Review

• 3 types of magnetism
  – Diamagnetism
  – Paramagnetism
  – Ferromagnetism

• Explain how materials exhibit different types of magnetism using electron’s spin and orbital magnetic dipole moments

• Placing material in an external $B$ field causes dipole moments to align creating an induced $B$ field

• Degree of alignment determines type of magnetism and amount of magnetization
Review

• Magnetic flux through an area $A$ in a $B$ field is

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

• Induced emf occurs when magnetic flux changes with time

\[ E = -N \frac{d\Phi_B}{dt} \]

• Changing $B$ field induces an $E$ field (Faraday’s law)

\[ \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]
Magnetism (18)

• Magnetic monopoles do not exist
• Express mathematically as

\[
\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0
\]

• Integral is taken over closed surface
• Net magnetic flux through closed surface is zero
  – As many \( B \) field lines enter leave the surface
Magnetism (19)

- Gauss’s law for $E$ fields

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- Gauss’s law for $B$ fields

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

- Both cases integrate over closed Gaussian surface
Magnetism (20)

- Faraday’s law of induction
  $E$ field is induced along a closed loop by a changing magnetic flux encircled by that loop

- Maxwell’s law of induction
  $B$ field is induced along a closed loop by a changing electric flux in region encircled by loop

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
Magnetism (21)

- Consider circular parallel-plate capacitor with $E$ field increasing at a steady rate.
- While $E$ field changing, $B$ fields are induced between plates, both inside and outside (point 1 and 2).
- If $E$ field stops changing, $B$ field disappears.

\[ \int \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
Magnetism (22)

- Two differences
  - Extra symbols, $\mu_0$ and $\varepsilon_0$, to preserve SI units
  - Minus sign – means induced $E$ field and induced $B$ field have opposite directions when produced in similar situations

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]
\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
Magnetism (23)

- Ampere’s law
  \[ \int \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \]

- Combine Ampere’s and Maxwell’s law
  \[ \int \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \]

- \( \vec{B} \) field can be produced by a current and/or a changing \( E \) field
  - Wire carrying constant current, \( d\Phi_E/dt = 0 \)
  - Charging a capacitor, no current so \( i_{\text{enc}} = 0 \)
Magnetism (24)

• What is the induced $B$ field inside a circular capacitor which is being charged?

\[ \oint B \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \]

• No current between capacitor plates so $i_{enc} = 0$ and equation becomes

\[ \oint B \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
Magnetism (25)

- For left-hand side of equation chose Amperian loop inside capacitor

\[ \oint \vec{B} \cdot d\vec{s} = \oint Bds \cos \theta \]

- \( B \) and \( ds \) are parallel and \( B \) is constant so

\[ \oint \vec{B} \cdot d\vec{s} = \oint Bds \cos 0 = B \oint ds = B(2\pi r) \]
Magnetism (26)

• For right-hand side of equation find $E$ flux Amperian loop

• $E$ uniform between plates $\perp$ to area $A$ of loop

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = EA \]

• Right-hand side of equation becomes

\[ \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt} (EA) = \mu_0 \varepsilon_0 A \frac{dE}{dt} \]
Magnetism (27)

- Equating two sides gives

\[ B(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt} \]

- A is area of loop

\[ A = \pi r^2 \]

- Solving for \( B \) field inside capacitor gives

\[ B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} \]

- \( B \) increases linearly with radius

- \( B = 0 \) at center and max at plate edges
Magnetism (28)

• What is the induced $B$ field outside a circular capacitor which is being charged?

\[ \oint B \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi}{dt} + \mu_0 i_{enc} \]

• Realize $i_{enc} = 0$ and find same relations

\[ \oint B \cdot d\vec{s} = B(2\pi r) \]

\[ \mu_0 \varepsilon_0 \frac{d\Phi}{dt} = \mu_0 \varepsilon_0 A \frac{dE}{dt} \]
Magnetism (29)

\[ B(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt} \]

- \( E \) field only exists between plates so area of \( E \) field is not full area of loop, only area of plates

\[ A = \pi R^2 \]

- \( B \) field becomes

\[ B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} \]

- Outside capacitor, \( B \) decreases with radial distance from a max value at \( r = R \)
Magnetism (30)

\[ \int \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \]

- Can represent change in electric flux with a fictitious current called the displacement current, \( i_d \)

\[ i_d = \varepsilon_0 \frac{d\Phi_E}{dt} \]

- Ampere-Maxwell’s law becomes

\[ \int \vec{B} \cdot d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc} \]
**Magnetism (31)**

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}
\]

- Think of displacement current as fictional current between plates

- Use right-hand rule to find direction of \( B \) field for both currents
Magnetism (32)

• Used Ampere’s law to calculate $B$ field inside a long straight wire with current $i$

$$B = \left( \frac{\mu_0 i}{2\pi R^2} \right) r$$

• Find $B$ field inside a circular capacitor just replace $i$ with displacement current, $i_d$

$$B = \left( \frac{\mu_0 i_d}{2\pi R^2} \right) r$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$
Magnetism (33)

• Used Ampere’s law to calculate $B$ field outside a long straight wire with current $i$

$$B = \frac{\mu_0 i}{2\pi r}$$

• Find $B$ field outside a circular capacitor just replace $i$ with displacement current, $i_d$

$$B = \frac{\mu_0 i_d}{2\pi r}$$
Magnetism (34)

- Checkpoint #6 – Parallel-plate capacitor of shape shown. Dashed lines are paths of integration. Rank the paths according to the magnitude of integral $Bds$ when capacitor is discharging, greatest first.

\[
\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}
\]

- Only displaced current in

\[
\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_0 i_{d,enc}
\]

- What is $i_d$ for each path?

b, c, d all tie, then a
Magnetism (35)

• Basis of all electrical and magnetic phenomena can be described by 4 equations called Maxwell’s equations

• As fundamental to electromagnetism as Newton’s law are to mechanics

• Einstein showed that Maxwell’s equations work with special relativity

• Maxwell’s equations basis for most equations studied since beginning of semester and will be basis for most of what we do the rest of the semester
Magnetism (36)

Maxwell’s 4 equations are

- Gauss’ Law
  \[ \oint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} \]

- Gauss’ Law for magnetism
  \[ \oint B \cdot dA = 0 \]

- Faraday’s Law
  \[ \oint E \cdot dS = -\frac{d\Phi_B}{dt} \]

- Ampere-Maxwell Law
  \[ \oint B \cdot dS = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0i_{\text{enc}} \]