Lecture 27

Chapter 32

Magnetism of Matter

Review

- 3 types of magnetism
  - Diamagnetism
  - Paramagnetism
  - Ferromagnetism
- Explain how materials exhibit different types of magnetism using electron’s spin and orbital magnetic dipole moments
- Placing material in an external B field causes dipole moments to align creating an induced B field
- Degree of alignment determines type of magnetism and amount of magnetization

Review

- Magnetic flux through an area *A* in a *B* field is

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

- Induced emf occurs when magnetic flux changes with time

$$E = -N \frac{d\Phi_B}{dt}$$

- Changing *B* field induces an *E* field (Faraday's law)

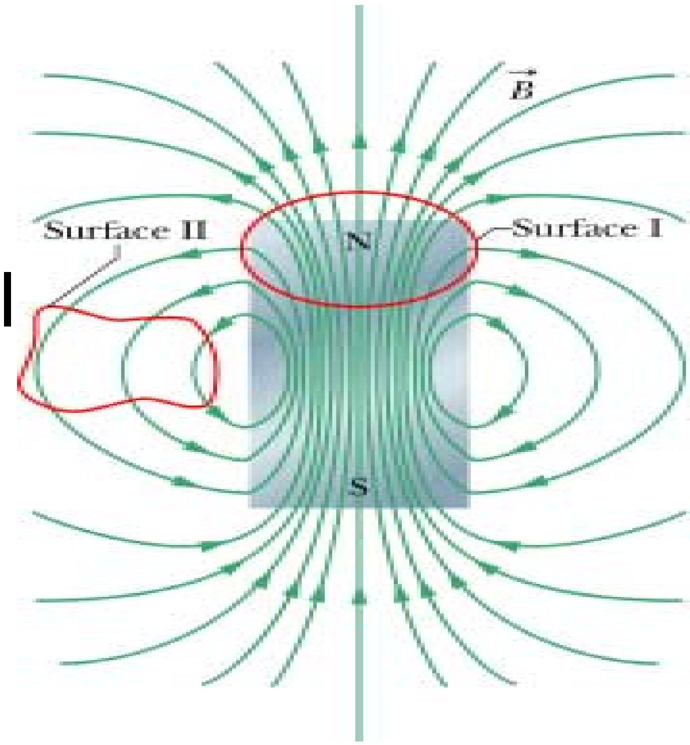
$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

Magnetism (18)

- Magnetic monopoles do not exist
- Express mathematically as

$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$

- Integral is taken over cl surface
- Net magnetic flux through closed surface is zero
  - As many *B* field lines enter leave the surface



Magnetism (19)

- Gauss's law for *E* fields

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- Gauss's law for *B* fields

$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$

- Both cases integrate over closed Gaussian surface

Magnetism (20)

- Faraday's law of induction
  E field is induced along a closed loop by a changing magnetic flux encircled by that loop

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

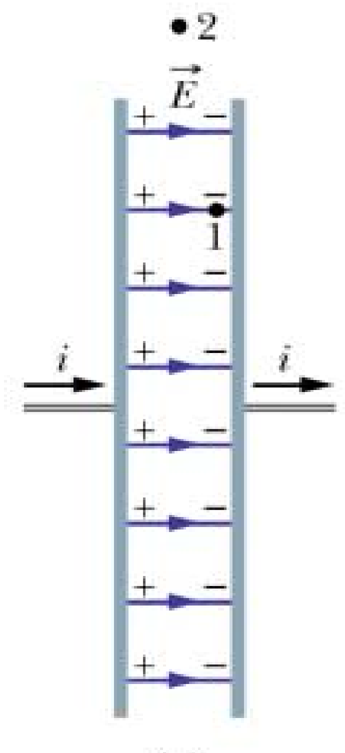
- Is the reverse true?
- Maxwell's law of induction
  B field is induced along a closed loop by a changing electric flux in region encircled by loop

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Magnetism (21)

- Consider circular parallel-plate capacitor with *E* field increasing at a steady rate
- While *E* field changing, *B* fields are induced between plates, both inside and outside (point 1 and 2).
- If *E* field stops changing, *B* field disappears

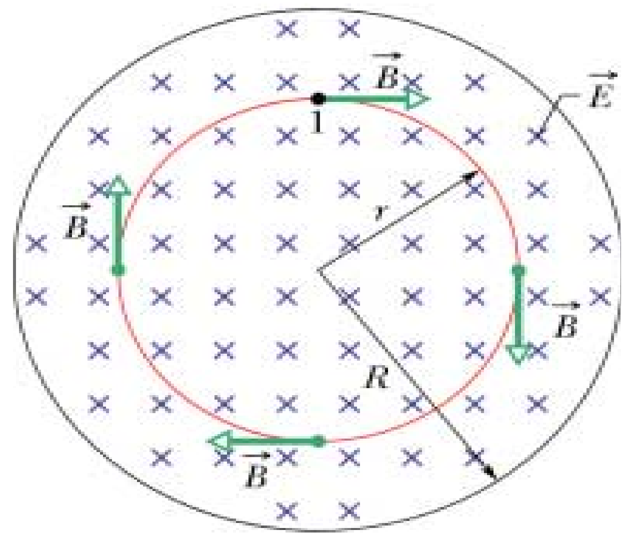
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

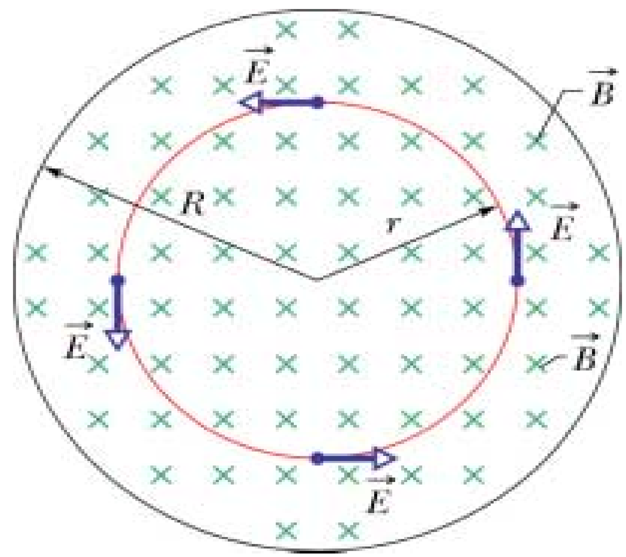


Magnetism (22)

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$





- Two differences
  - Extra symbols, μ_0 and ε_0, to preserve SI units
  - Minus sign – means induced *E* field and induced *B* field have opposite directions when produced in similar situations

Magnetism (23)

- Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Combine Ampere's and Maxwell's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- *B* field can be produced by a current and/or a changing *E* field
  - Wire carrying constant current, $d\Phi_E/dt = 0$
  - Charging a capacitor, no current so $i_{enc} = 0$

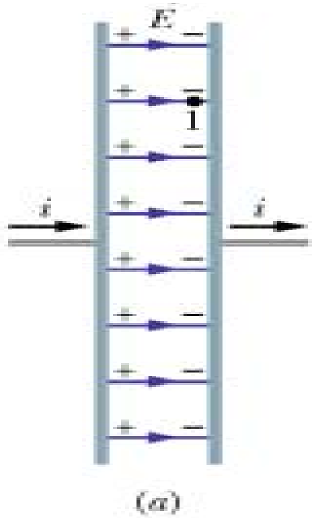
Magnetism (24)

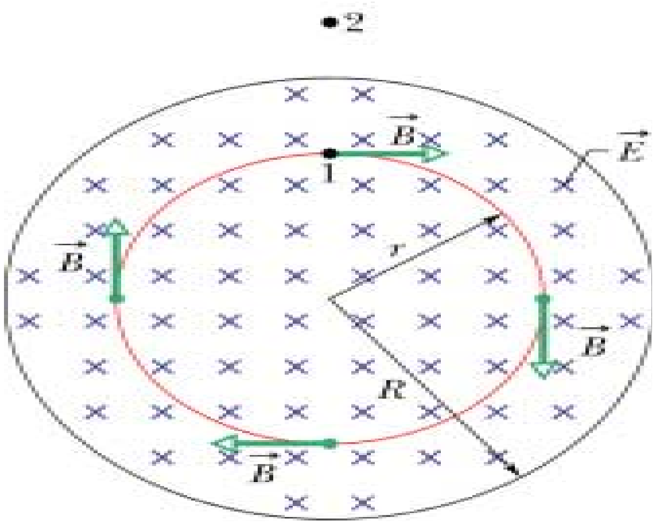
- What is the induced *B* field inside a circular capacitor which is being charged?

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- No current between capacitor plates so $i_{enc} = 0$ and equation becomes

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

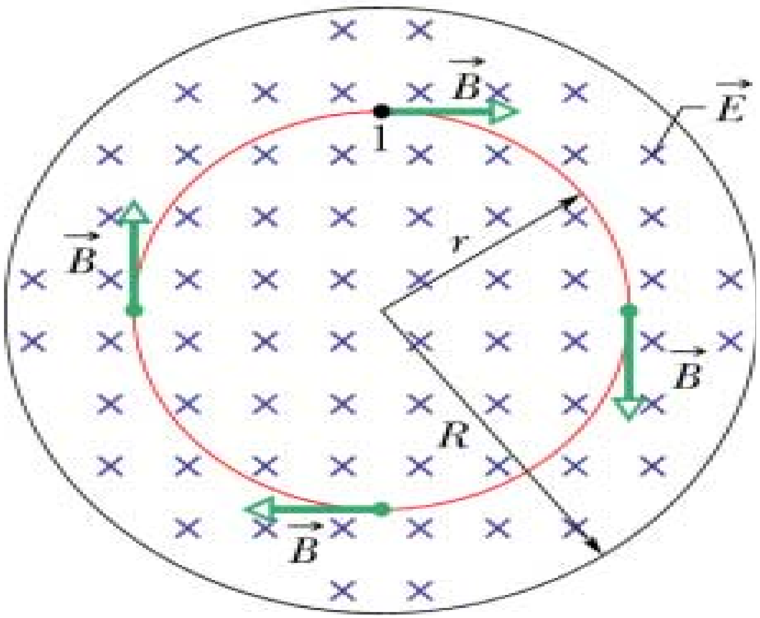




Magnetism (25)

- For left-hand side of equation chose Amperian loop inside capacitor

$$\oint \vec{B} \bullet d\vec{s} = \oint Bds\cos\theta$$



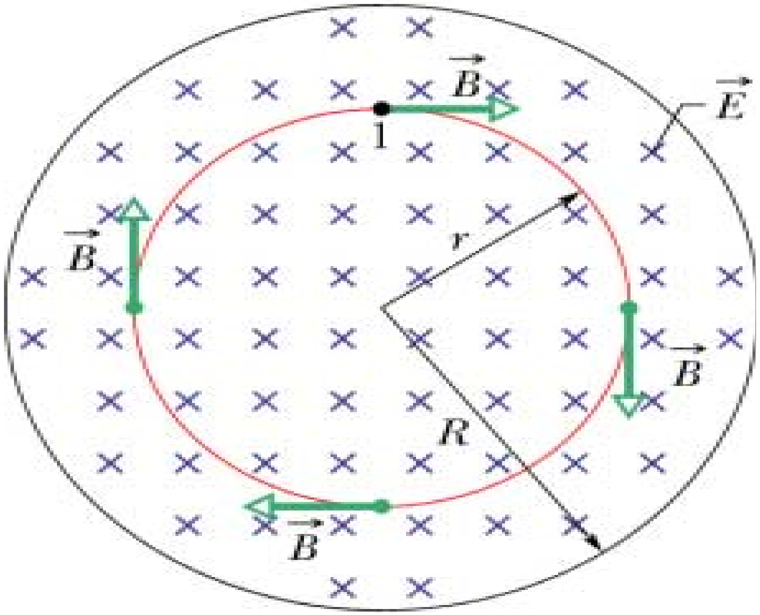
- *B* and *ds* are parallel and *B* is constant so

$$\oint \vec{B} \bullet d\vec{s} = \oint Bds\cos 0 = B\oint ds = B(2\pi r)$$

Magnetism (26)

- For right-hand side of equation find E flux Amperian loop
- E uniform between plates $\perp$ to area A of loop

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = EA$$



- Right-hand side of equation becomes

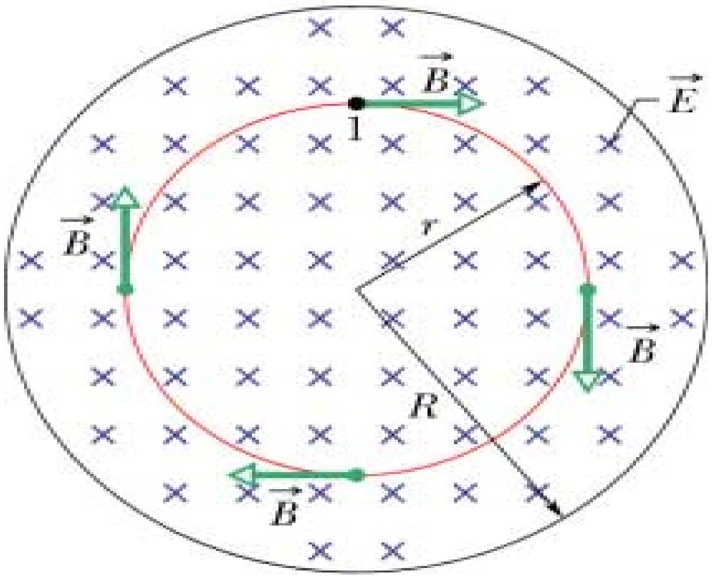
$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt}(EA) = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

Magnetism (27)

- Equating two sides gives

$$B(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

- A is area of loop $A = \pi r^2$



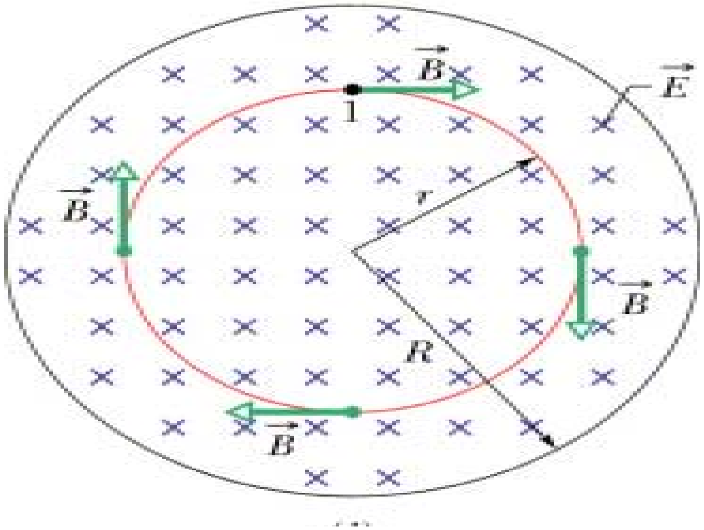
- Solving for *B* field inside capacitor gives

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}$$

- *B* increases linearly with radius
- *B = 0* at center and max at plate edges

Magnetism (28)

- What is the induced *B* field outside a circular capacitor which is being charged?



$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

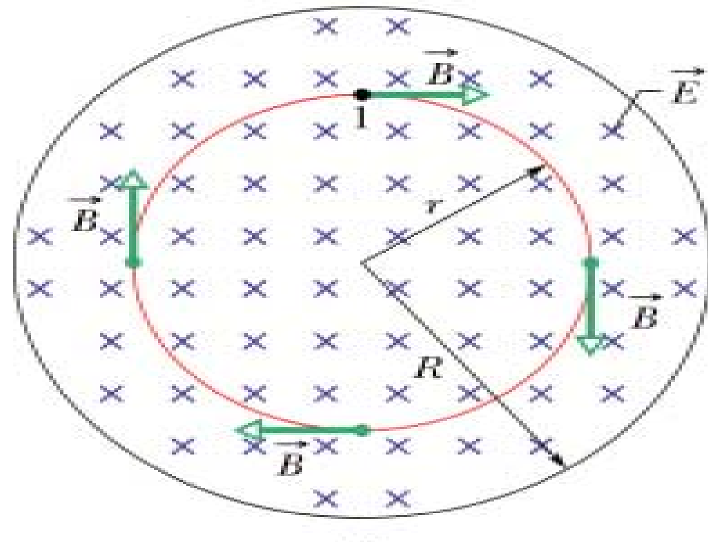
- Realize $i_{enc} = 0$ and find same relations

$$\oint \vec{B} \bullet d\vec{s} = B(2\pi r)$$

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

Magnetism (29)

$$B(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$



- *E* field only exists between plates so area of E field is not full area of loop, only area of plates

$$A = \pi R^2$$

- *B* field becomes

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}$$

- Outside capacitor, *B* decreases with radial distance from a max value at $r = R$

Magnetism (30)

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- Can represent change in electric flux with a fictitious current called the displacement current, i_d

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Ampere-Maxwell's law becomes

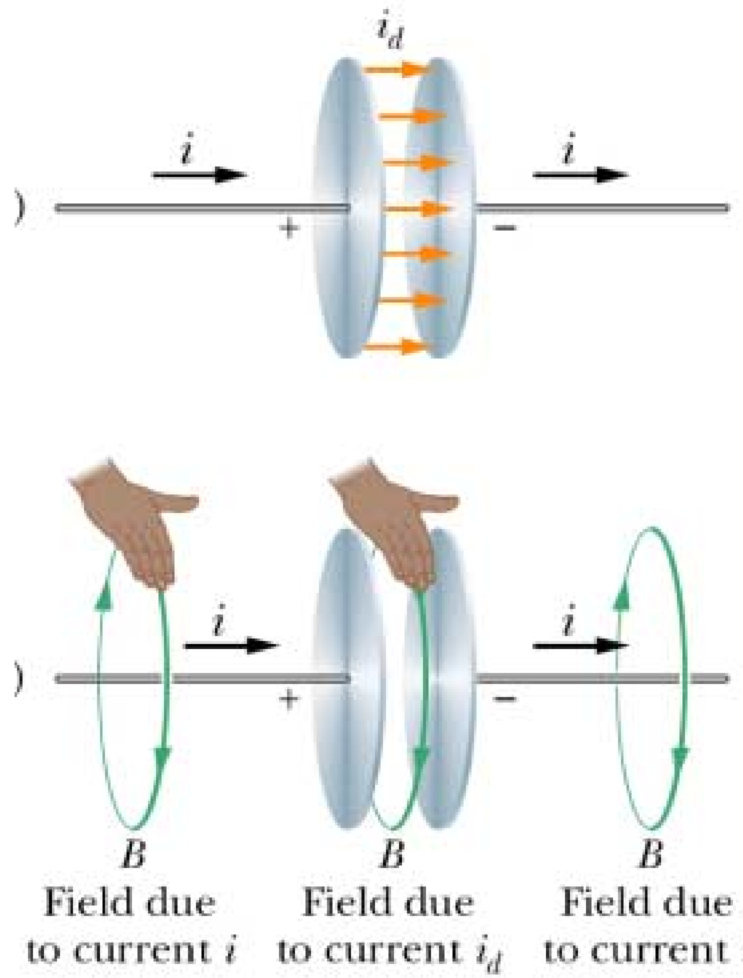
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

Magnetism (31)

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

- Think of displacement current as fictional current between plates

- Use right-hand rule to find direction of *B* field for both currents



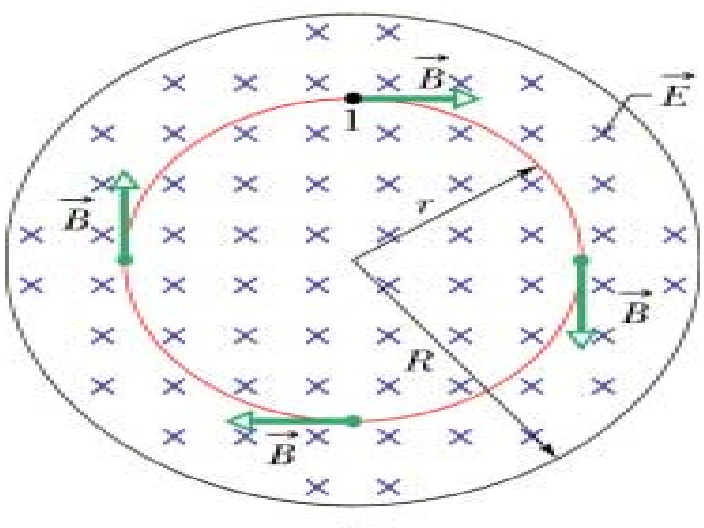
Magnetism (32)

- Used Ampere's law to calculate *B* field inside a long straight wire with current *i*

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r$$

- Find *B* field inside a circular capacitor just replace *i* with displacement current, i_d



$$B = \left(\frac{\mu_0 i_d}{2\pi R^2} \right) r$$

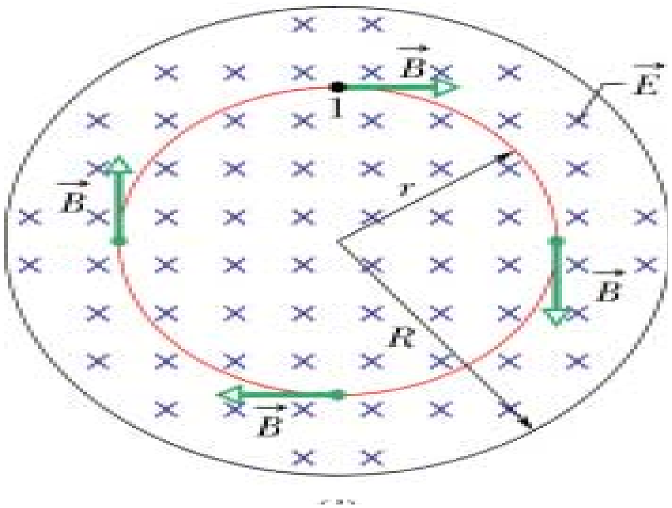
Magnetism (33)

- Used Ampere's law to calculate B field outside a long straight wire with current i

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$B = \frac{\mu_0 i}{2\pi r}$$

- Find B field outside a circular capacitor just replace i with displacement current, i_d

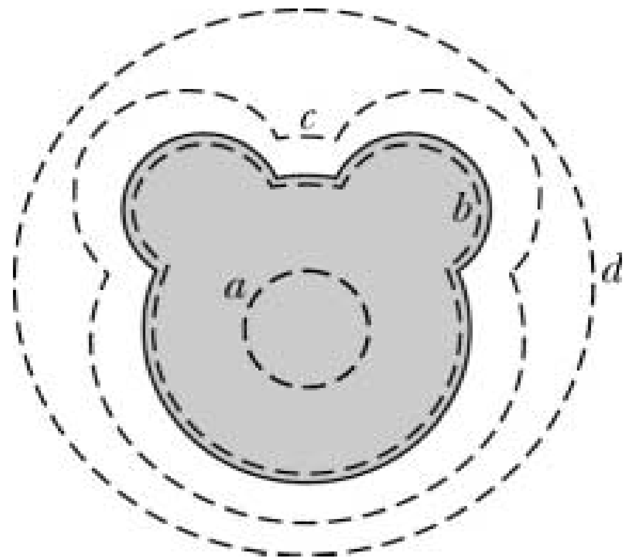


$$B = \frac{\mu_0 i_d}{2\pi r}$$

Magnetism (34)

- Checkpoint #6 – Parallel-plate capacitor of shape shown. Dashed lines are paths of integration. Rank the paths according to the magnitude of integral *Bds* when capacitor is discharging, greatest first.

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$



- Only displaced current in

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc}$$

- What is i_d for each path?

b, c, d all tie, then a

Magnetism (35)

- Basis of all electrical and magnetic phenomena can be described by 4 equations called Maxwell’s equations
- As fundamental to electromagnetism as Newton’s law are to mechanics
- Einstein showed that Maxwell’s equations work with special relativity
- Maxwell’s equations basis for most equations studied since beginning of semester and will be basis for most of what we do the rest of the semester

Magnetism (36)

Maxwell's 4 equations are

- Gauss' Law $$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
- Gauss' Law for magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$
- Faraday's Law $$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$
- Ampere-Maxwell Law $$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$