Lecture 24

Chapter 31

Induction and Inductance

Review

- Can produce an induced current and induced emf in a loop of wire when the number of magnetic field lines passing through the loop is changing
- Magnetic flux

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

- Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Lenz's law – An induced emf gives rise to a current whose *B* field opposes the change in flux that produced it

Review

- Induced emf of a conductor moving with velocity, *v*, in a $\perp B$ field is given by

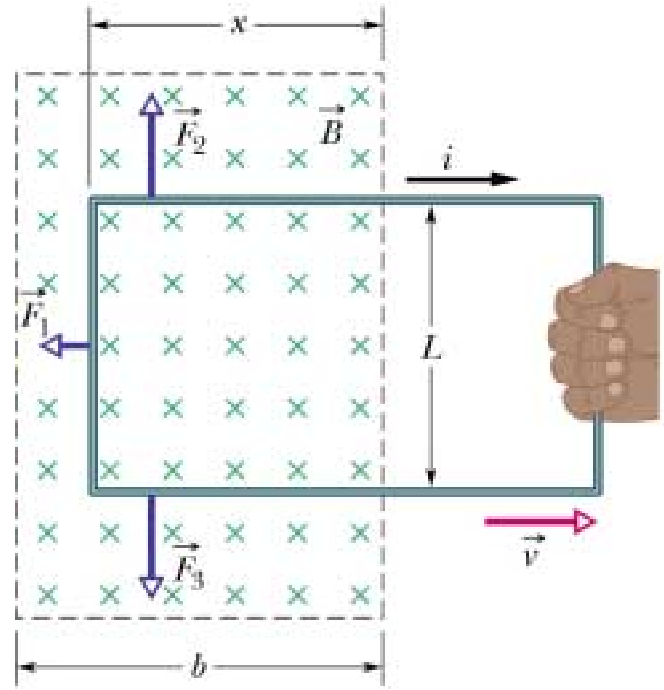
$$\boxed{\mathcal{E} = BLv}$$

- Induced current in loop in a *B* field experiences a force

$$\boxed{F_B = i\vec{L} \times \vec{B}}$$

- Found F_1 opposes your force F_{app}

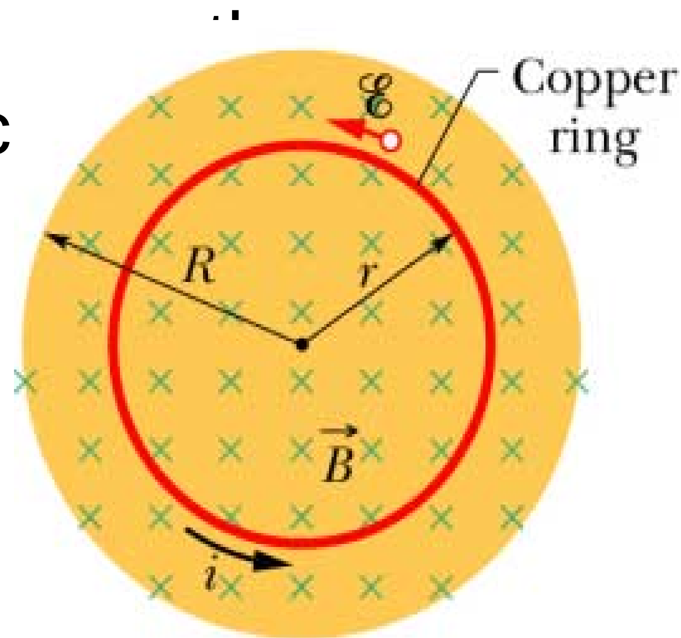
$$\boxed{\vec{F}_{app} = -\vec{F}_1}$$



- Work you do in pulling the loop appears as thermal energy in the loop

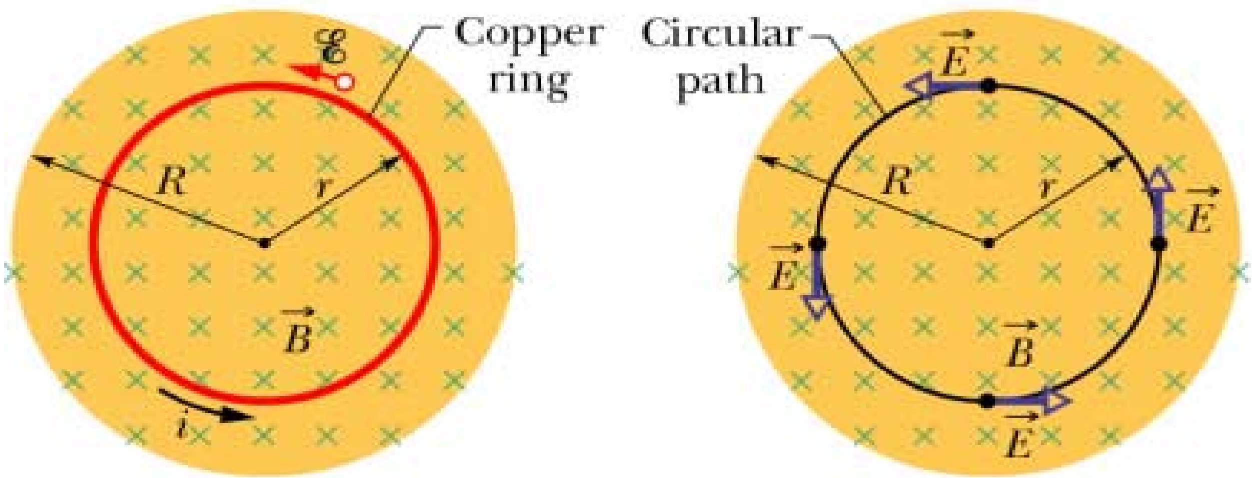
Inductance (20)

- Put a copper ring in a uniform *B* field which is increasing in time so magnetic flux through the c ring is changing
- By Faraday's law an induced and current are produced
- From Lenz's law their direction counterclockwise
- If there is a current there must be an *E* field present to move the conduction electrons around ring



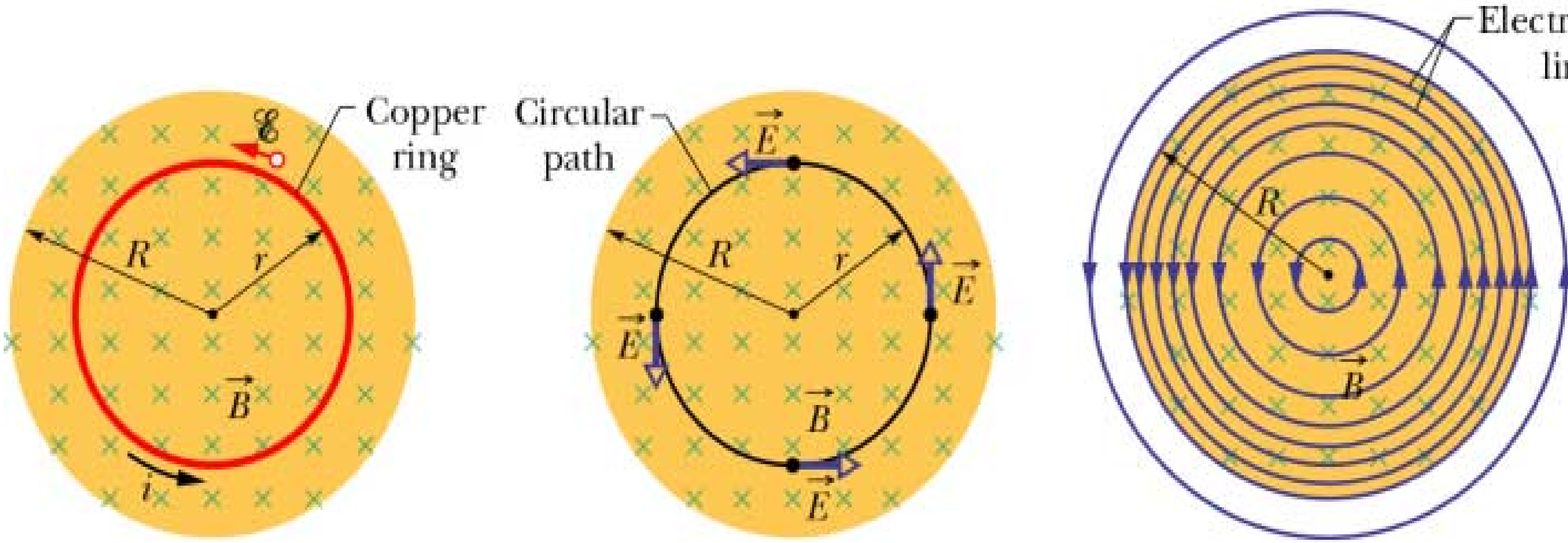
Inductance (21)

- Induced *E* field is same as *E* field produced by static charges, it will exert a force, *F=qE*, on a charged particle
- Restate Faraday's law – A changing *B* field produces an *E* field
- True even if no copper ring



Inductance (22)

- Electric field lines produced by a changing *B* field are set of concentric circles
- If *B* field increasing or decreasing field lines are present (decreasing have opposite direction)
- If *B* field is constant with time, no *E* field

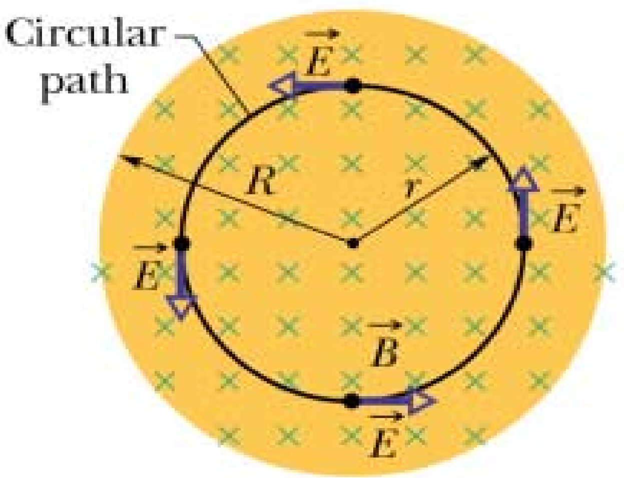


Inductance (23)

- Calculate work done on particle by induced *E* field
- Remember

$$\mathcal{E} = \frac{dW}{dq}$$

$$W = q_0\mathcal{E}$$



- For q_0 moving along closed path the work is defined as

$$W = \oint \vec{F} \bullet d\vec{s}$$

- But F_E is

$$F_E = q_0 E$$

- Equating equations for work gives

$$q_0\mathcal{E} = \oint q_0\vec{E} \bullet d\vec{s}$$

Inductance (24)

- Canceling q_0 find that

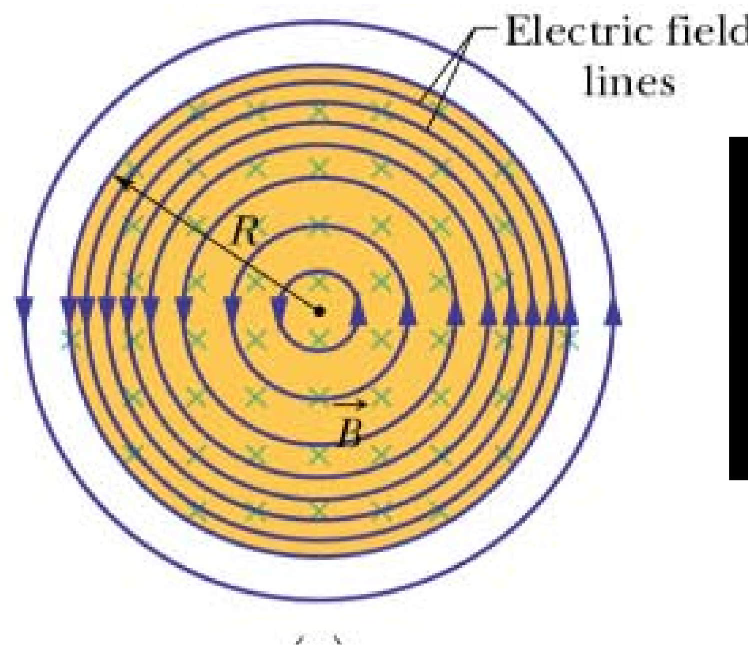
$$\mathcal{E} = \oint \vec{E} \bullet d\vec{s}$$

- But from Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- So Faraday's law can be written as

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$



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- A changing B field induces an E field

Inductance (25)

- Inductor is a device used to produce a desired *B* field (e.g. solenoid)
- A current, *i*, in an inductor with N turns produces a magnetic flux, Φ_B, in its central region
- Inductance, *L* is defined as

$$L = \frac{N\Phi_B}{i}$$

- SI unit is henry, H

$$1H = 1T \cdot m^2 / A$$

Inductance (26)

- What is inductance per unit length near the middle of a long solenoid?

$$L = \frac{N\Phi_B}{i}$$

- First find flux of single loop in solenoid

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA$$

- Remember # turns (N) per unit length (l) is

$$n = N / l$$

$$L = \frac{nlBA}{i}$$

Inductance (27)

- *B* field from a solenoid

$$B = \mu_0 i n$$

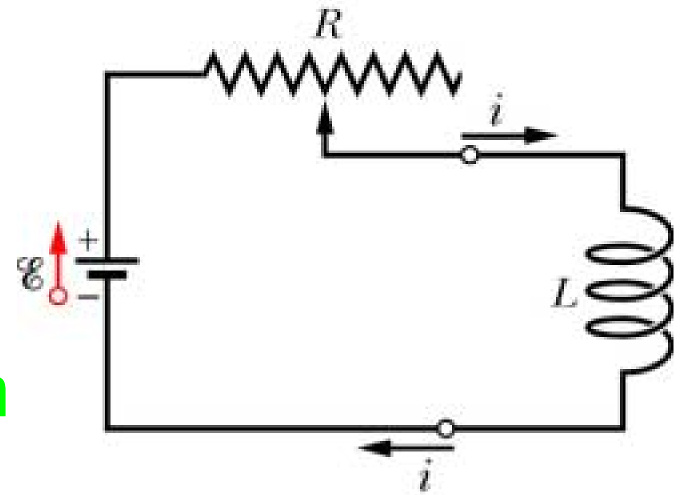
$$L = \frac{nlBA}{i} = \frac{nl(\mu_0 i n)A}{i} = l\mu_0 n^2 A$$

- Inductance per unit length is
- Depends only on geometry of device (like capacitance)

$$\frac{L}{l} = \mu_0 n^2 A$$

Inductance (28)

- A changing current in a coil generates a self-induced emf, $\mathcal{E}_L$ in the coil
- Process is called self-induction
- Change current in coil using a variable resistor, $\mathcal{E}_L$, will appear in coil only while the current is changing



$$L = \frac{N\Phi_B}{i}$$

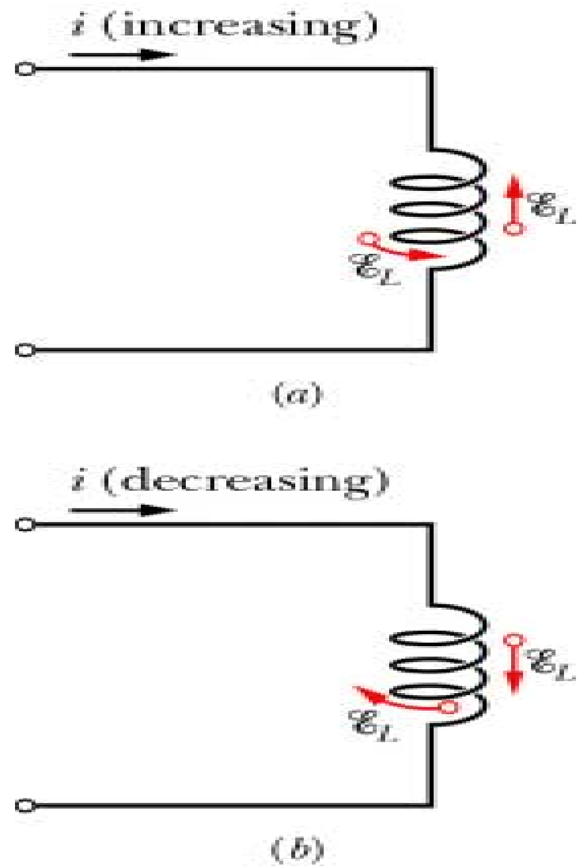
$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt}$$

Inductance (29)

$$\mathcal{E}_L = -L\frac{di}{dt}$$

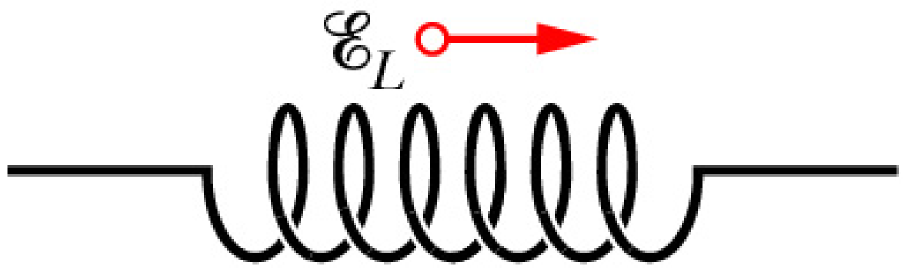
- Induced emf only depends on rate of change of current, not its magnitude
- Determine direction of $\mathcal{E}_L$ using Lenz's law
- Self-induced V_L across inductor
  - Ideal inductor $$V_L = \mathcal{E}_L$$
  - Real inductor (like real battery) has some internal resistance

$$V_L = \mathcal{E}_L - iR$$



Inductance (30)

- Checkpoint #4 – Have an induced emf in a coil. What can we tell about the current through the coil? Is it moving right or left and is it constant, decreasing or increasing?



Decreasing and rightward (answer d)

OR

Increasing and leftward (answer e)

Inductance (31)

- RL circuit is a resistor and inductor in series
- Similar to RC circuit (resistor & capacitor in series)
  - Charging up a capacitor

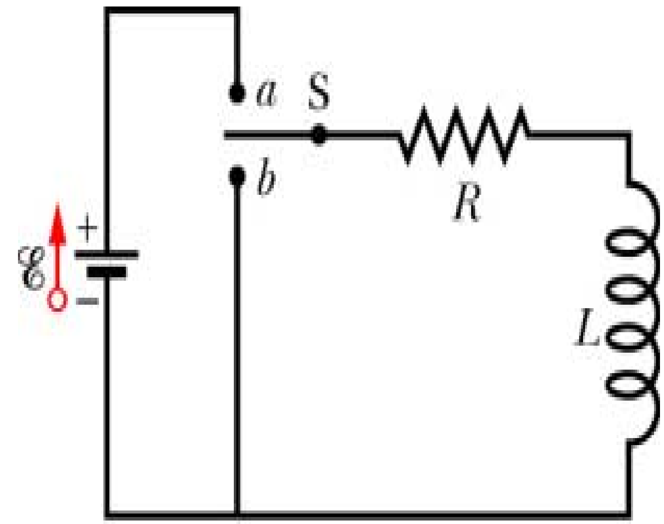
$$q = C\mathcal{E}(1 - e^{-t/\tau_c})$$

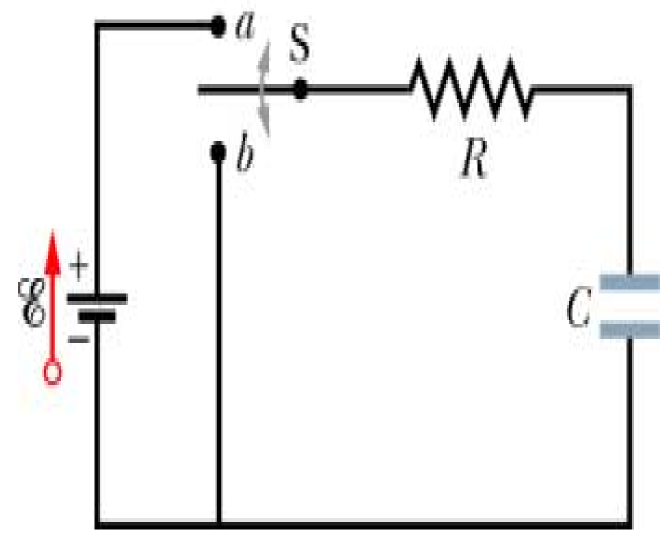
  - Discharging capacitor

$$q = q_0 e^{-t/\tau_c}$$

  - where

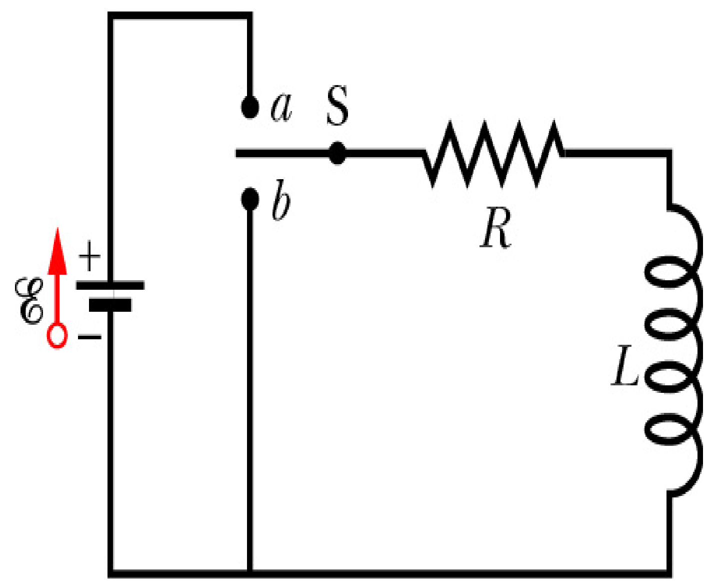
$$\tau_C = RC$$





Inductance (32)

- Analogous time dependence on rise (or fall) of current if introduce an emf into (or removie it from) an RL circuit

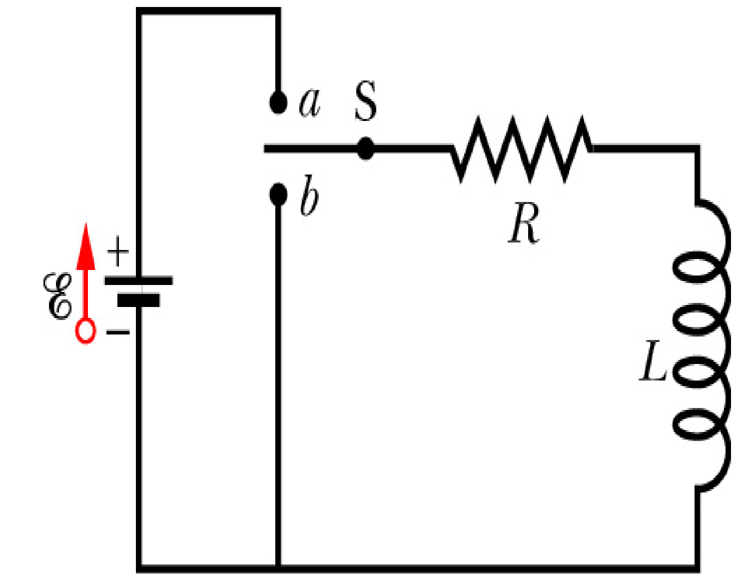


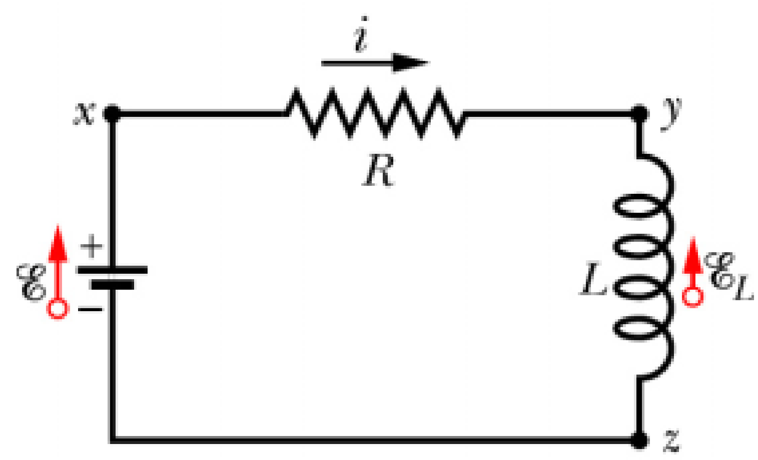
- Initially close switch *i* is increasing through inductor so $\mathcal{E}_L$ opposes rise
- *i* through *R* will be $i < \mathcal{E}/R$
- Long time later *i* is constant so $\mathcal{E}_L$=0 and *i* in circuit is $i = \mathcal{E}/R$

Inductance (33)

- Initially an inductor acts to oppose changes in current through it
- Long time later inductor acts like ordinary conducting wire
- Apply loop rule right after switch has been closed at a
- Starting at *x*, go clockwise

$$-iR - L\frac{di}{dt} + \mathcal{E} = 0$$





Inductance (34)

- Differential equation similar to capacitors

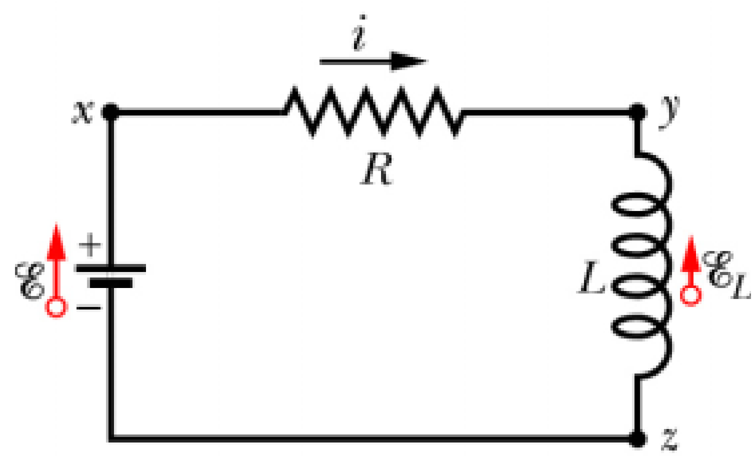
$$\mathcal{E} = iR + L\frac{di}{dt}$$

- Solution is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

- Where inductive time constant is

$$\tau_L = \frac{L}{R}$$



- Satisfies conditions
- At t=0, $i = 0$
- At t=∞, $i = \mathcal{E}/R$

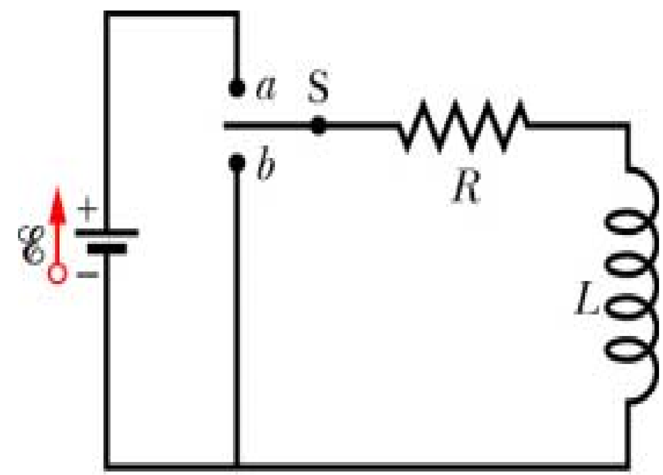
Inductance (35)

- Now move switch to position b so battery is out of system
- Current will decrease with time and loop rule gives

$$iR + L\frac{di}{dt} = 0$$

- Solution is

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

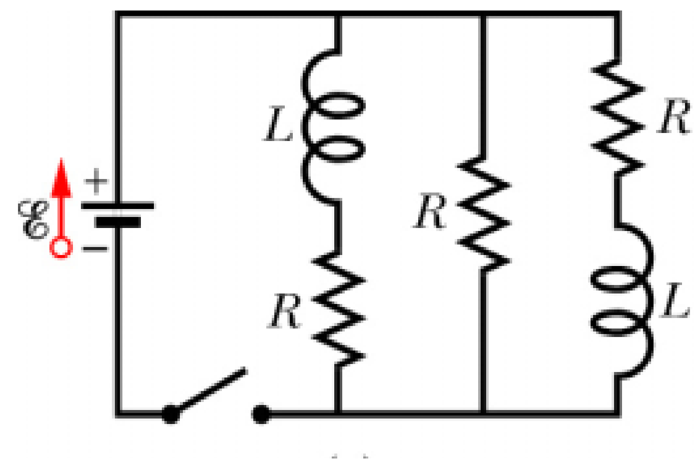


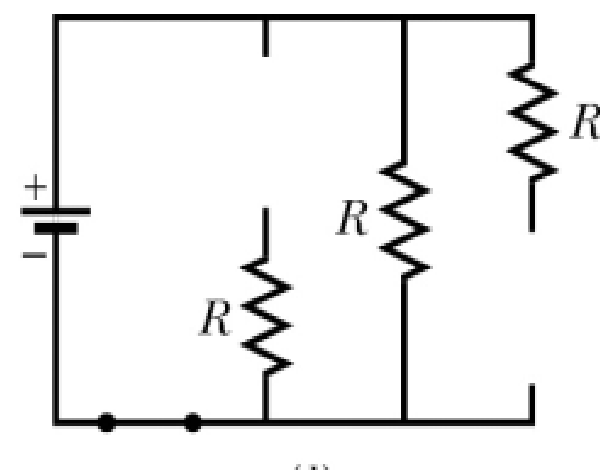
- Satisfies conditions
- At t=0, $i = i_0 = \mathcal{E}/R$
- At t=∞, $i = 0$

$$\tau_L = \frac{L}{R}$$

Inductance (36)

- Have a circuit with resistors and inductors
- What is the current through the battery just after close the switch?
- Inductor oppose change in current through it
- Right after switch is closed, current through inductor is 0
- Inductor acts like broken wire





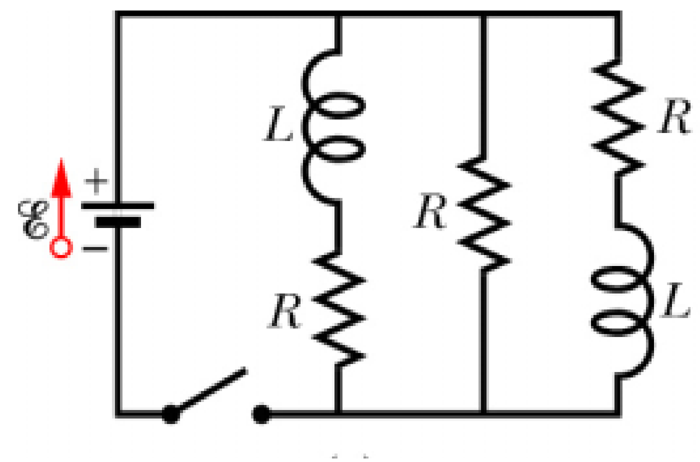
Inductance (37)

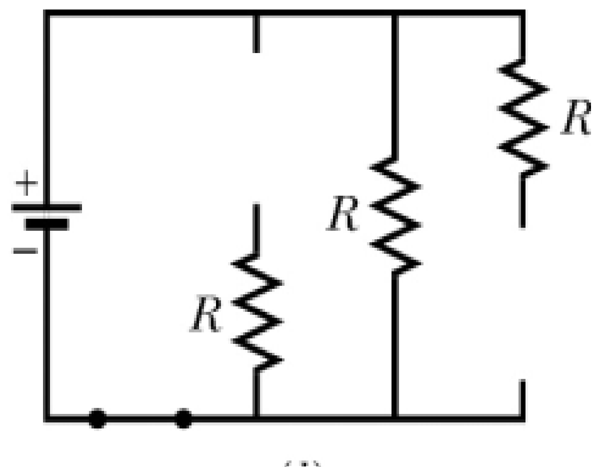
- Apply loop rule

$$\boxed{\mathcal{E} - iR = 0}$$

- Immediately after switch closed current through the battery is

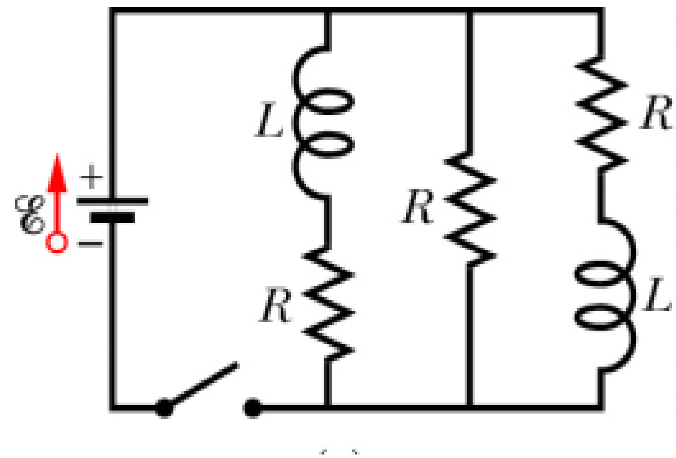
$$\boxed{i = \frac{\mathcal{E}}{R}}$$





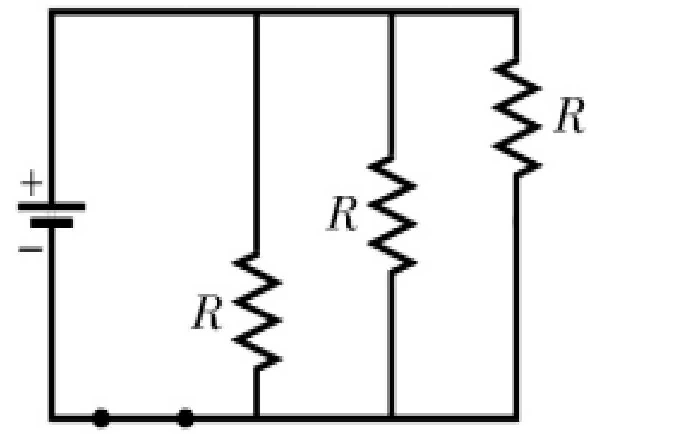
Inductance (38)

- What is the current through the battery long time after the switch has been closed?
- Currents in circuit have reached equilibrium so inductor acts like simple wire
- Circuit is 3 resistors in parallel



$$i = \frac{\mathcal{E}}{R_{eq}}$$

$$R_{eq} = \frac{R}{3}$$



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