Lecture 23

Chapter 31

Induction and Inductance

Review

- Forces due to *B* fields
  - C ge
    $$F_B = q\vec{v} \times \vec{B}$$
  - C
    $$F_B = i\vec{L} \times \vec{B}$$
- Current carrying coil feels a torque
  $$\vec{\tau} = \vec{\mu} \times \vec{B}$$
  $$\mu = NiA$$

- A current generates a *B* field
  - Biot-Savart law
    $$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$$
  - Ampere's law
    $$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

Review

- Calculated *B* field for
  - Long, straight wire

$$B = \frac{\mu_0 i}{2\pi r}$$

  - At center of loop

$$B = \frac{\mu_0 i}{2R}$$

  - Solenoid

$$B = \mu_0 i n$$

- Force on a wire carrying current, i_1, due to *B* of another parallel wire with current i_2

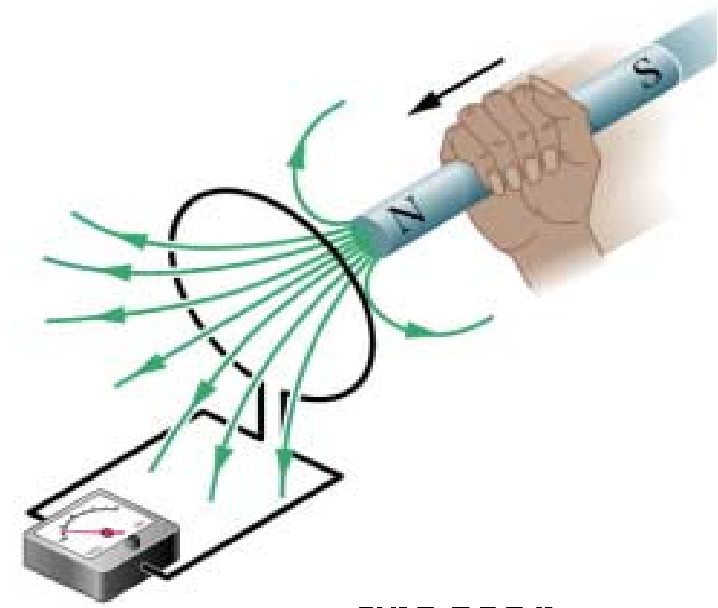
$$F = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

- Force is attractive (repulsive) if current in both wires are same (opposite) directions

$$n = N / L$$

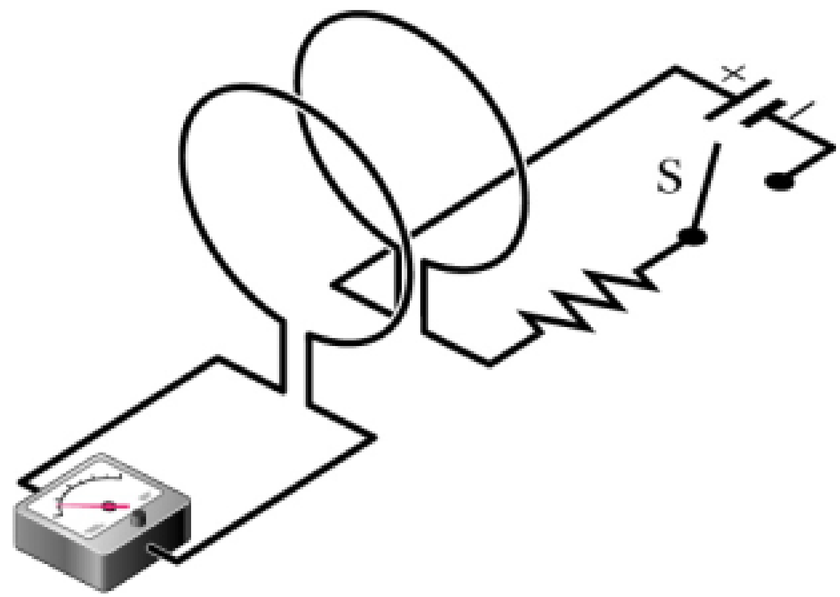
Inductance (1)

- A current can produce a *B* field
- Can a *B* field generate a current?
- Move a bar magnet in and out of loop of wire
  - Moving magnet towards loop current in loop
  - Current disappears when stops
  - Move magnet away from loop again appears but in direction
  - Faster motion produces a greater current



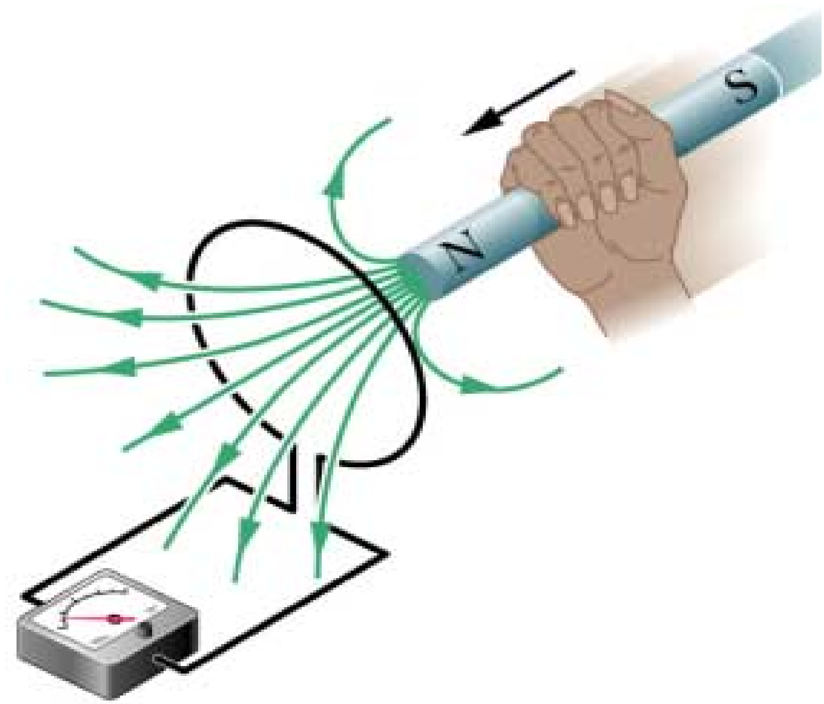
Inductance (2)

- Have 2 conducting loops near each other
  - Close switch so current flows in one loop, briefly register a current in other loop
  - Open switch, again briefly register current in other loop but in opposite direction



Inductance (3)

- Current produced in the loop is called induced current
- The work done per unit charge to produce the current is called an induced emf
- Process of producing the current and emf is called induction

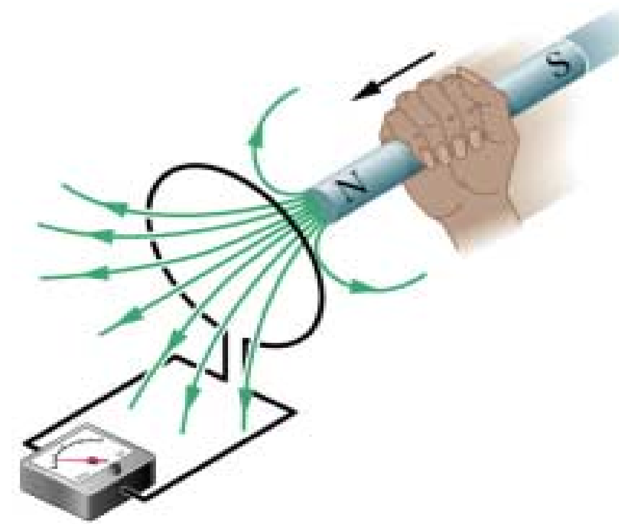


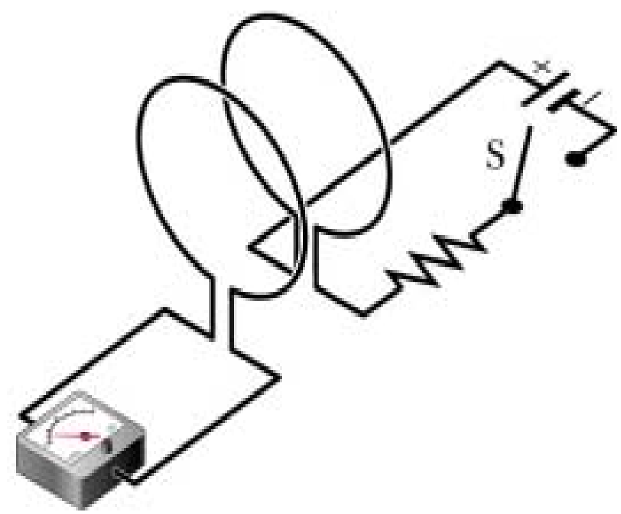
Inductance (4)

- Faraday observed that an induced current (and an induced emf) can be generated in a loop of wire by:
  - Moving a permanent magnet in or out of the loop
  - Holding it close to a coil (solenoid) and changing the current in the coil
  - Keep the current in the coil constant but move the coil relative to the loop
  - Rotate the loop in a steady B field
  - Change the shape of the loop in a B field

Inductance (5)

- Faraday concluded that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop
- Need to calculate the amount of magnetic field through the loop so define magnetic flux analogous to electric flux





Inductance (6)

- Magnetic flux through area *A*

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

- $d\vec{A}$ is vector of magnitude that is ⊥ to the differential area, *dA*

- If *B* is uniform and ⊥ to *A* then

$$\Phi_B = BA$$

- SI unit is the weber, Wb

$$1Wb = 1T \cdot m^2$$

Inductance (7)

- Faraday's law of induction – induced emf in loop is equal to the rate at which the magnetic flux changes with time

$$E = -\frac{d\Phi_B}{dt}$$

- Minus signs means induced emf tends to oppose the flux change
- If magnetic flux is through a closely packed coil of N turns

$$E = -N\frac{d\Phi_B}{dt}$$

Inductance (8)

- Can change the magnetic flux through a loop (or coil) by

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

  - If *B* is constant within coil

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

  - Change magnitude of *B* field within coil

$$\mathcal{E} = -NA\cos\theta \frac{dB}{dt}$$

  - Change area of coil, or portion of area within field

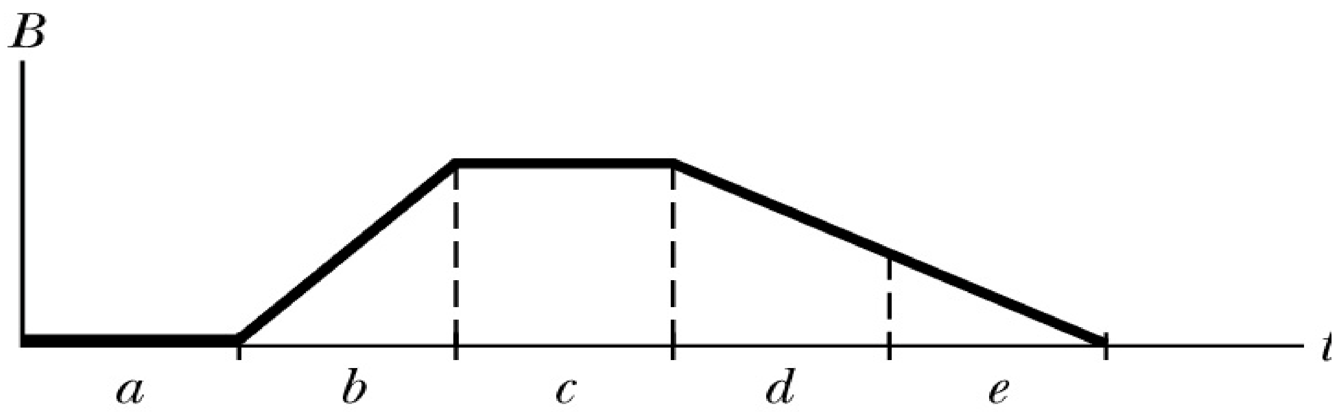
$$\mathcal{E} = -NB\cos\theta \frac{dA}{dt}$$

  - Change angle between *B* field and area of coil (e.g. rotating coil)

$$\mathcal{E} = -NBA \frac{d(\cos\theta)}{dt}$$

Inductance (9)

- Checkpoint #1 – Graph shows magnitude *B(t)* of uniform *B* field passing through loop, ⊥ to plane of the loop. Rank the five regions according to magnitude of emf induced in loop, greatest first.

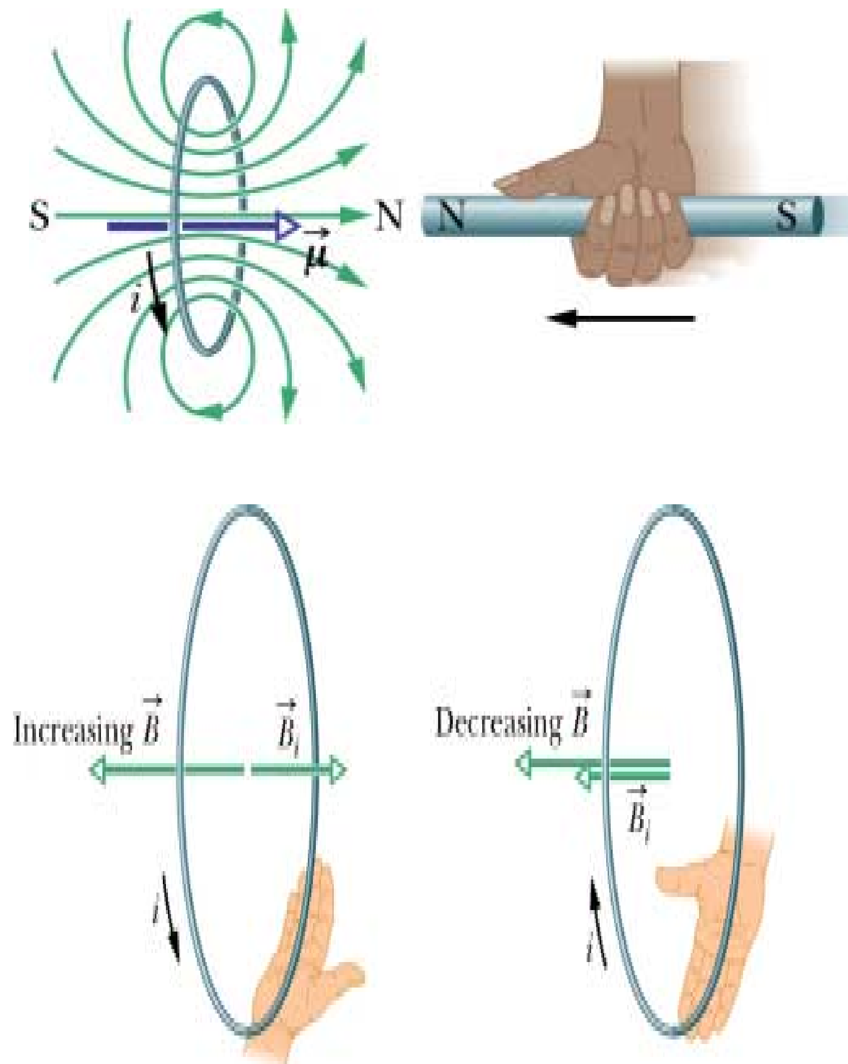


$$E = -NA\cos\theta\frac{dB}{dt} = -NA\frac{dB}{dt}$$

b, then d & e tie, then a & c (zero)

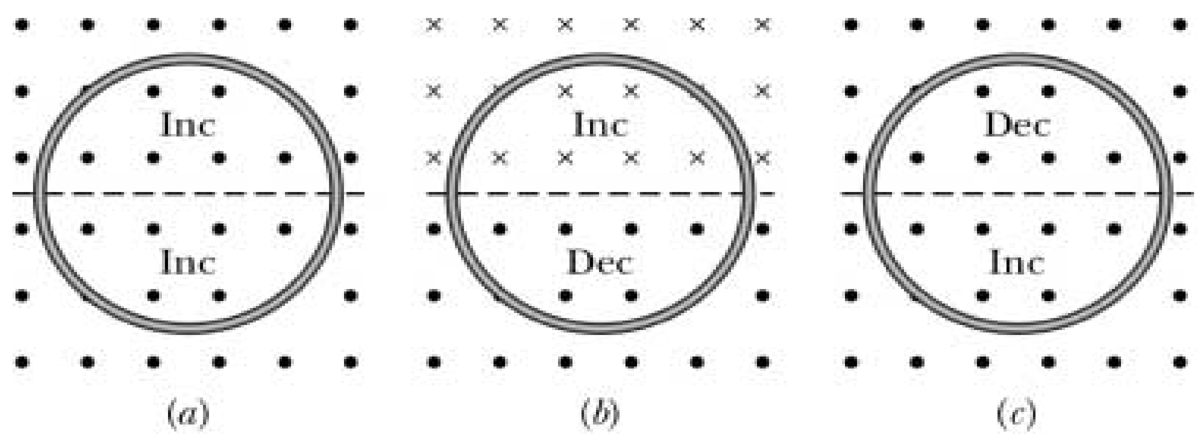
Inductance (10)

- Lenz's law – An induced emf gives rise to a current whose *B* field opposes the change in flux that produced it
  - Magnet moves towards loop the flux in loop increases so induced current sets up *B* field opposite direction
  - Magnet moves away from loop the flux decreases so induced current have *B* field in same direction to



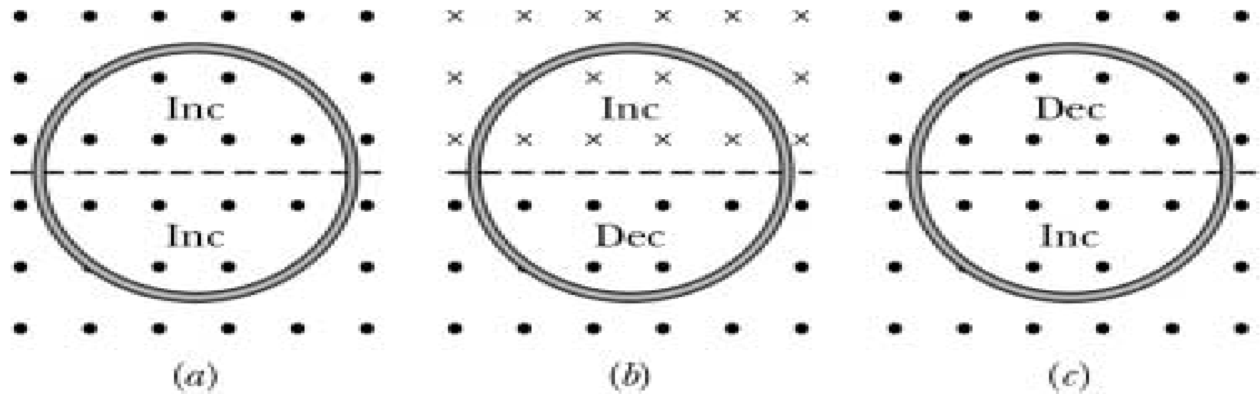
Inductance (11)

- Checkpoint #2 – Three identical circular conductors in uniform *B* fields that are either increasing or decreasing in magnitude at identical rates. Rank according to magnitude of current induced in loop, greatest first.



  - Use Lenz's law to find direction of B_i
  - Use right-hand rule to find direction of current

Inductance (12)

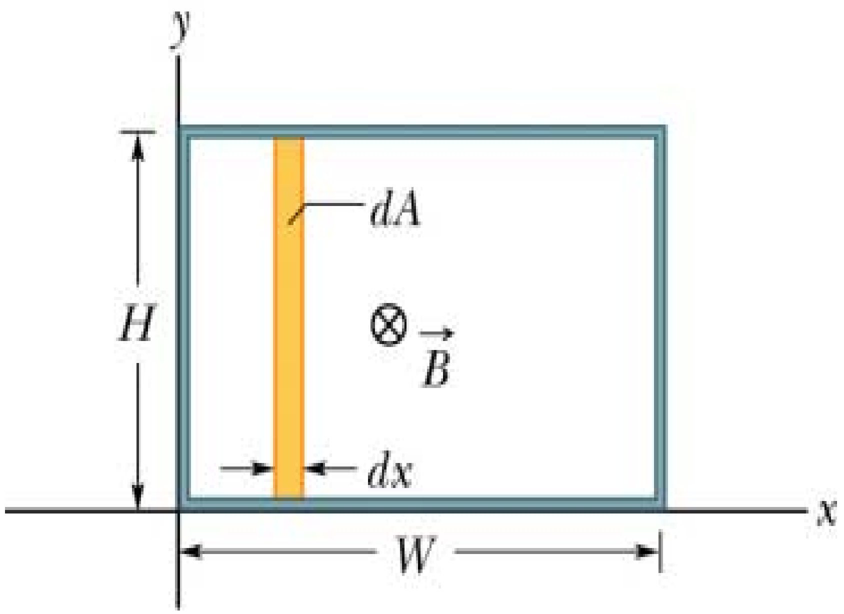


- Situation a –
  - From Lenz's law, B_i from induced current opposes increasing B so B_i is into page
  - From right-hand rule, induced current is clockwise in both sections of circle
- Do same for situation b and c

a & b tie, then c (zero)

Inductance (13)

- What is magnitude and direction of induced emf around loop at t=0.10s?
- Loop has width W=3.0m height H=2.0m
- Loop in non-uniform and *B* field ⊥ to loop into the page



$$B = 4t^2x^2$$

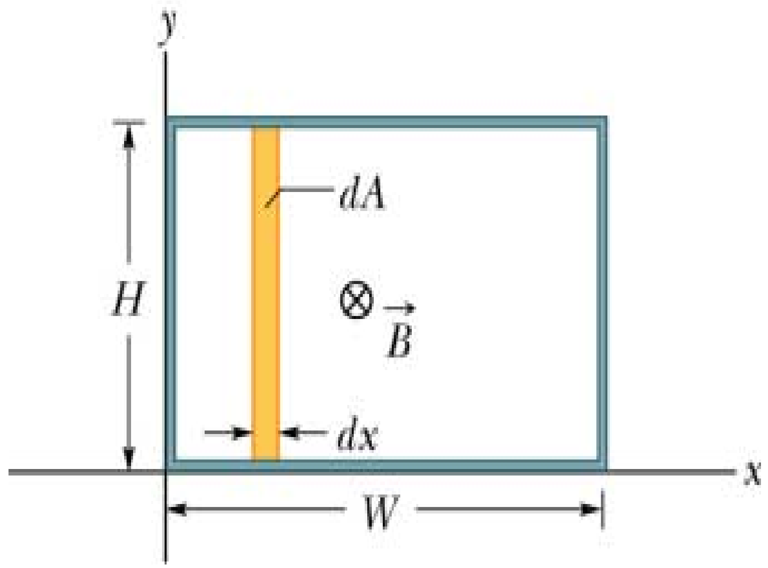
- Since magnitude *B* is changing in time, flux through the loop is changing so use Fara[day's] law to calculate induced emf

$$E = -\frac{d\Phi_B}{dt}$$

Inductance (14)

- *B* is not uniform so need to calculate magnetic flux using

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$



- *B* $\perp$ to plane of loop and only changes in x direction

$$\vec{B} \bullet d\vec{A} = BdA = BHdx$$

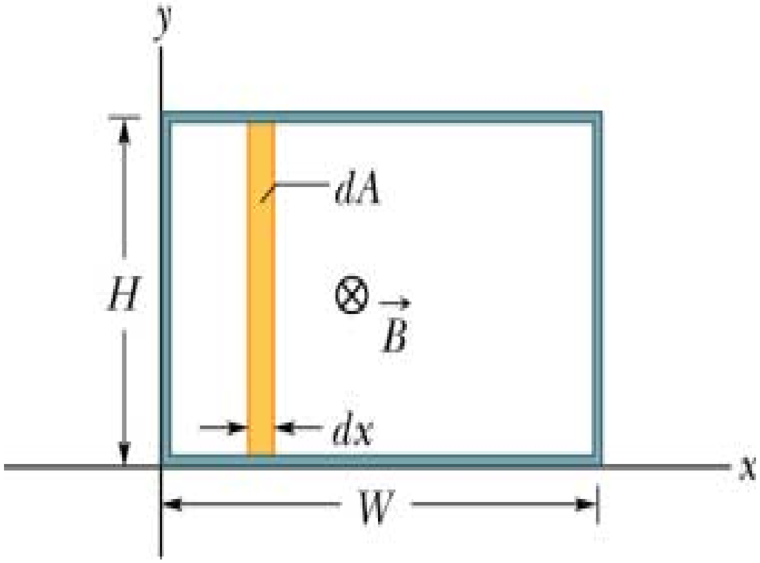
- Treat time as constant so

$$\Phi_B = \int BHdx = 4t^2 H \int_0^3 x^2 dx = 4t^2 H \left[\frac{x^3}{3} \right]_0^3 = 72\, t^2$$

Inductance (15)

- Now use Faraday's law to find the magnitude of the induced emf

$$E = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t$$

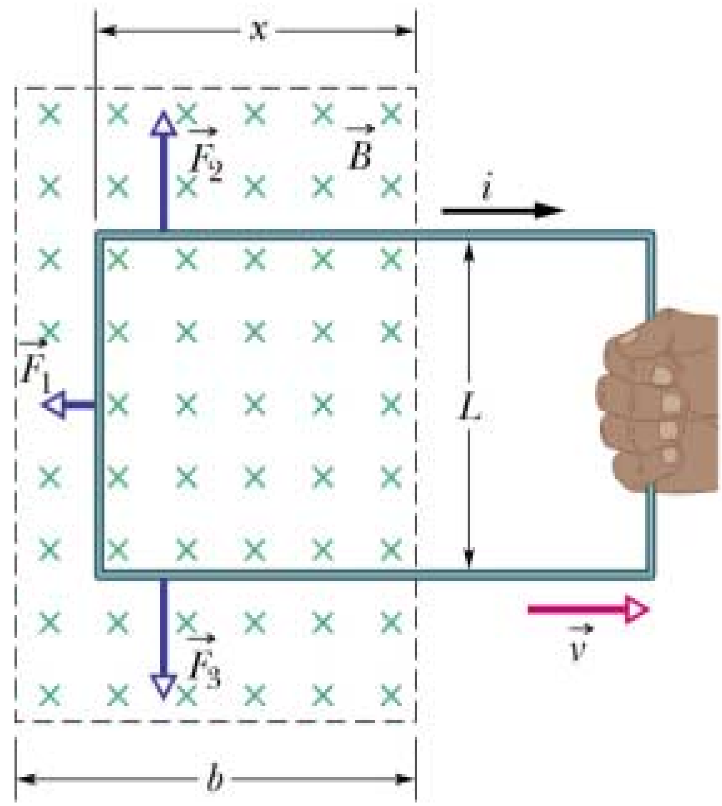


- At t=0.10s, emf = 14 V

- Find direction of emf by Lenz's law
  - *B* is increasing so B_i is in opposite direction - out of the page
  - Right-hand rule – current (and emf) are counterclockwise

Inductance (16)

- If you pull a loop at a constant velocity, *v*, through a *B* field, you must apply a constant force, *F*
- As move loop to right, less area is in B field so magnetic flux decreases and current is induced in loop
- Magnetic flux when *B* is ⊥ and constant to area is



$$\Phi_B = BA = BLx$$

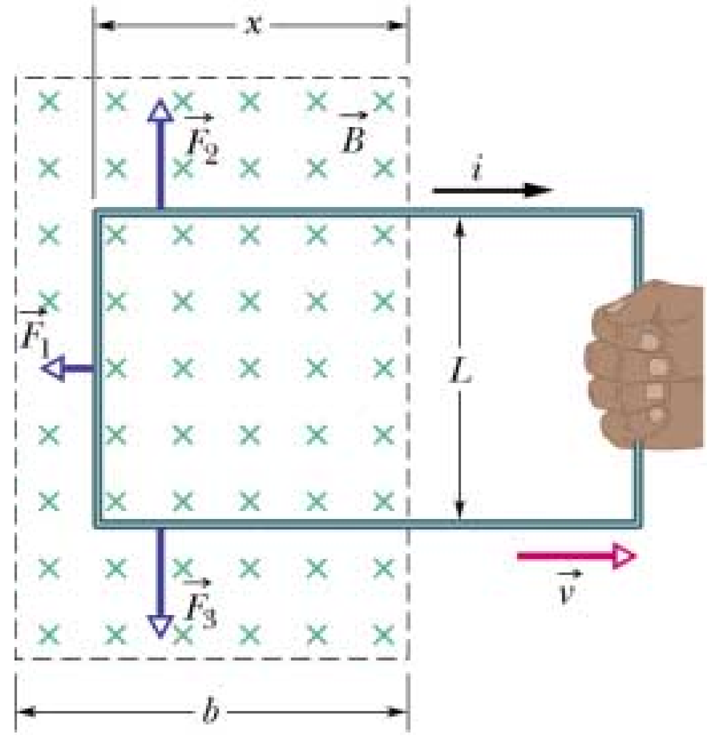
Inductance (17)

- Using Faraday's law

$$E = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BL\frac{dx}{dt}$$

- Remember $v = dx/dt$ so

$$E = BLv$$

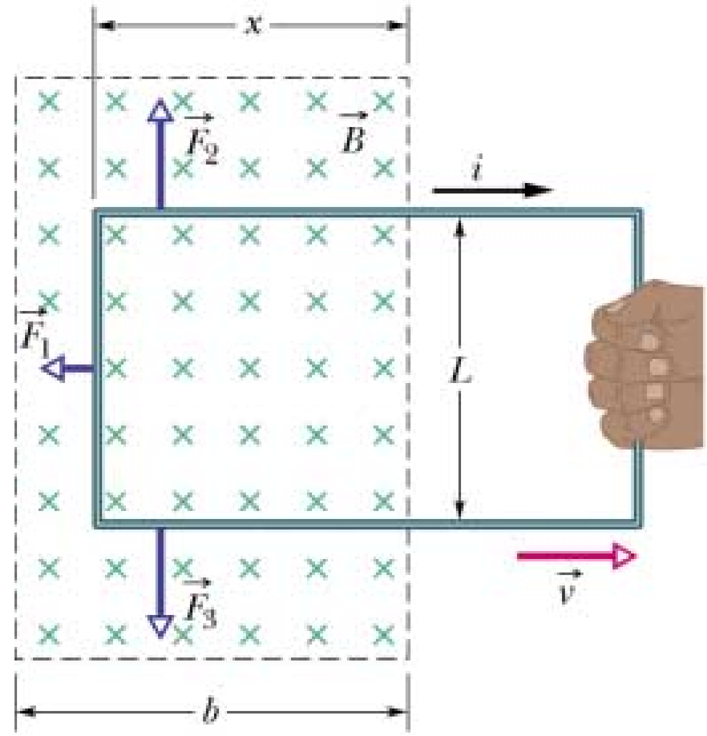


- where L is the length of the loop and v is ⊥ to B field
- B is decreasing so B_i is in same direction (into page) and current is clockwise

Inductance (18)

- Since loop carries current through a *B* field there is force given by

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

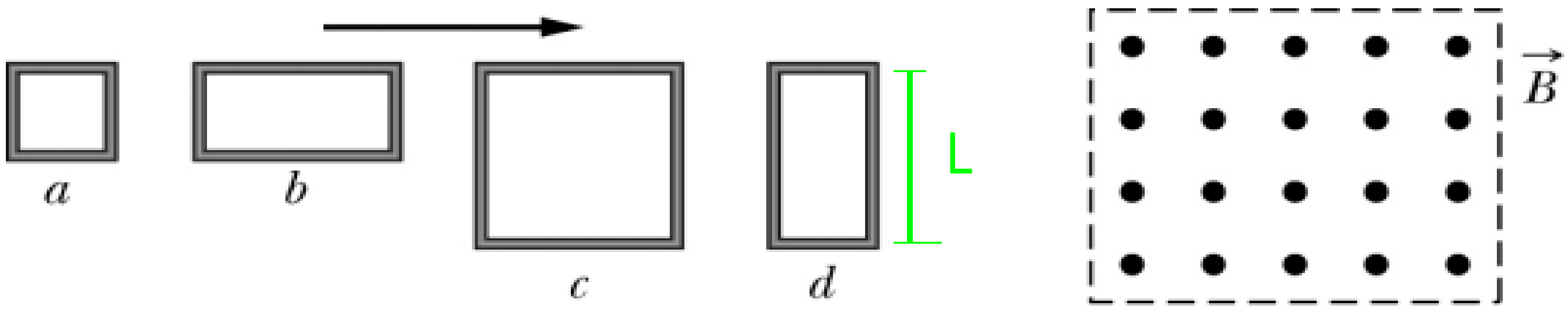


- Use right-hand rule to find direction of F_B on segments loop in *B* field
- Find forces, F2 and F3, cancel each other
- Force, F_1 opposes your force

$$\vec{F}_{app} = -\vec{F}_1$$

Inductance (19)

- Checkpoint #3 – Four wire loops with edge lengths of either L or 2L. All loops move through uniform *B* field at same velocity. Rank the four loops according to maximum magnitude of induced emf, greatest first.



$$E = BLv$$

c & d tie, then
a & b tie