Lecture 23

Chapter 31
Induction and Inductance
Review

- Forces due to $B$ fields
  - On a moving charge
    \[ F_B = q\vec{v} \times \vec{B} \]
  - On a current
    \[ F_B = i\vec{L} \times \vec{B} \]

- Current carrying coil feels a torque
  \[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
  \[ \mu = NiA \]

- A current generates a $B$ field
  - Biot-Savart law
    \[ d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3} \]
  - Ampere’s law
    \[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \]
Review

• Calculated $B$ field for
  – Long, straight wire
    \[ B = \frac{\mu_0 i}{2\pi r} \]
  – At center of loop
    \[ B = \frac{\mu_0 i}{2R} \]
  – Solenoid
    \[ B = \mu_0 in \]

• Force on a wire carrying current, $i_1$, due to $B$ of another parallel wire with current $i_2$

\[ F = \frac{\mu_0 L i_1 i_2}{2\pi d} \]

• Force is attractive (repulsive) if current in both wires are same (opposite) directions

\[ n = \frac{N}{L} \]
Inductance (1)

• A current can produce a $B$ field
• Can a $B$ field generate a current?
• Move a bar magnet in and out of loop of wire
  – Moving magnet towards loop current in loop
  – Current disappears when stops
  – Move magnet away from loop again appears but in direction
  – Faster motion produces a greater current
Inductance (2)

• Have 2 conducting loops near each other
  – Close switch so current flows in one loop, briefly register a current in other loop
  – Open switch, again briefly register current in other loop but in opposite direction
Inductance (3)

- Current produced in the loop is called **induced current**
- The work done per unit charge to produce the current is called an **induced emf**
- Process of producing the current and emf is called **induction**
Inductance (4)

- Faraday observed that an induced current (and an induced emf) can be generated in a loop of wire by:
  - Moving a permanent magnet in or out of the loop
  - Holding it close to a coil (solenoid) and changing the current in the coil
  - Keep the current in the coil constant but move the coil relative to the loop
  - Rotate the loop in a steady $B$ field
  - Change the shape of the loop in a $B$ field
Inductance (5)

- Faraday concluded that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop.
- Need to calculate the amount of magnetic field through the loop so define **magnetic flux** analogous to electric flux.
Inductance (6)

- Magnetic flux through area $A$

- $d\vec{A}$ is vector of magnitude that is $\perp$ to the differential area, $dA$

- If $B$ is uniform and $\perp$ to $A$ then

- SI unit is the weber, Wb

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\Phi_B = BA$$

$$1\text{Wb} = 1T \cdot m^2$$
Inductance (7)

- Faraday’s law of induction – induced emf in loop is equal to the rate at which the magnetic flux changes with time

- Minus signs means induced emf tends to oppose the flux change

- If magnetic flux is through a closely packed coil of N turns

\[ E = - \frac{d\Phi_B}{dt} \]

\[ E = -N \frac{d\Phi_B}{dt} \]
Inductance (8)

• Can change the magnetic flux through a loop (or coil) by
  – If \(B\) is constant within coil
    \[ \Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta \]
  – Change magnitude of \(B\) field within coil
  – Change area of coil, or portion of area within field
  – Change angle between \(B\) field and area of coil (e.g. rotating coil)

\[
E = -N \frac{d\Phi_B}{dt}
\]

\[
E = -N A \cos \theta \frac{dB}{dt}
\]

\[
E = -N B \cos \theta \frac{dA}{dt}
\]

\[
E = -N B A \frac{d(\cos \theta)}{dt}
\]
Inductance (9)

• Checkpoint #1 – Graph shows magnitude $B(t)$ of uniform $B$ field passing through loop, $\perp$ to plane of the loop. Rank the five regions according to magnitude of emf induced in loop, greatest first.

$$E = -N A \cos \theta \frac{dB}{dt} = -N A \frac{dB}{dt}$$

b, then d & e tie, then a & c (zero)
Inductance (10)

- **Lenz’s law** – An induced emf gives rise to a current whose $B$ field opposes the change in flux that produced it
  - Magnet moves towards loop the flux in loop increases so induced current sets up $B$ field opposite direction
  - Magnet moves away from loop the flux decreases so induced current have $B$ field in same direction to oppose.

\[ \text{Diagram of induction process} \]
Inductance (11)

• Checkpoint #2 – Three identical circular conductors in uniform $B$ fields that are either increasing or decreasing in magnitude at identical rates. Rank according to magnitude of current induced in loop, greatest first.

• Use Lenz’s law to find direction of $B_i$
• Use right-hand rule to find direction of current
Inductance (12)

• Situation a –
  – From Lenz’s law, $B_i$ from induced current opposes increasing $B$ so $B_i$ is into page
  – From right-hand rule, induced current is clockwise in both sections of circle

• Do same for situation b and c
  a & b tie, then c (zero)
Inductance (13)

• What is magnitude and direction of induced emf around loop at t=0.10s?
• Loop has width \( W = 3.0 \text{m} \) and height \( H = 2.0 \text{m} \)
• Loop in non-uniform and varying \( B \) field \( \perp \) to loop into the page

\[
B = 4t^2 x^2
\]

• Since magnitude \( B \) is changing in time, flux through the loop is changing so use Faraday law to calculate induced emf

\[
E = -\frac{d\Phi_B}{dt}
\]
Inductance (14)

• $B$ is not uniform so need to calculate magnetic flux using

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

• $B \perp$ to plane of loop and only changes in x direction

$$\vec{B} \cdot d\vec{A} = BdA = BHdx$$

• Treat time as constant so

$$\Phi_B = \int BHdx = 4t^2H \int_0^3 x^2 dx = 4t^2H \left[ \frac{x^3}{3} \right]_0^3 = 72t^2$$
Inductance (15)

• Now use Faraday’s law to find the magnitude of the induced emf

\[ E = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t \]

• At \( t=0.10\)s, \( \text{emf} = 14 \) V

• Find direction of emf by Lenz’s law
  – \( B \) is increasing so \( B_i \) is in opposite direction - out of the page
  – Right-hand rule – current (and emf) are counterclockwise
Inductance (16)

- If you pull a loop at a constant velocity, $v$, through a $B$ field, you must apply a constant force, $F$.
- As move loop to right, less area is in $B$ field so magnetic flux decreases and current is induced in loop.
- Magnetic flux when $B$ is $\perp$ and constant to area is

$$\Phi_B = BA = BLx$$
Inductance (17)

- Using Faraday’s law

$$E = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt}$$

- Remember $v = \frac{dx}{dt}$ so

$$E = BLv$$

- where $L$ is the length of the loop and $v$ is $\perp$ to $B$ field

- $B$ is decreasing so $B_i$ is in same direction (into page) and current is clockwise
Inductance (18)

- Since loop carries current through a $B$ field there is force given by
  \[ \vec{F}_B = i\vec{L} \times \vec{B} \]

- Use right-hand rule to find direction of $\vec{F}_B$ on segments loop in $B$ field

- Find forces, $F_2$ and $F_3$, cancel each other

- Force, $F_1$, opposes your force

\[ \vec{F}_{app} = -\vec{F}_1 \]
Inductance (19)

- Checkpoint #3 – Four wire loops with edge lengths of either L or 2L. All loops move through uniform $B$ field at same velocity. Rank the four loops according to maximum magnitude of induced emf, greatest first.

\[ E = BLv \]

- $c$ & $d$ tie, then $a$ & $b$ tie