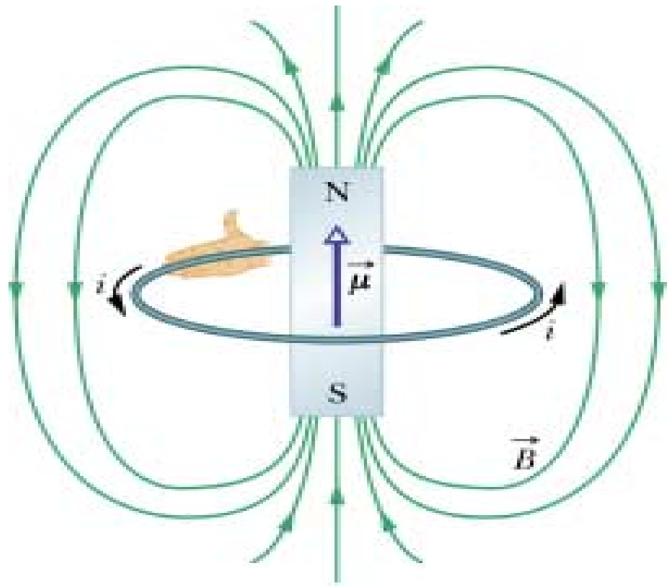
Lecture 22

Chapter 30

Magnetic Fields Due to Currents

Review

- Used Biot-Savart law to calculate *B* field from a loop at a point along the z axis



$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

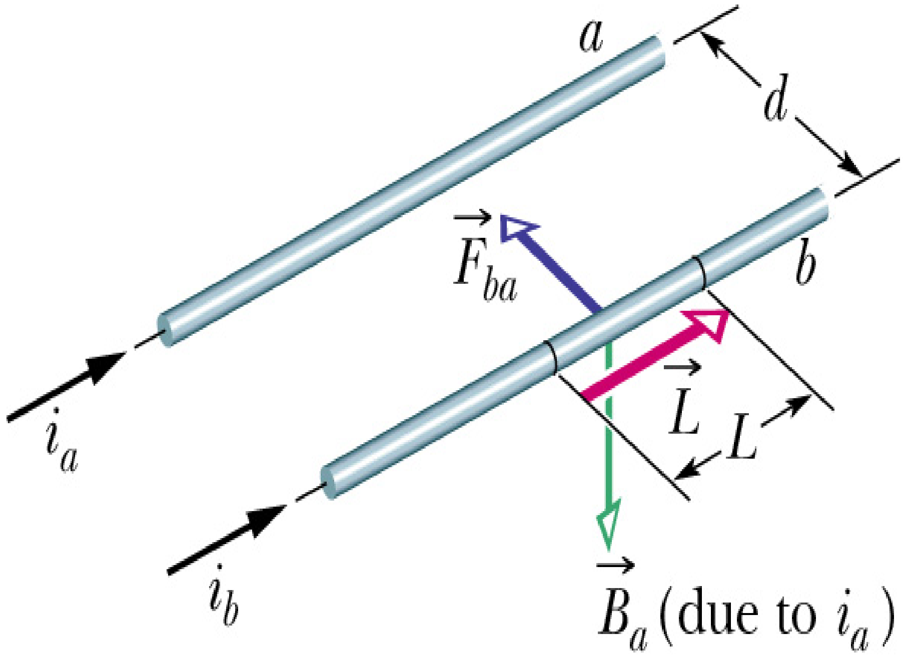
- At point z>>R, rewrite

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$

- Current-carrying coil acts as a magnetic dipole
  - Experiences a torque in an external *B* field
  - Generates its own intrinsic *B* field

Review

- Current carrying wires will exert a force on one another
- Calculate *B* field from wire a at site of wire b



$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

- Force on b from a is

$$F_{ba} = i_b L B_a = \frac{\mu_0 L i_a i_b}{2\pi d}$$

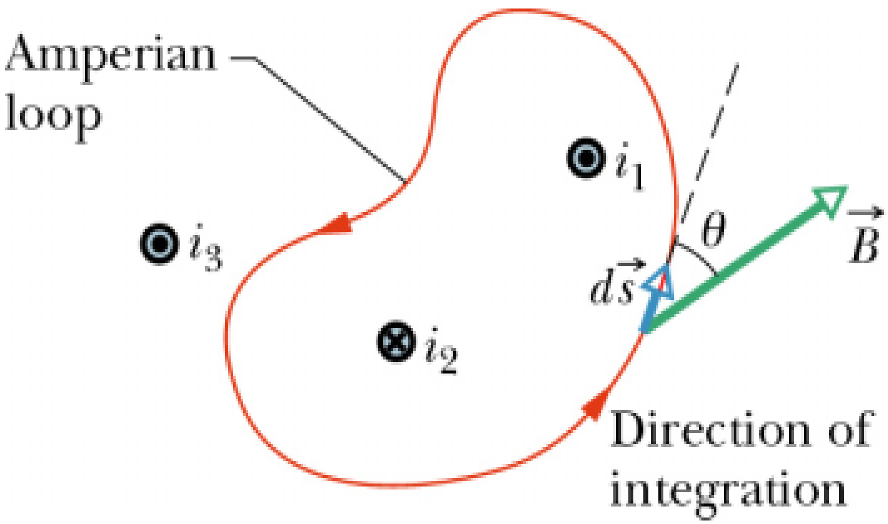
- Parallel currents attract, anti-parallel currents repel

B Fields from Currents (50)

- For certain symmetric distributions of charge able to use Gauss' law to calculate *E* field

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- For symmetric distributions of charge use Ampere's law to calculate *B* field

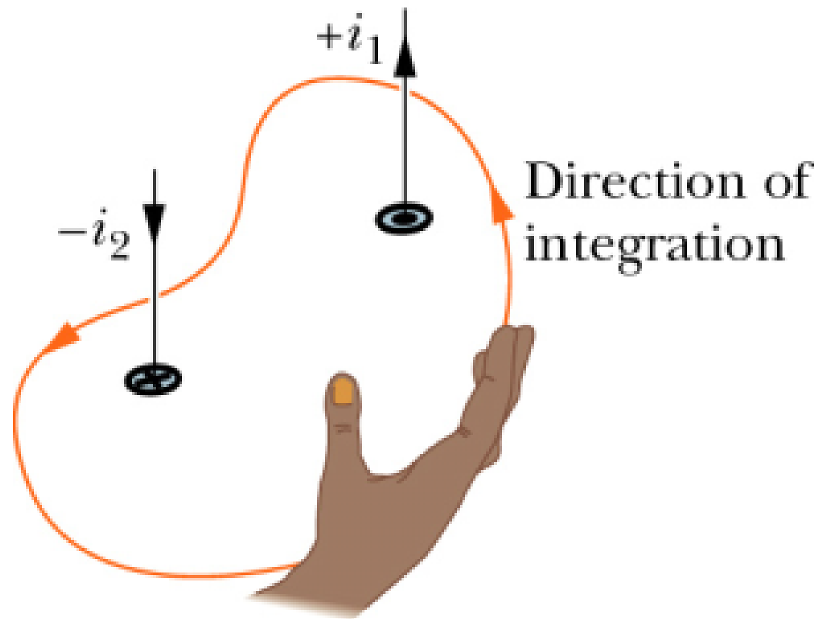


- Integrate around closed loop called Amperian loop

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

B Fields from Currents (51)

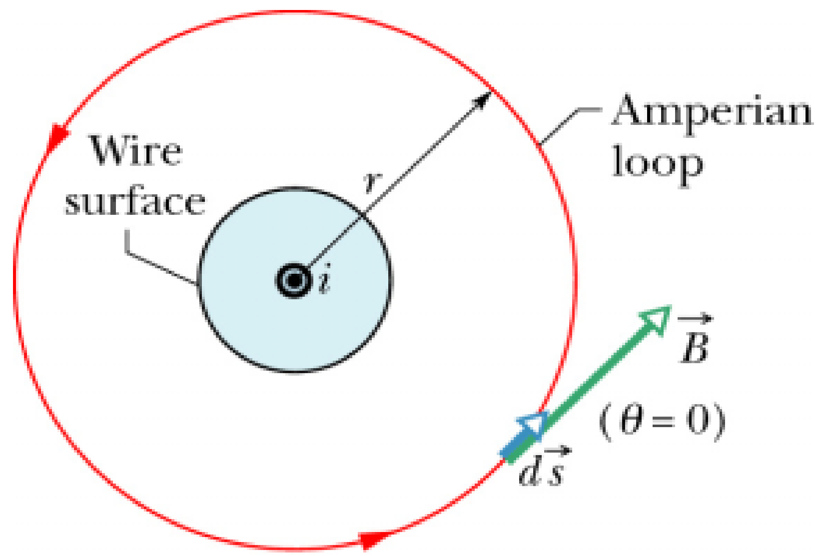
- Use the right-hand rule to determine the signs for the currents encircled by the Amperian loop
- Curl right hand around Amperian loop with fingers pointing in direction of integration
- Current going through loop in general (opposite) direction of thumb is positive (negative)



B Fields from Currents (52)

- Use Ampere's law to calculate *B* field from long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



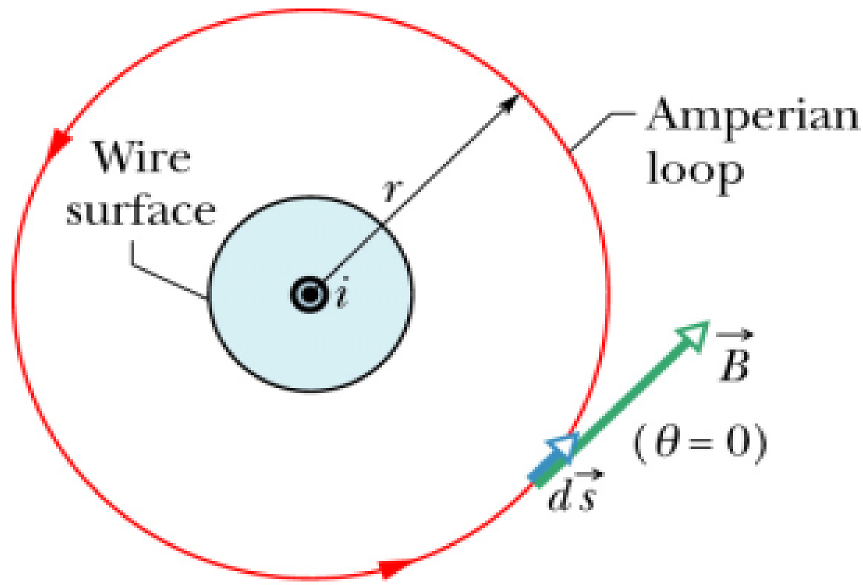
- Draw Amperian loop as a circle surrounding the wire
- Remind you of the magnetic field lines
- At every point of the loop
  - Magnitude of *B* is constant
  - *B* and *ds* are tangent

B Fields from Currents (53)

- *B* and *ds* are || so

$$\cos\theta = \cos 0 = 1$$

$$\vec{B} \bullet d\vec{s} = B\,ds$$



- *B* constant on loop so

$$\oint \vec{B} \bullet d\vec{s} = B \oint ds$$

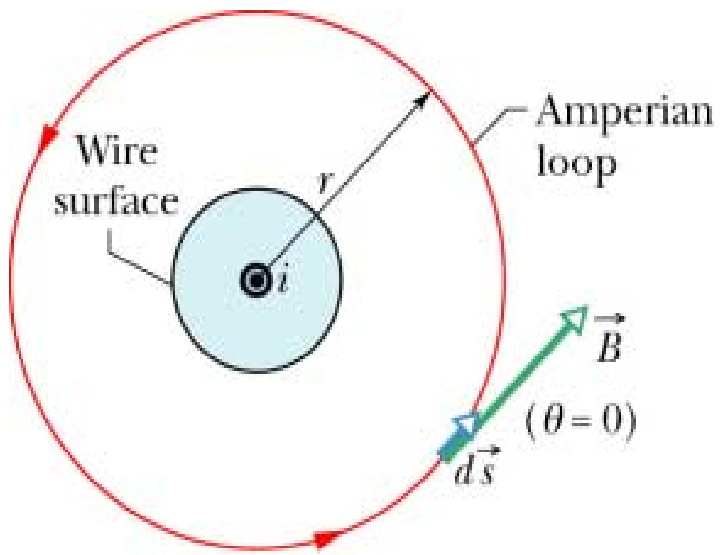
$$\oint ds = 2\pi r$$

$$\oint \vec{B} \bullet d\vec{s} = B(2\pi r)$$

B Fields from Currents (54)

- Ampere's law becomes

$$B(2\pi r) = \mu_0 i_{enc}$$



- Current enclosed is just *i* so

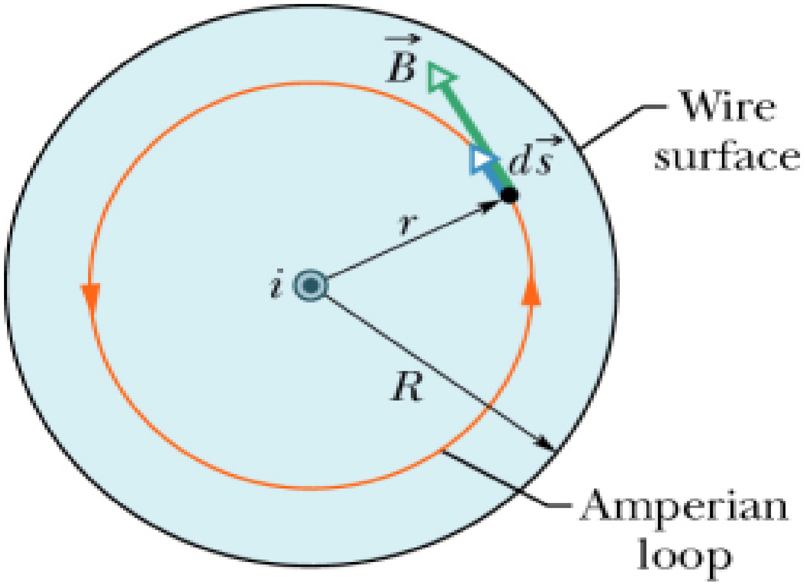
$$B = \frac{\mu_0 i}{2\pi r}$$

- Same result as with Biot-Savart law

B Fields from Currents (55)

- Calculate *B* field inside a long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



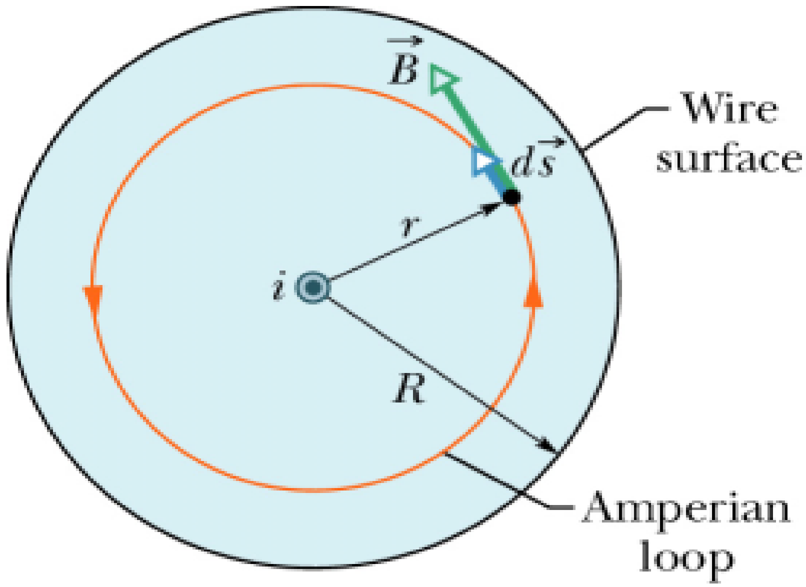
- Again *B* and *ds* are || and *B* is a constant so

$$\oint \vec{B} \bullet d\vec{s} = B\oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i_{enc}$$

B Fields from Currents (56)

- Need to find i_{enc}
- Current is uniformly distributed so i enclosed by loop is α to area enclosed



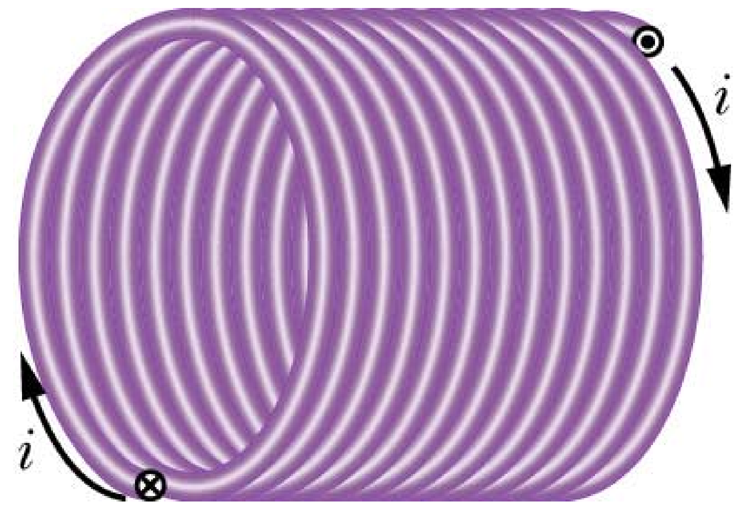
$$i_{enc} = i\frac{\pi r^2}{\pi R^2}$$

$$B(2\pi r) = \mu_0 i\frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right)r$$

B Fields from Currents (57)

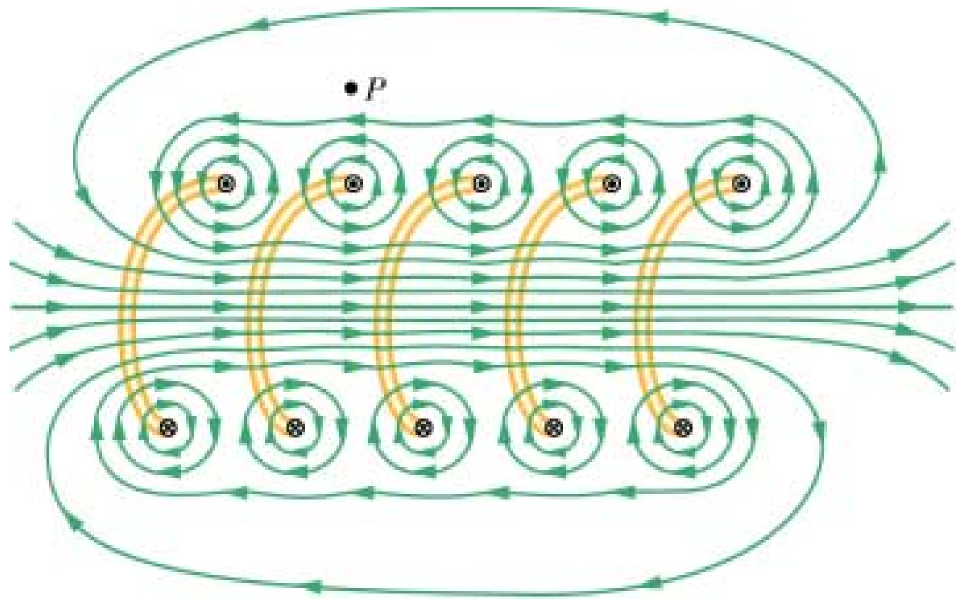
- What happens if there are several loops of wire put together?
- A long, tightly wound helical coil of wire is called a solenoid
- Bend solenoid so ends meet to make a hollow donut gives a toroid



- Use Ampere's law to calculate *B* field for a solenoid and a toroid

B Fields from Currents (58)

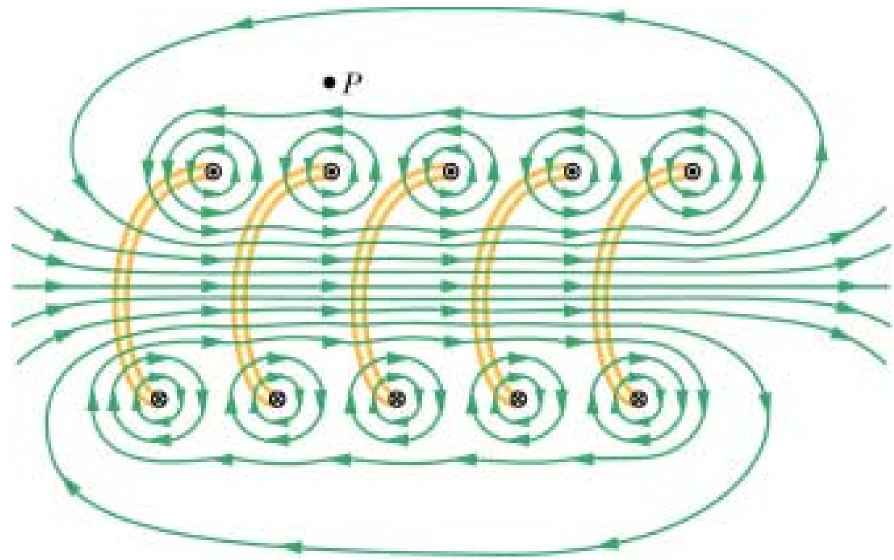
- Solenoid's *B* field is vector sum of fields produced by each turn (loop) in solenoid
- Near loop acts as infinite straight wire
- Between the loops fields tend to cancel
- Inside the solenoid, far from the wire, *B* field is parallel to axis



- An ideal solenoid
  - is infinity long with closely packed turns of wire
  - has uniform *B* field which is parallel to solenoid axis

B Fields from Currents (59)

- For points outside the solenoid *B* fields from the upper parts of the turns tend to cancel the lower
- Ideal solenoid $B_{outside}=0$
- For a real solenoid can assume $B_{outside}=0$ if
  - length >> diameter
  - Only consider points not near ends of solenoid

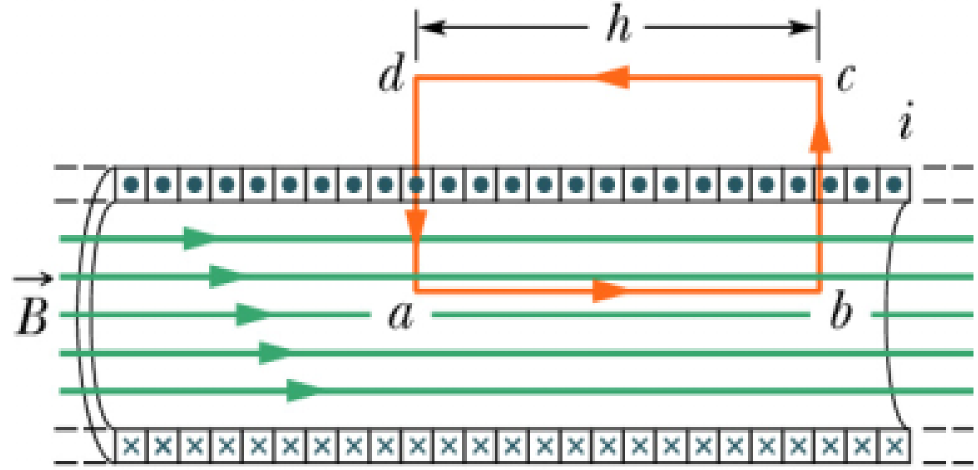


- Use right-hand rule to find direction of *B* field
  - Grasp solenoid so fingers follow direction of *i* in loops, thumb points in *B*

B Fields from Currents (60)

- Use Ampere's law to calculate *B* field of ideal solenoid

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



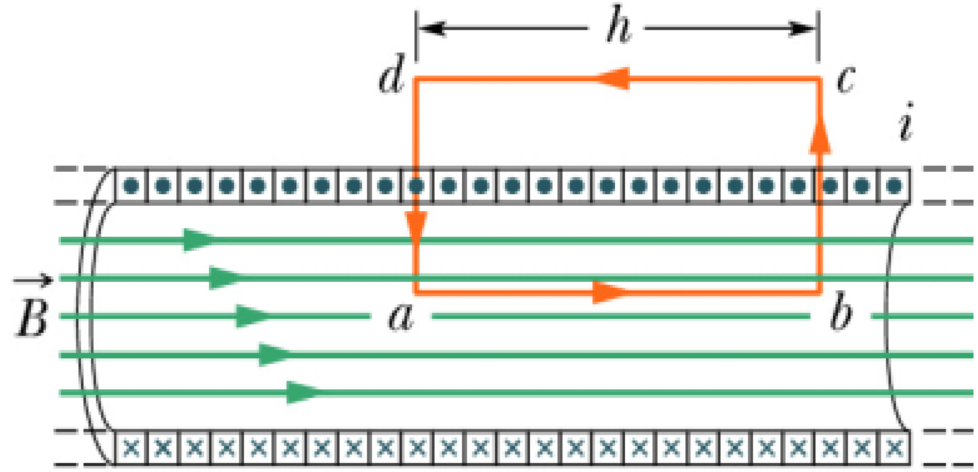
- Draw Amperian loop abcda intersecting solenoid
- Integral can be written as sum of 4 integrals, one for each side

$$\oint \vec{B} \bullet d\vec{s} = \int_a^b \vec{B} \bullet d\vec{s} + \int_b^c \vec{B} \bullet d\vec{s} + \int_c^d \vec{B} \bullet d\vec{s} + \int_d^a \vec{B} \bullet d\vec{s}$$

B Fields from Currents (61)

- First integral *B* field is || to *ds*

$$\int_a^b \vec{B} \bullet d\vec{s} = B\left[s\right]_a^b = Bh$$



- For sides *bc* and *da* *B* is ⊥ to *ds* so

$$\int_b^c \vec{B} \bullet d\vec{s} = \int_d^a \vec{B} \bullet d\vec{s} = 0$$

- For the length outside the solenoid B = 0

$$\int_c^d \vec{B} \bullet d\vec{s} = 0$$

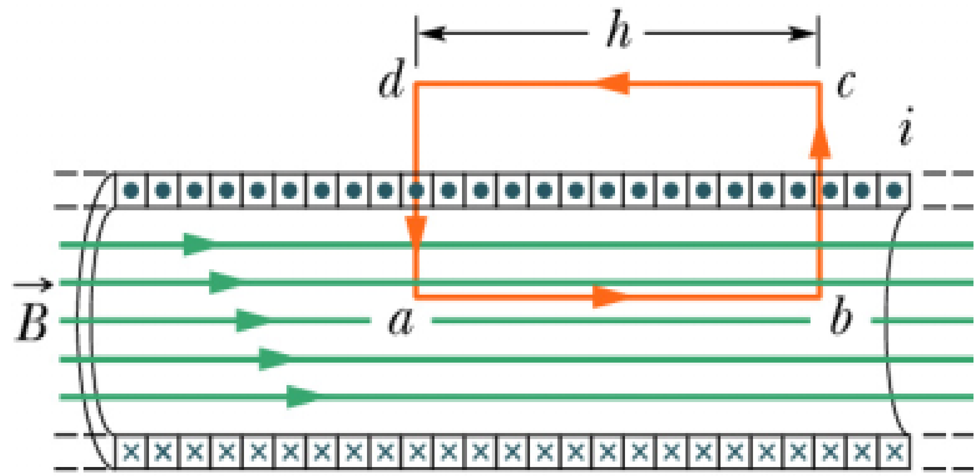
$$\oint \vec{B} \bullet d\vec{s} = Bh$$

B Fields from Currents (62)

- Now need to find amount of current enclosed

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Single coil has current *i*
- But Amperian loop encloses several coils so total current is
- *n* is the number of turns per unit length



$$i_{enc} = inh$$

$$n = \frac{N}{L}$$

- N = total # of turns
- L = length

B Fields from Currents (63)

- Substituting into Ampere's law

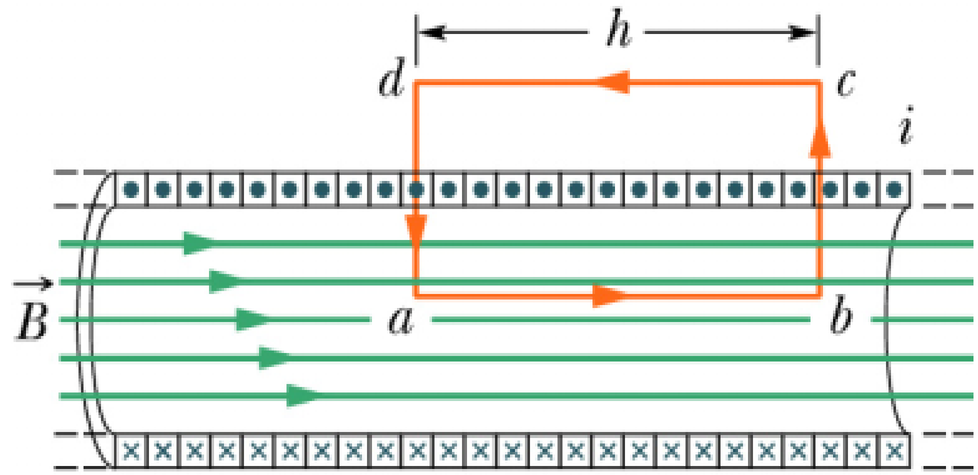
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$Bh = inh$$

- For ideal solenoid:

$$B = \mu_0 in$$

- n is # turns/length



- *B* field of solenoid
  - does not depend on diameter or length of solenoid
  - is uniform over its cross section

B Fields from Currents (64)

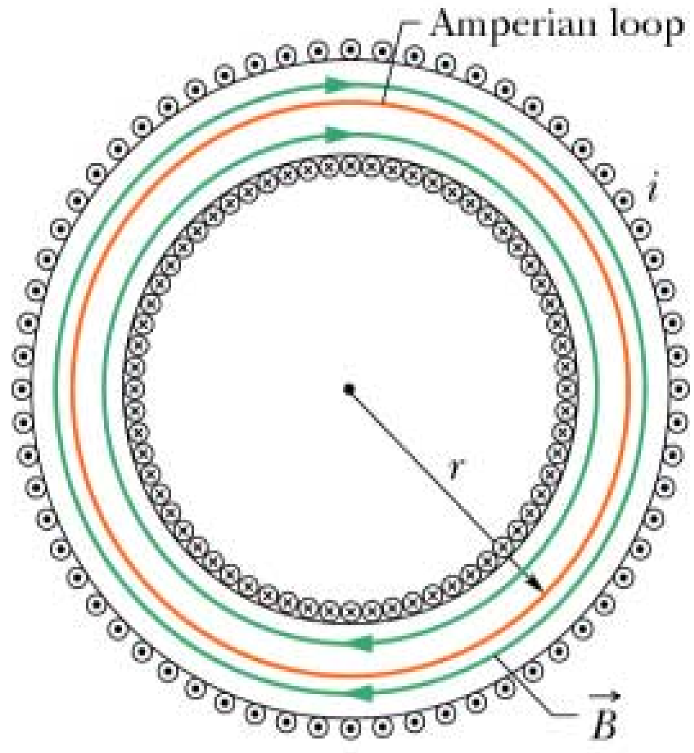
- Calculate *B* field for a toroid using Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Choose Amperian loop to be a concentric circle inside toroid
- *B* and *ds* are parallel along entire loop so

$$\oint \vec{B} \bullet d\vec{s} = B\int ds = B(2\pi r)$$

(a)



(b)

B Fields from Currents (65)

- Current enclosed by loop is

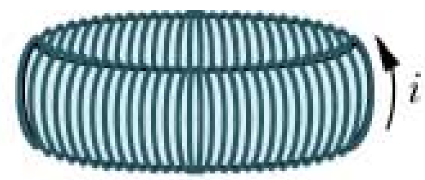
$$i_{enc} = iN$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

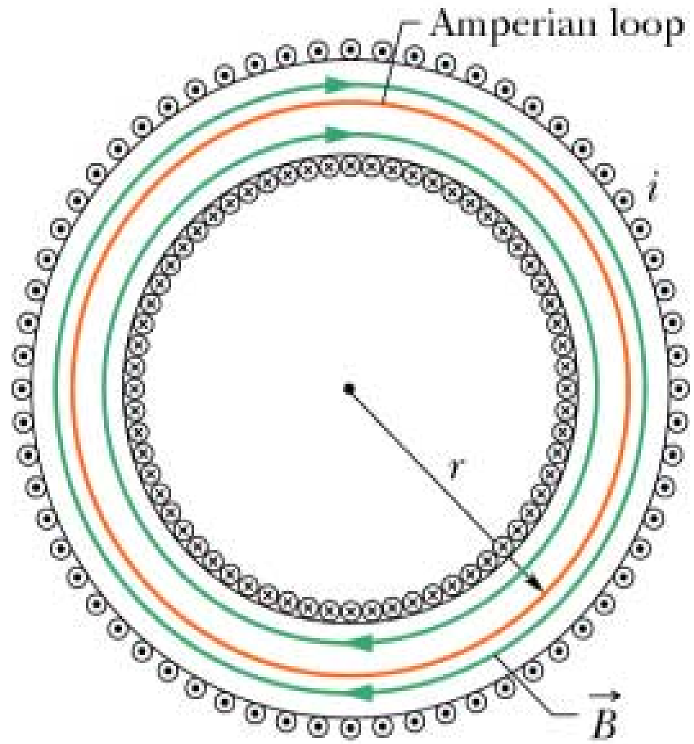
$$B(2\pi r) = \mu_0 iN$$

- *B* field for toroid is

$$B = \frac{\mu_0 iN}{2\pi r}$$



(a)



(b)

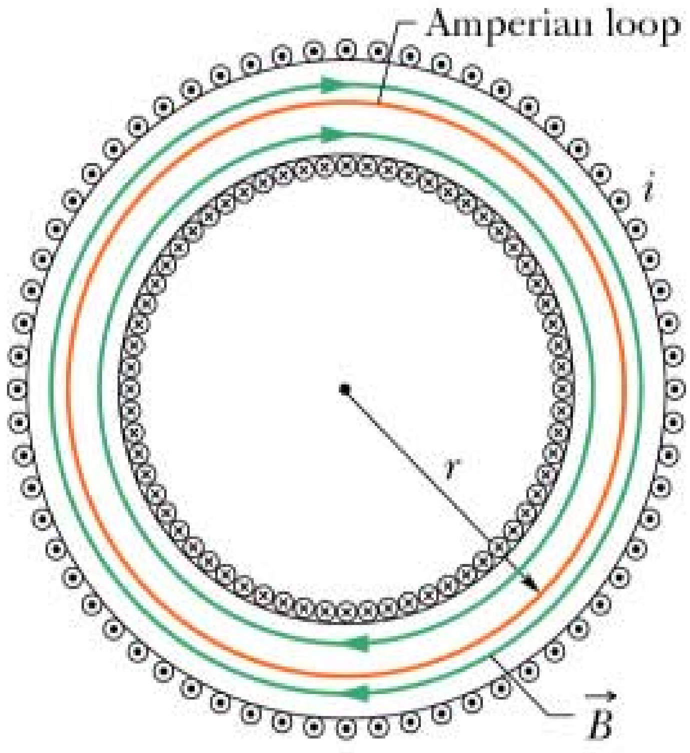
B Fields from Currents (66)

- Toroid – *B* field is not constant over its cross section

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

- N = total # of turns
- Use right-hand rule to find direction of *B* field
  - Grasp toroid with fingers in direction of current in windings, thumb points in B
- *B* = 0 outside toroid

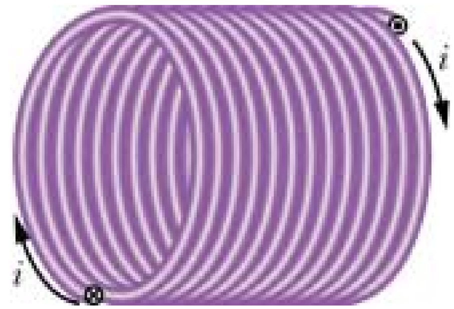
(*a*)



(*b*)

B Fields from Currents (67)

- Solenoids are practical way to setup a known uniform *B* field
  - Like parallel plate capacitor to generate known uniform *E* field
- Many everyday devices use solenoids
- Example – Tevatron at Fermilab

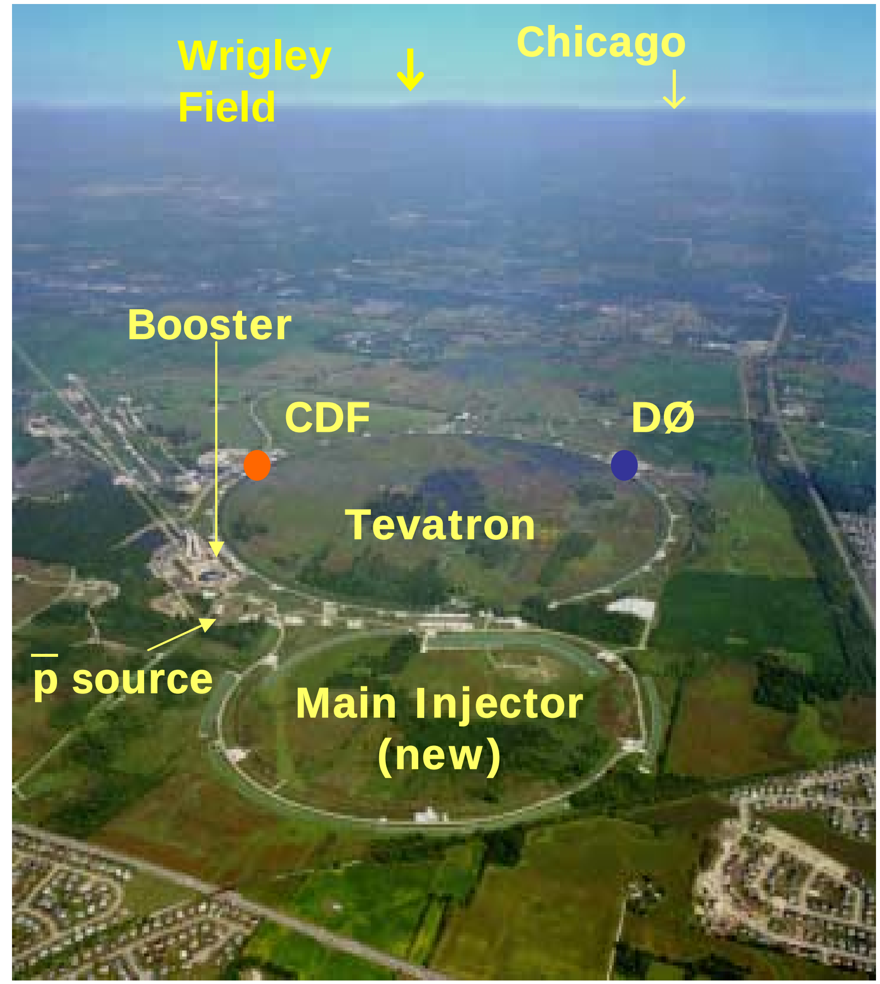


$$B = \mu_0 i n$$

B Fields from Currents (68)

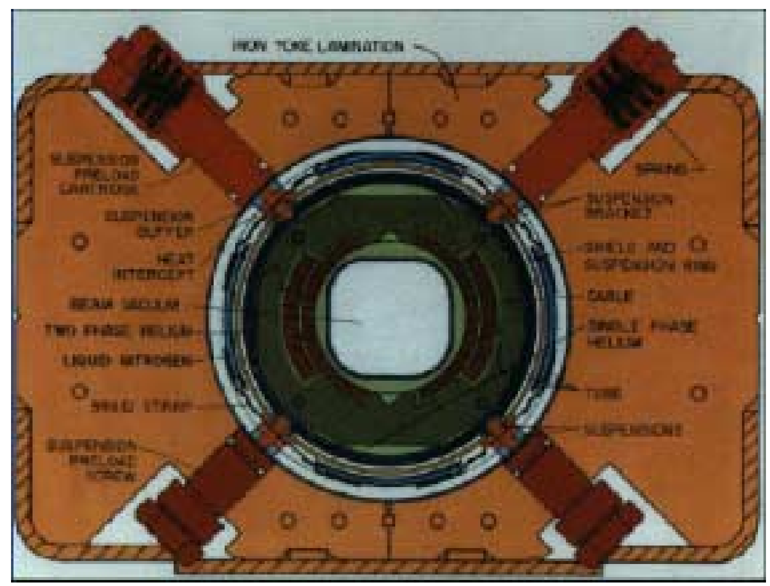
- Tevatron is the largest of 6 synchrotons at Fermilab
- Accelerates protons and anti-protons up to 1 TeV (1 TeV=10^{12} eV)
- Remember a synchroton accelerates charged particles in a circular path of fixed radius by varying the *B* field

$$r = \frac{mv}{qB}$$



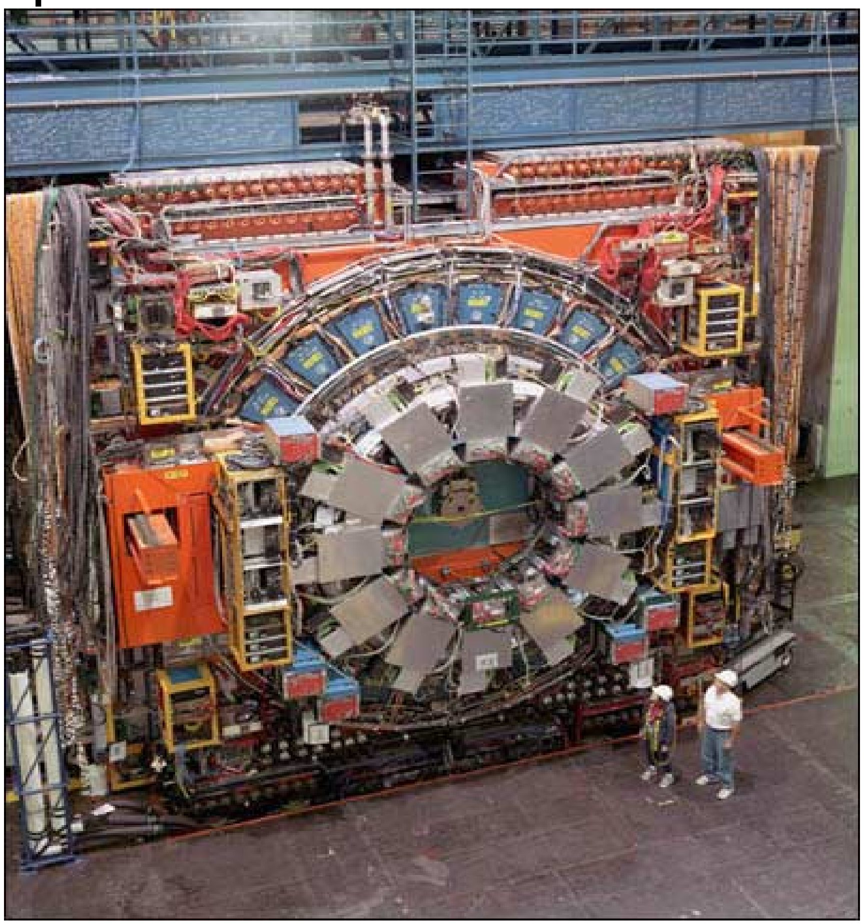
B Fields from Currents (69)

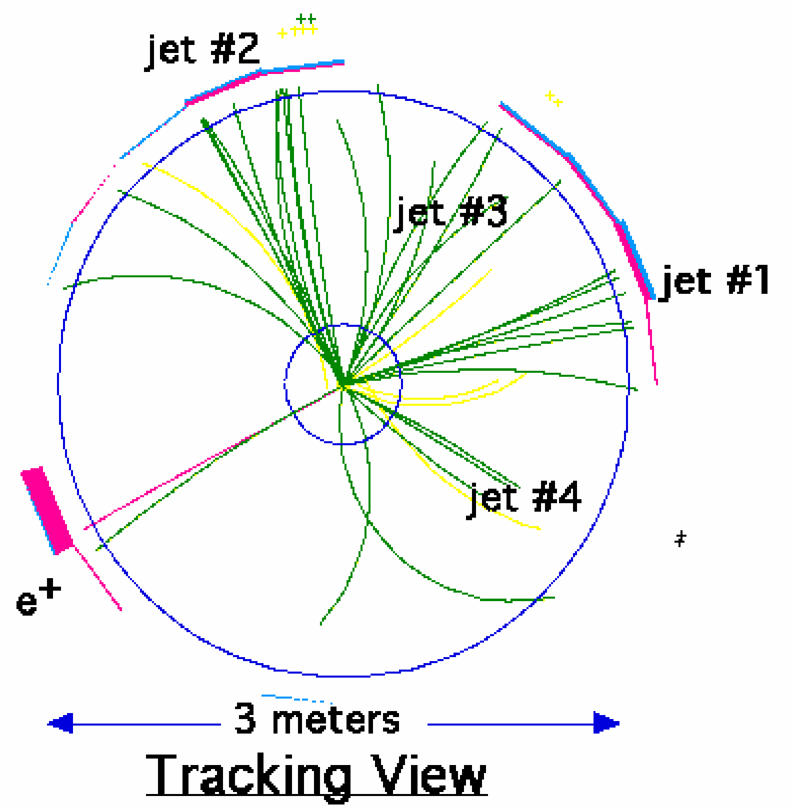
- Tevatron uses 1000 magnets with *B* fields of 4.2 Tesla
  - Small bar magnet 10^{-2} T
  - Earth is 3×10^{-4} T
- Magnets are solenoids
  - Niobium-titanium alloy
  - N = 11 million
  - Current = 4000 A



B Fields from Currents (70)

- Collider Detector at Fermilab (CDF) also uses solenoid
- Measures momentum and charge of particles by their path in a B field





B Fields from Currents (71)

- CDF solenoid
  - Niobium-titanium, copper and Al
  - Length = 5 m
  - Diameter = 3 m
  - N = 1164
  - Current = 5000 A

$$B = \mu_0 i n = \mu_0 i \frac{N}{L}$$

$$B = 1.5T$$