# Lecture 25

Chapter 31

Induction and Inductance

# Review

- Magnetic flux

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

- Faraday's law

$$E = -\frac{d\Phi_B}{dt}$$

- Lenz's law – induced emf gives rise to a current whose *B* field opposes the change in flux that produced it

- Changing *B* field produces an *E* field

- Restate Faraday's law

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

- Inductance, L defined

$$L = \frac{N\Phi_B}{i}$$

# Review

- Inductor – device produces known *B* field
- Solenoid is an inductor with inductance per unit length of

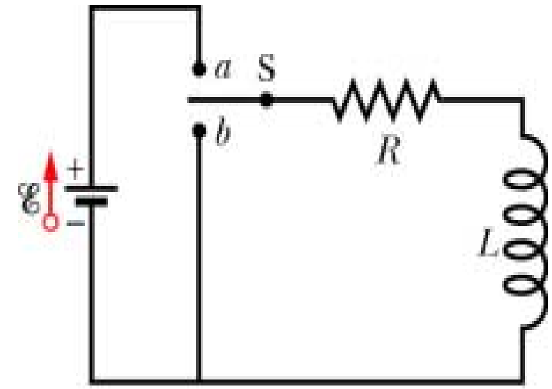
$$\frac{L}{l} = \mu_0 n^2 A$$

- Self-induce emf, $E_L$ appears in any coil in which the current is changing

$$E_L = -L\frac{di}{dt}$$

- Direction of $E_L$ follows Lenz's law and opposes the change in current

# Review




- RL circuit – resistor and inductor in series
- Time dependence on current in RL circuit
- Initially inductor acts to oppose changes in current through it
- Long time later, inductor acts like simple wire

- Rise of current

$$i = \frac{E}{R}\left(1 - e^{-t/\tau_L}\right)$$

- Decay of current

$$i = \frac{E}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

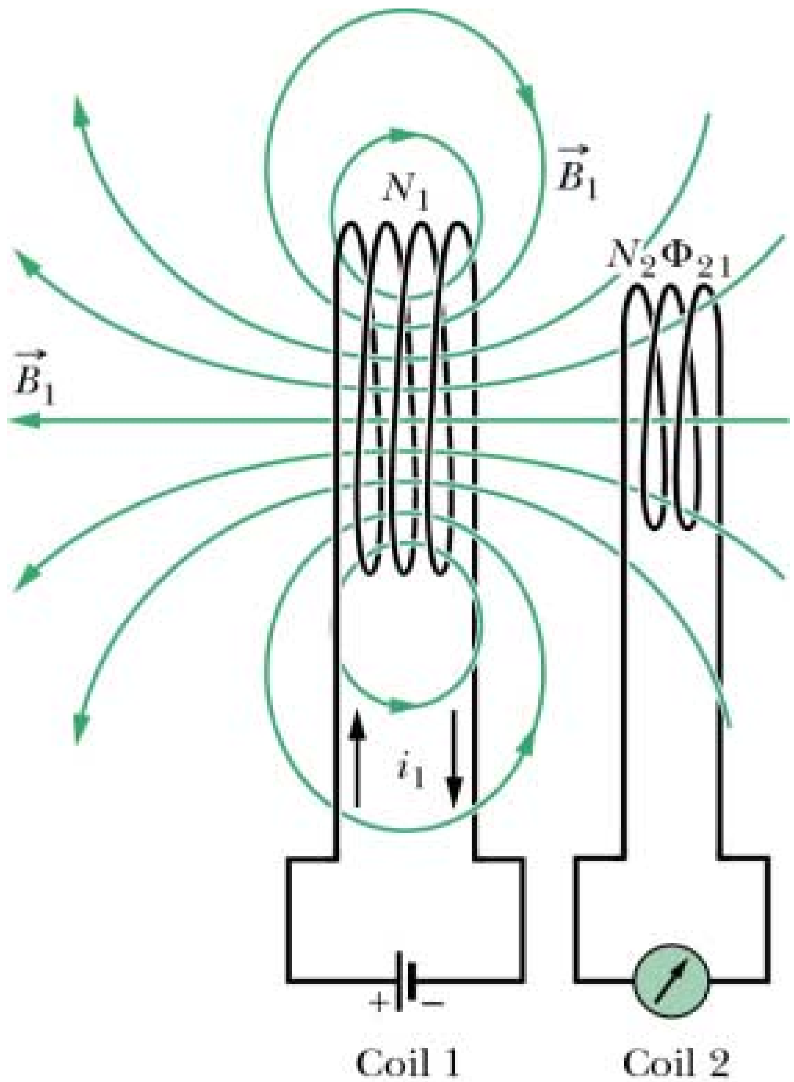
- Inductive time constant

$$\tau_L = \frac{L}{R}$$

# Inductance (39)

- Mutual induction – current in one coil induces emf in other coil
- Distinguish from self-induction
- Mutual inductance, $M_{21}$ of coil 2 with respect to coil 1 is

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$



# Inductance (40)

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$

- Rearrange equation

$$M_{21} i_1 = N_2 \Phi_{21}$$

- Vary $i_1$ with time

$$M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}$$

- Faraday's law

$$E_2 = -N_2 \frac{d\Phi_{21}}{dt}$$

- Induced emf in coil 2 due to $i$ in coil 1 is

$$E_2 = -M_{21} \frac{di_1}{dt}$$

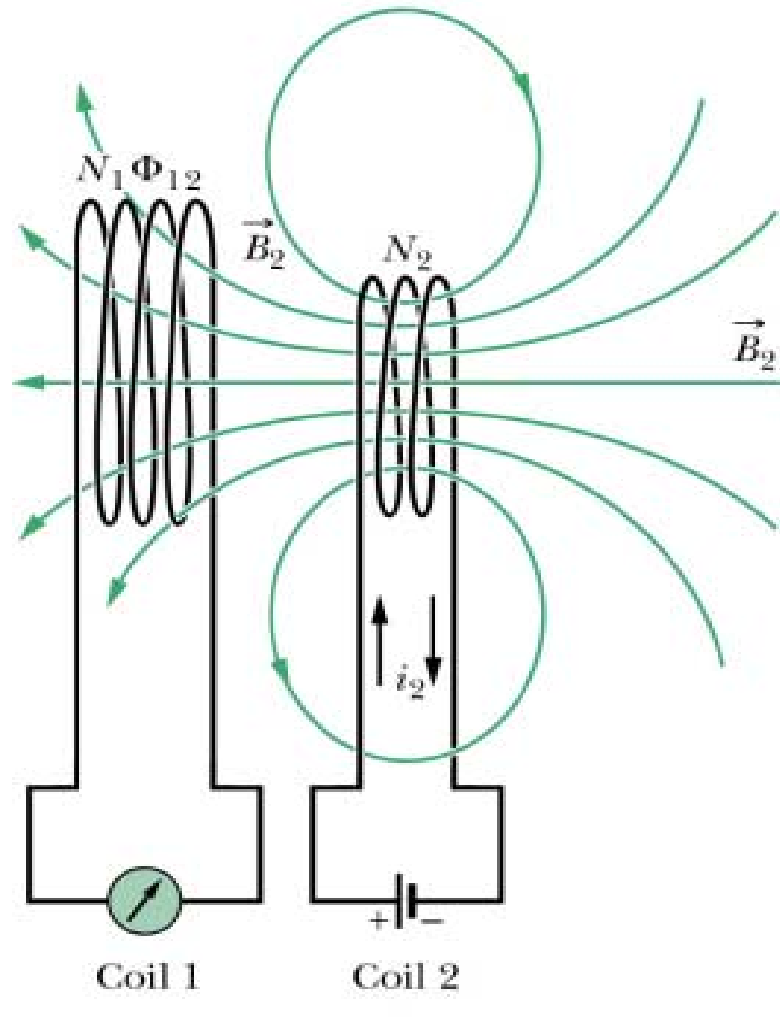
- Obeys Lenz's law (minus sign)

# Inductance (41)

- Reverse roles of coils
- What is induced emf in coil 1 from a changing current in coil 2?
- Same game as before

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2}$$

$$E_1 = -M_{12} \frac{di_2}{dt}$$



# Inductance (42)

- The mutual inductance terms are equal

$$M_{21} = M_{12} = M$$

- Rewrite emfs as

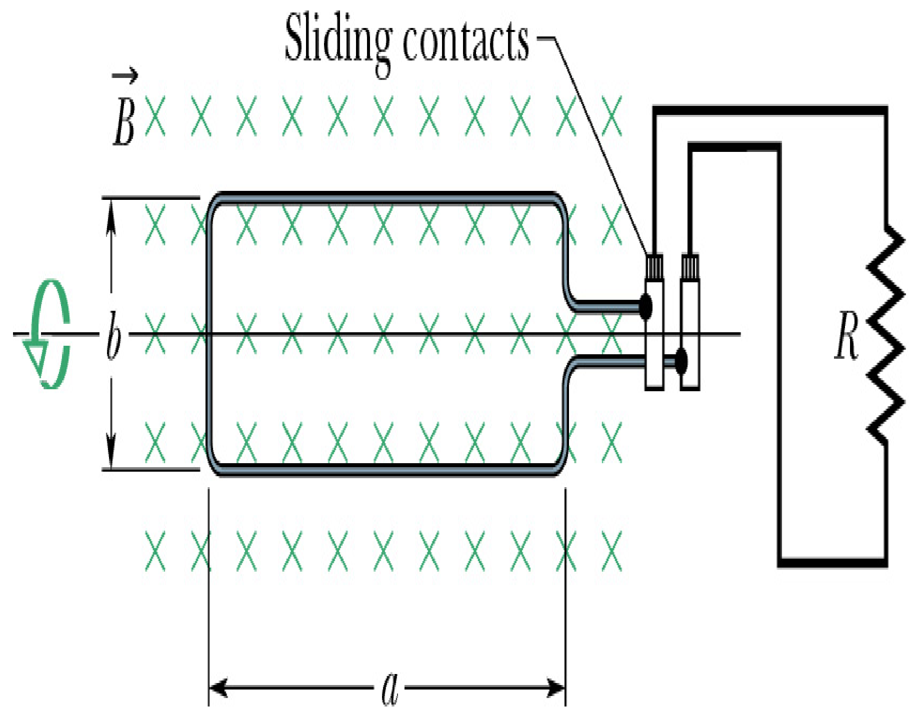
$$E_2 = -M\frac{di_1}{dt} \qquad E_1 = -M\frac{di_2}{dt}$$

- Notice same form as self-induced emf

$$E_L = -L\frac{di}{dt} \qquad L = \frac{N\Phi_B}{i}$$

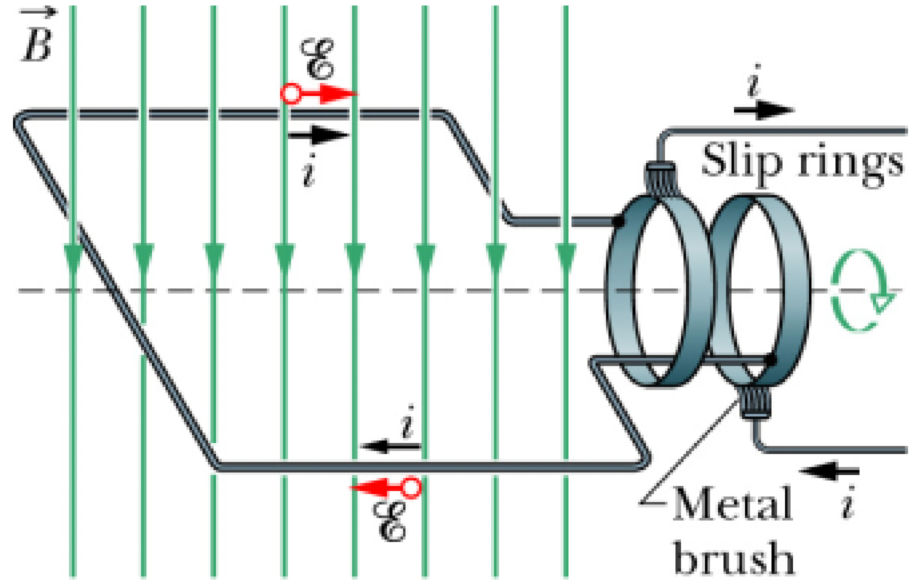
# Inductance (43)

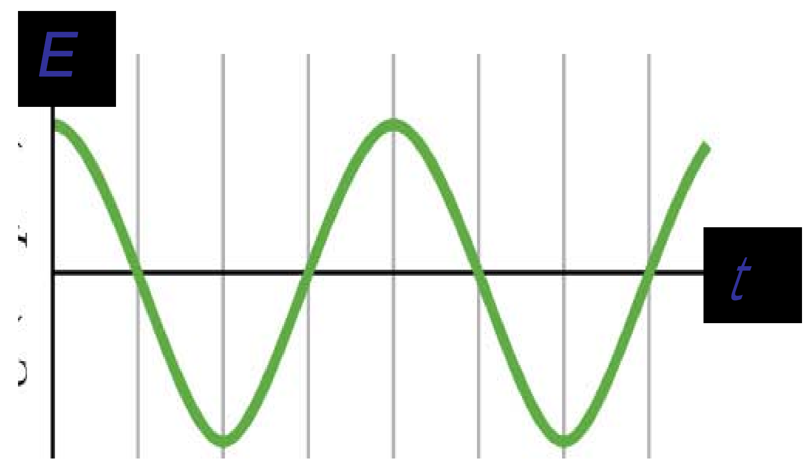
- Generators – convert mechanical energy to electrical energy
- External agent rotates loop of wire in *B* field
  - Hydroelectric plant
  - Coal burning plant
- Changing $\Phi_B$ induces an emf and current in an external circuit



# Inductance (44)

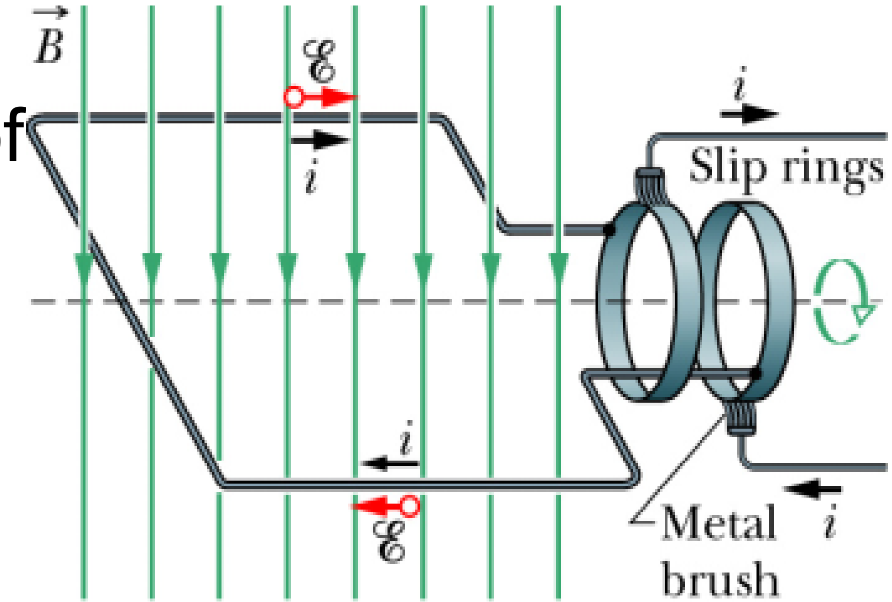
- Alternating current (ac) generator
  - Ends of wire loop are attached to slip rings which rotate with loop
  - Stationary metal brushes are in contact with slip rings and connected to external circuit
  - emf and current in circuit alternate in time



# Inductance (45)

- Calculate emf for generator with N turns of area A and rotating with constant angular velocity, ω




- Magnetic flux is

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

- Relate angular displacement to angular velocity

$$\theta = \omega t$$

- Flux through one loop is

$$\Phi_B = BA\cos\omega t$$

# Inductance (46)

- Faraday's law says

$$E = -N\frac{d\Phi_B}{dt}$$

- Substitute

$$\Phi_B = BA\cos\omega t$$

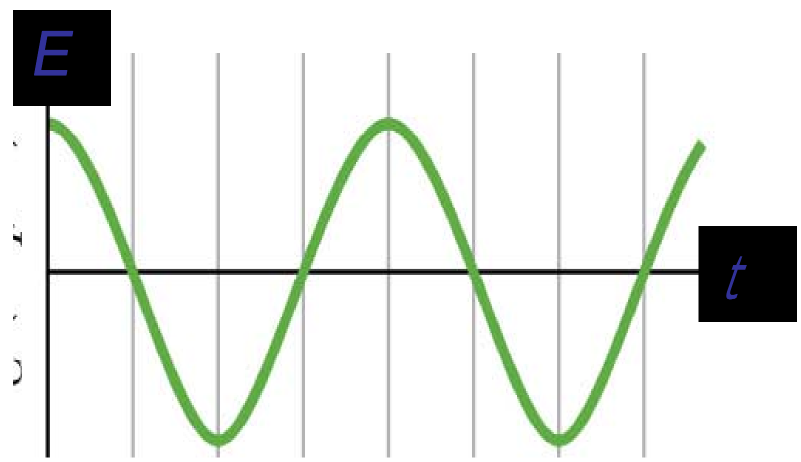
$$E = -NBA\frac{d}{dt}(\cos\omega t)$$

$$E = NBA\omega\sin\omega t$$

- Maximum emf is when $\omega t$ = 90 or 270 degrees

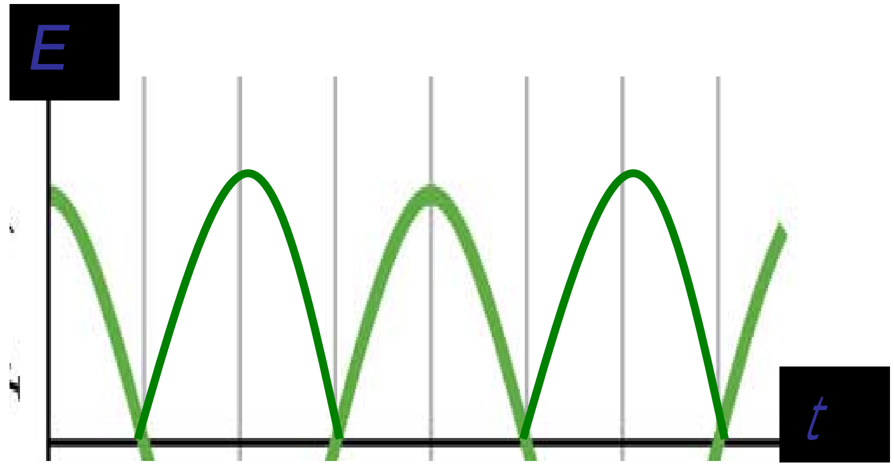
$$E_{\max} = NBA\omega$$

- Emf is 0 when $\omega t$ = 0 or 180 degrees



# Inductance (47)

- Direct current (dc) generator
  - Ends of loop are connected to a single split ring
  - Metal brush contacts to split ring reverse their roles every half cycle
  - Polarity of induced emf reverses but polarity of split ring remains the same



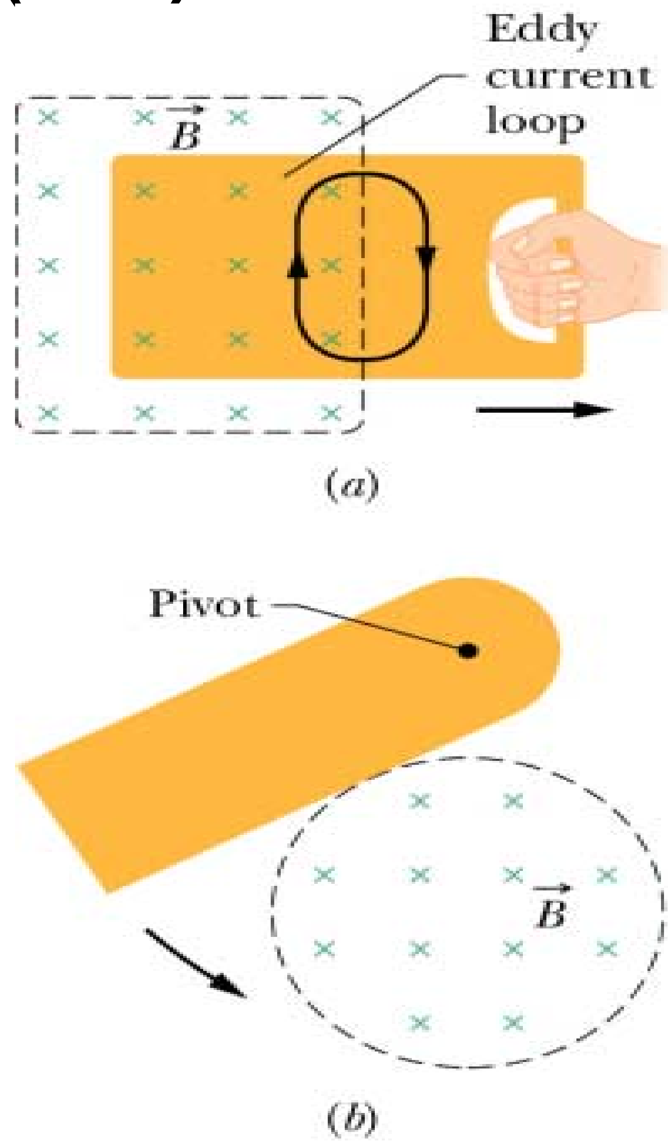

- Not suitable for most applications
  - Can use to charge batteries
- Commercial dc gen. use out of phase coils

# Inductance (48)

- Motors – converts electrical energy to mechanical energy
  - Generator run in reverse
  - Current is supplied to loop and the torque acting on the current-carrying loop causes it to rotate
  - Do mechanical work by using the rotating armature
  - As loop rotates, changing $B$ field induces an emf
  - Induced emf (back emf) reduces the current in the loop – remember Lenz's law
  - Power requirements are greater for starting a motor and for running it under heavy loads

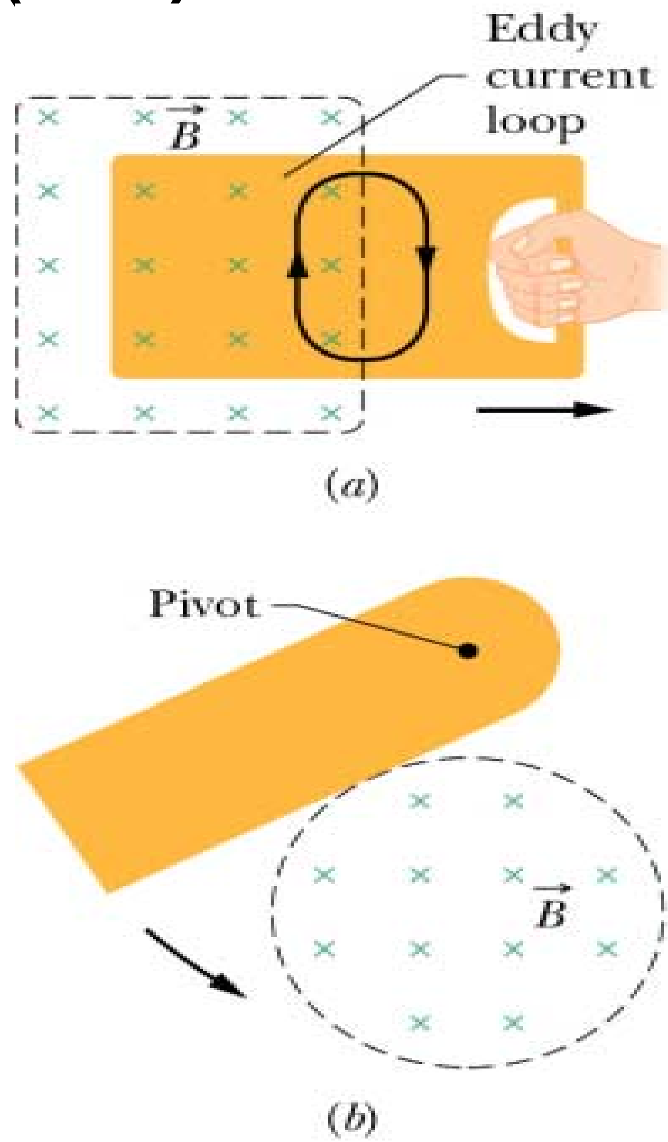
# Inductance (49)

- Instead of a loop of wire, what happens when a bulk piece of metal moves through a *B* field?
- Free electrons in metal move in circles as if caught in a whirlpool called eddy currents
- A metal plate swinging through a *B* field will generate eddy currents



# Inductance (50)

- Eddy currents will oppose the change that caused them – Lenz’s law
- Induced eddy currents will always produce a retarding force when plate enters or leaves *B* field causing the plate to come to rest
- Cutting slots in metal plate will greatly reduce the eddy currents



# Inductance (51)

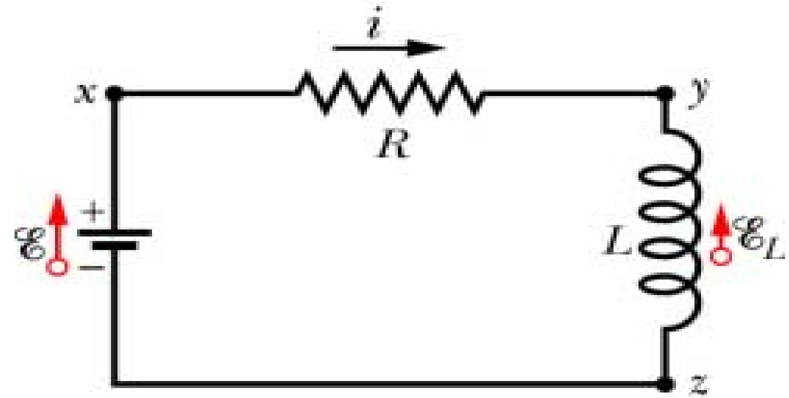
- Induction and eddy currents are used for braking systems on some subways and rapid transit cars
- Moving vehicle has electromagnet (e.g. solenoid) which is positioned near steel rails
- Current in electromagnet generates $B$ field
- Relative motion of $B$ field to rails induces eddy currents in rails
- Direction of eddy currents produce a drag force on the moving vehicle
- Eddy currents decrease steadily as car slows giving a smooth stop

# Inductance (52)

- Eddy currents often undesirable since they dissipate energy in form of heat
- Moving conducting parts often laminated
  - Build up several thin layers separated by nonconducting material
  - Layered structure confines eddy currents to individual layers
- Used in transformers and motors to minimize eddy currents and improve efficiency

# Inductance (53)

- How much energy is stored in a *B* field?
- Conservation of energy expressed in loop rule




$$E = L\frac{di}{dt} + iR$$

- Multiply each side by *i*

$$Ei = Li\frac{di}{dt} + i^2R$$

- *Ei* is rate at which emf device delivers energy to rest of circuit
- $i^2R$ is rate at which energy appears as thermal energy in resistor

# Inductance (54)

- Middle term represents the rate $dU_B/dt$ at which energy is stored in the *B* field

$$Ei = Li\frac{di}{dt} + i^2 R$$

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

$$dU_B = Lidi$$

- Integrating gives

$$\int_0^{U_B} dU_B = \int_0^i Lidi$$

- Energy stored in *B* field

$$U_B = \frac{1}{2}Li^2$$

- Similar to $U_E$

$$U_E = \frac{1}{2}\frac{q^2}{C}$$

# Inductance (55)

- What is the energy density of *B* field?
- Energy density, $u_B$ is energy per unit volume
- Volume is area x length

$$u_B = \frac{U_B}{Al}$$

$$U_B = \frac{1}{2} L i^2$$

- Substituting $U_B$ gives

$$u_B = \frac{Li^2}{l2A}$$

- For a solenoid

$$\frac{L}{l} = \mu_0 n^2 A$$

- Energy density is

$$u_B = \frac{1}{2} \mu_0 n^2 i^2$$

# Inductance (56)

- Remember $B$ field from a solenoid is

$$B = \mu_o i n$$

$$u_B = \frac{1}{2}\mu_0 n^2 i^2$$

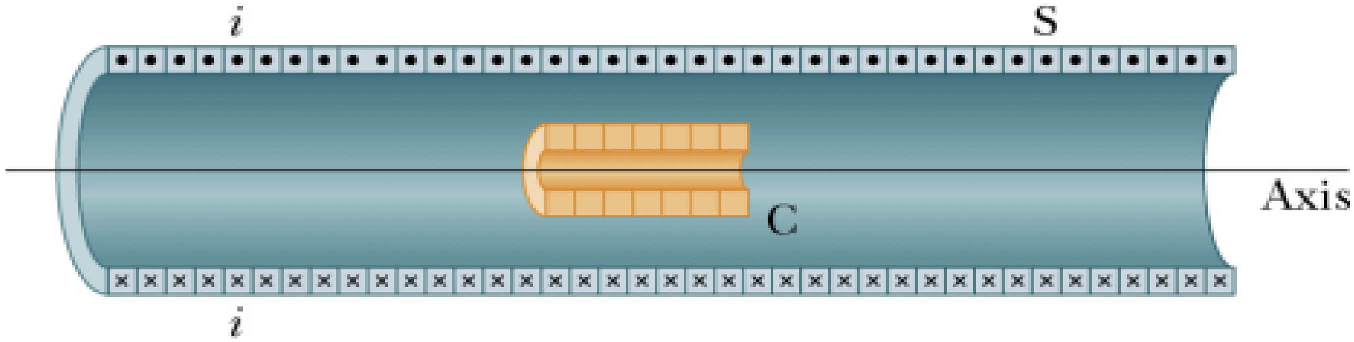
- Substituting $B$ gives magnetic energy density

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

- Similar to electric energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

# Inductance (57)

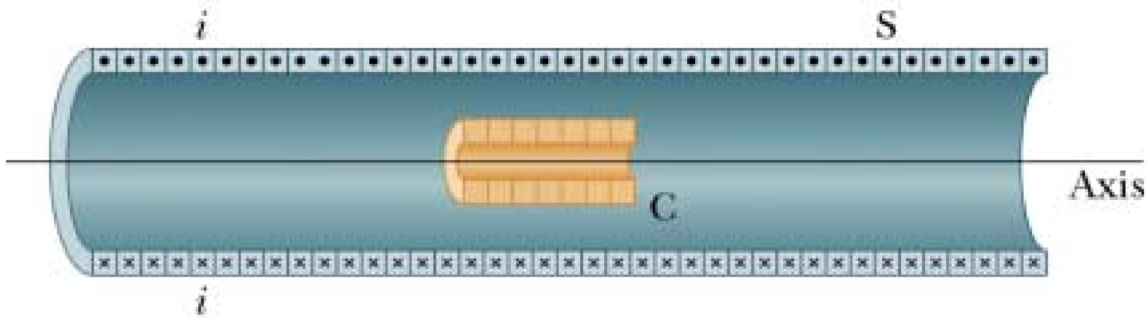



- Place coil C at center of long solenoid which has a steadily decreasing current. What is the magnitude of the induced emf in coil C?
- Solenoid generates uniform *B* field of

$$B = \mu_o i n$$

- Current is decreasing so *B* field decreases

# Inductance (58)



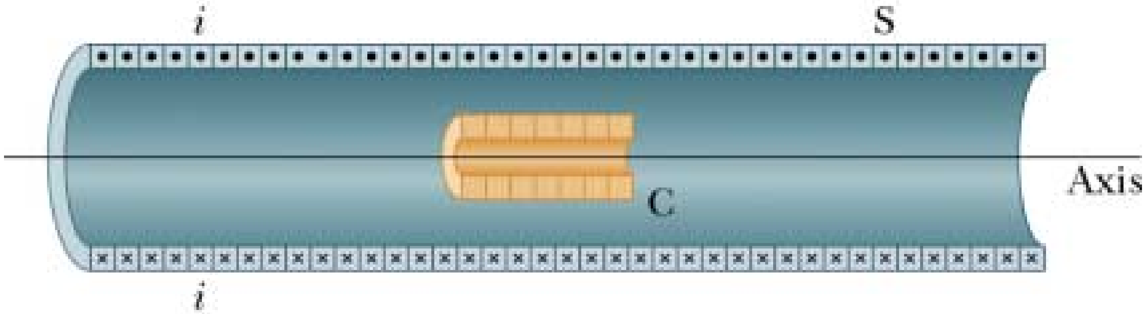

- Since *B* field decreases the flux decreases and an emf is induced in coil C (Faraday's law)

$$\Phi_B = \int \vec{B} \bullet d\vec{A} \qquad E = -N \frac{d\Phi_B}{dt}$$

- The current is decreasing at a steady rate so flux also decreases at steady rate and write

$$\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t}$$

# Inductance (59)



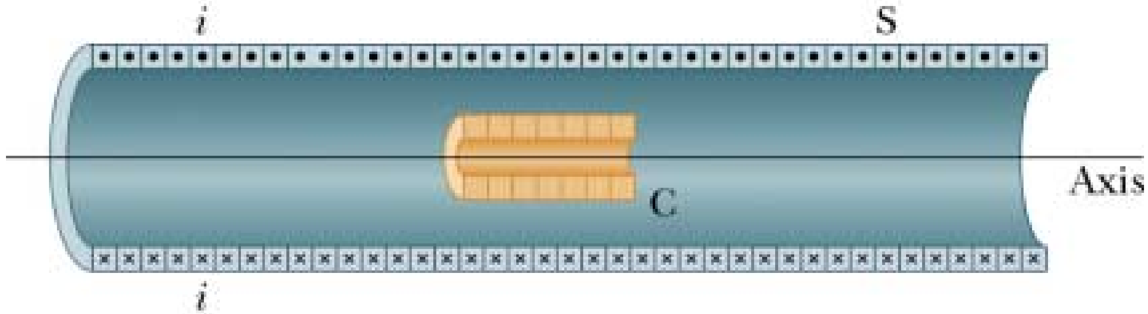

- Need to find initial and final flux
- Current decreases to zero so final flux = 0
- Initial flux

$$\Phi_{B,i} = \int \vec{B} \bullet d\vec{A}$$

- Solenoid has uniform *B* field and it is directed ⊥ to area of coil C

$$\Phi_{B,i} = BA = \mu_0 inA$$

# Inductance (60)




- Substituting into equation for emf

$$E = -N\frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} = N\frac{0 - \mu_0 i n_S A_C}{\Delta t}$$

- Only want magnitude of emf so can ignore minus sign
- $n_S$ is N/L of solenoid but $A_C$ is area of coil C