated by a distance, *d*, and have currents, i_a and i_b



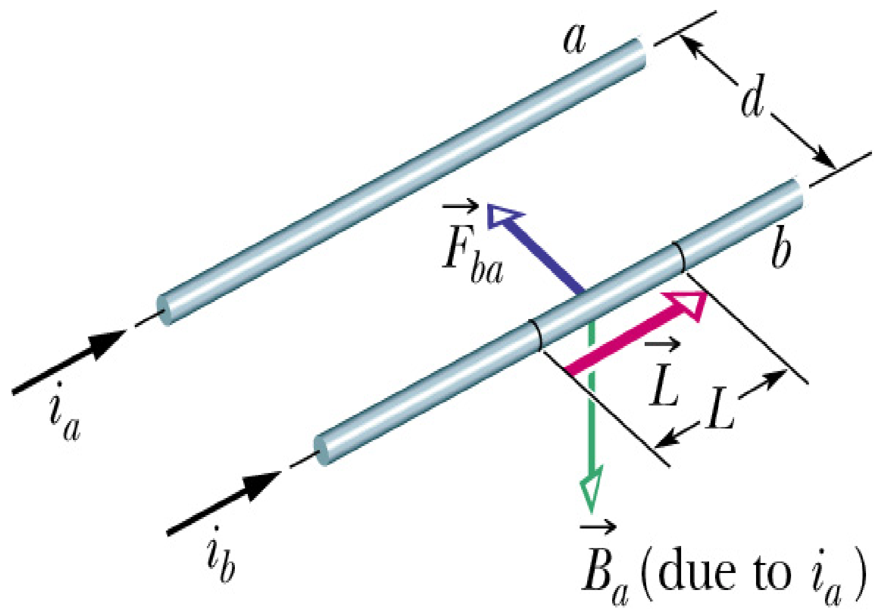
- First calculate *B* field from wire a at the site of wire b (ignoring wire b)

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

B Fields from Currents (40)

- Using right-hand rule find that the B_a field is directed down

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$



- Now calculate force, F_{ba}, on wire b using

$$F_{ba} = i_b \vec{L} \times \vec{B}_a$$

- Where the current, i_b, is current in wire b
- B field is from wire a
- L is length of wire b

B Fields from Currents (41)

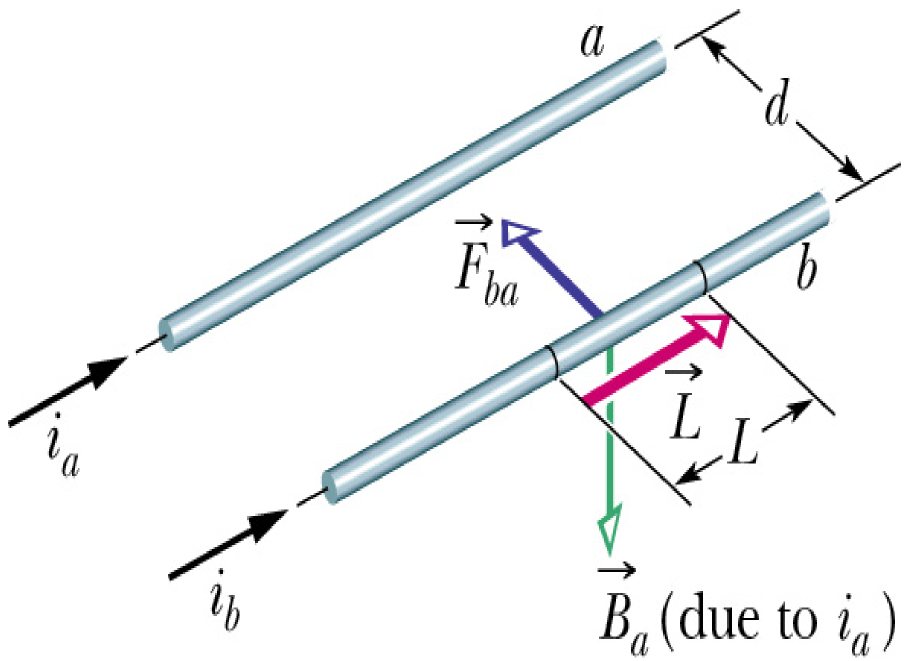
- Current, i_b , and B_a are ⊥ to each other

$$F_{ba} = i_b L B_a \sin 90$$

- Substituting B_a

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

$$F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}$$



- Applying right-hand rule, direction of F_{ba} is towards wire a

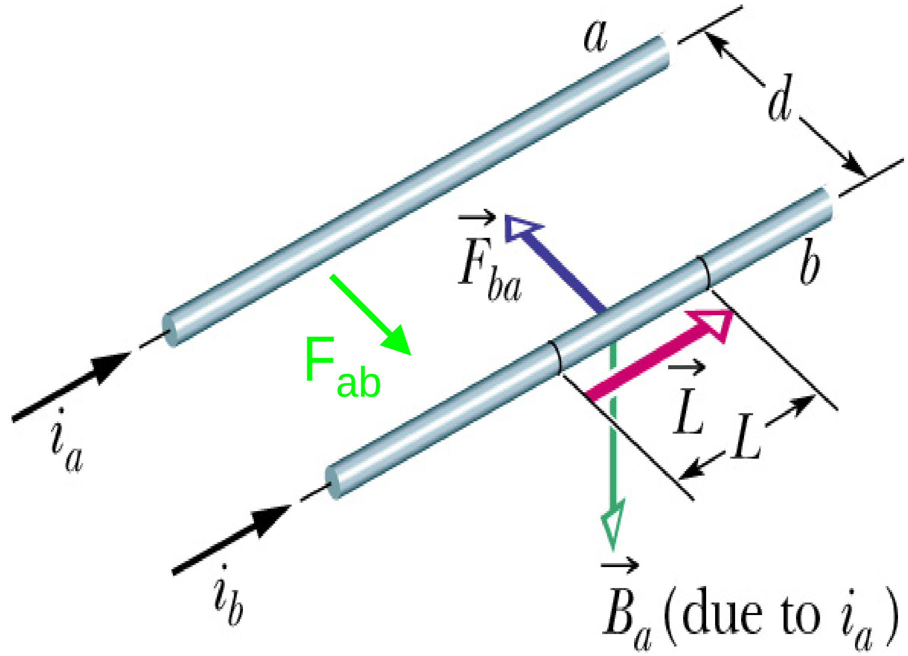
B Fields from Currents (42)

- What is the force, F_{ab}, on wire a due to the current in wire b?
- Calculate B field from wire b at site of wire a

$$B_b = \frac{\mu_0 i_b}{2\pi d}$$

- Force on a from b is

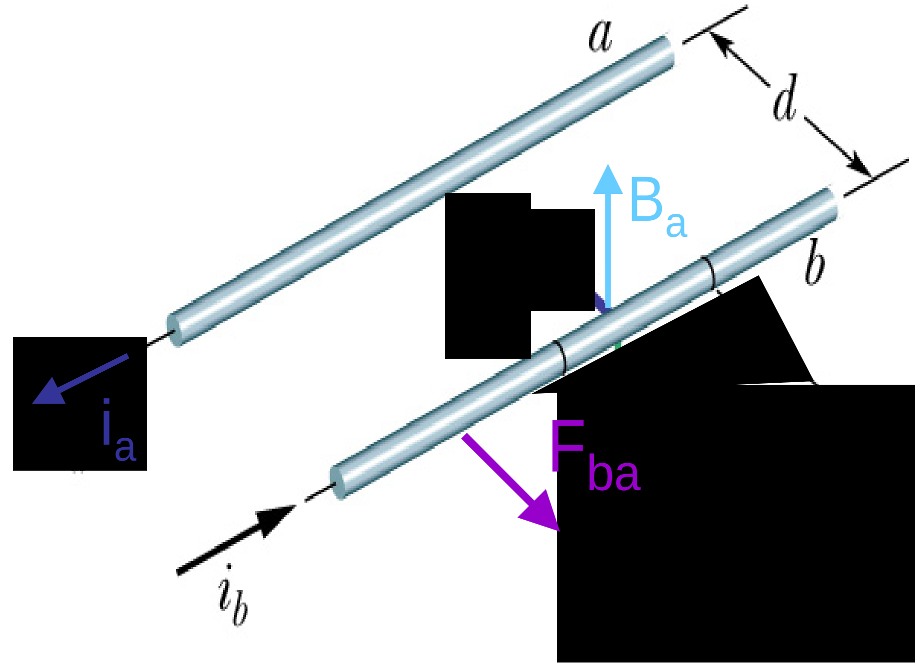
$$F_{ab} = i_a L B_b = \frac{\mu_0 L i_a i_b}{2\pi d}$$



- Apply right-hand rule and find, F_{ab}, has same magnitude as F_{ba} but opposite direction

B Fields from Currents (43)

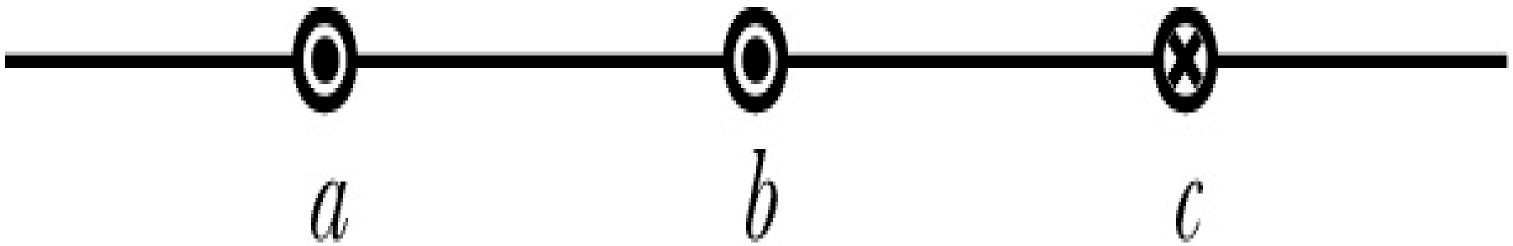
- What happens if we flip current in wire a its moving opposite to current in wire b?



- Use right-hand rules
- B_a points up
- F_{ba} points away from wire a

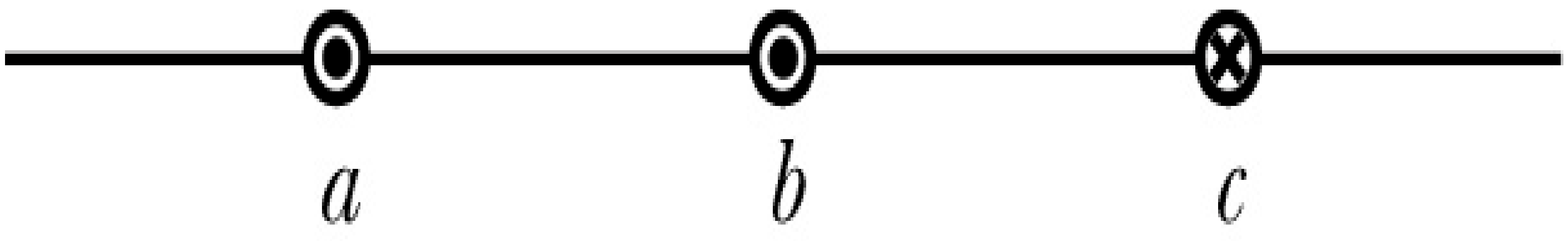
- Parallel currents attract, anti-parallel currents repel

B Fields from Currents (44)



- Checkpoint #2 – Three long, straight, parallel wires are equally spaced with identical currents, either into or out of page. Rank the wires according to the magnitude of the force on each wire due to the currents in the other two wires, greatest first.

B Fields from Currents (45)



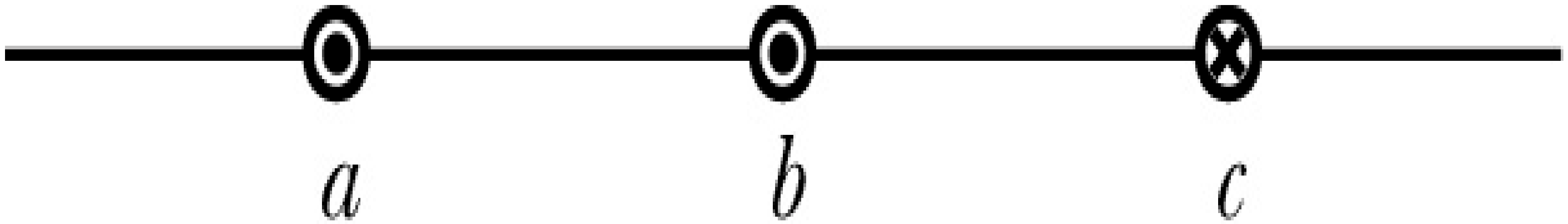
- What's the force on wire a due to wires b and c?
- First find net *B* field at a from wires b and c
  - Calculate magnitude for B_b and B_c
  - Use right-hand rule to find direction B_b and B_c
  - Add B_b and B_c as vectors to get B_{bc}

$$B = \frac{\mu_0 i}{2\pi d}$$

- Find force on wire a with

$$F_{abc} = i_a \vec{L} \times \vec{B}_{bc}$$

B Fields from Currents (46)



- What's the force on wire a due to wires b and c?
- B_b at a is down
- B_c at a is up

$$B_b = \frac{\mu_0 i_b}{2\pi d} \qquad B_c = \frac{\mu_0 i_c}{2\pi(2d)}$$

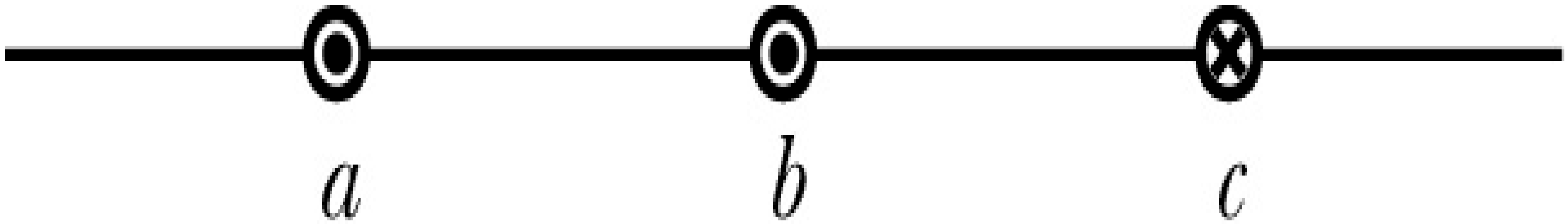
- Net *B* field is

$$B_{bc} = \frac{\mu_o i}{2\pi d} - \frac{\mu_0 i}{4\pi d} = \frac{\mu_0 i}{4\pi d}$$

- Force is

$$F_{abc} = i_a L B_{bc} = \frac{\mu_0 i^2 L}{4\pi d}$$

B Fields from Currents (47)



- What's the force on wire b due to wires a and c?
- B_a at a is up
- B_c at a is up

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

$$B_c = \frac{\mu_0 i_c}{2\pi d}$$

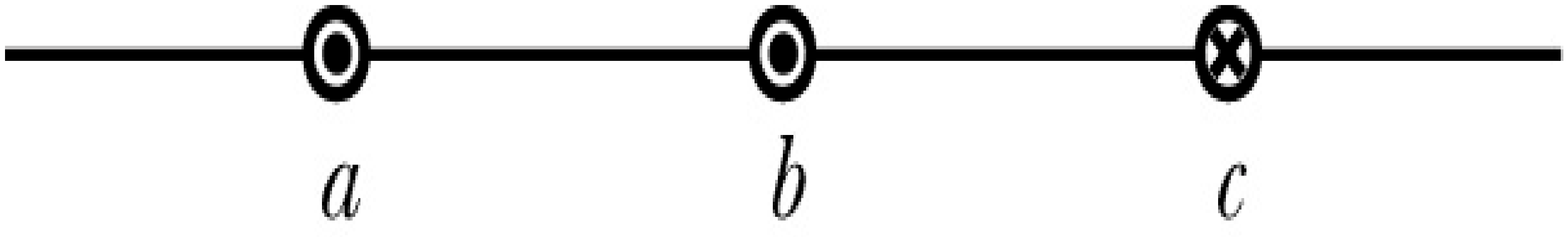
- Net *B* field is

$$B_{ac} = \frac{\mu_o i}{2\pi d} + \frac{\mu_0 i}{2\pi d} = \frac{\mu_0 i}{\pi d}$$

- Force is

$$F_{bac} = i_b L B_{ac} = \frac{\mu_0 i^2 L}{\pi d}$$

B Fields from Currents (48)



- What's the force on wire c due to wires a and b?
- B_a at a is up
- B_b at a is up

$$B_a = \frac{\mu_0 i_a}{2\pi(2d)} \qquad B_b = \frac{\mu_0 i_b}{2\pi d}$$

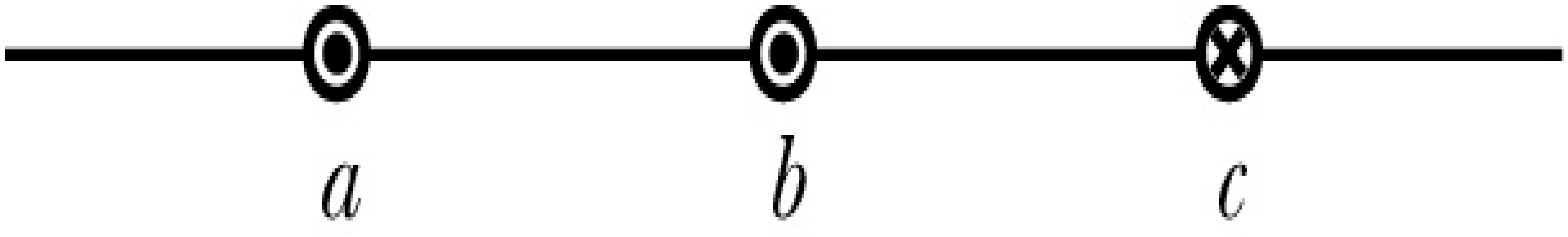
- Net B field is

$$B_{ab} = \frac{\mu_o i}{2\pi d} + \frac{\mu_0 i}{4\pi d} = \frac{3\mu_0 i}{4\pi d}$$

- Force is

$$F_{cab} = i_c L B_{ab} = \frac{3\mu_0 i^2 L}{4\pi d}$$

B Fields from Currents (49)



- Forces on each wire due to other two are:

$$F_{abc} = i_a L B_{bc} = \frac{\mu_0 i^2 L}{4\pi d}$$

$$F_{bac} = i_b L B_{ac} = \frac{\mu_0 i^2 L}{\pi d}$$

$$F_{cab} = i_c L B_{ab} = \frac{3\mu_0 i^2 L}{4\pi d}$$

b, c, a