Lecture 28

Chapter 33

EM Oscillations and AC

Review

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- Can represent change in electric flux with a fictitious current called the displacement current, i_d

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Ampere-Maxwell's law becomes

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

Review

Maxwell's 4 equations are

- Gauss' Law $$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
- Gauss' Law for magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$
- Faraday's Law $$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$
- Ampere-Maxwell Law $$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

EM Oscillations (1)

- In this chapter study an RLC circuit

- Review RC and RL circuits
- Charge, current and potential difference grow and decay exponentially given by time constant, τ_C or τ_L

- Look at LC circuit

EM Oscillations (2)

- RC circuit - resistor & capacitor in series
  - Charging up a capacitor

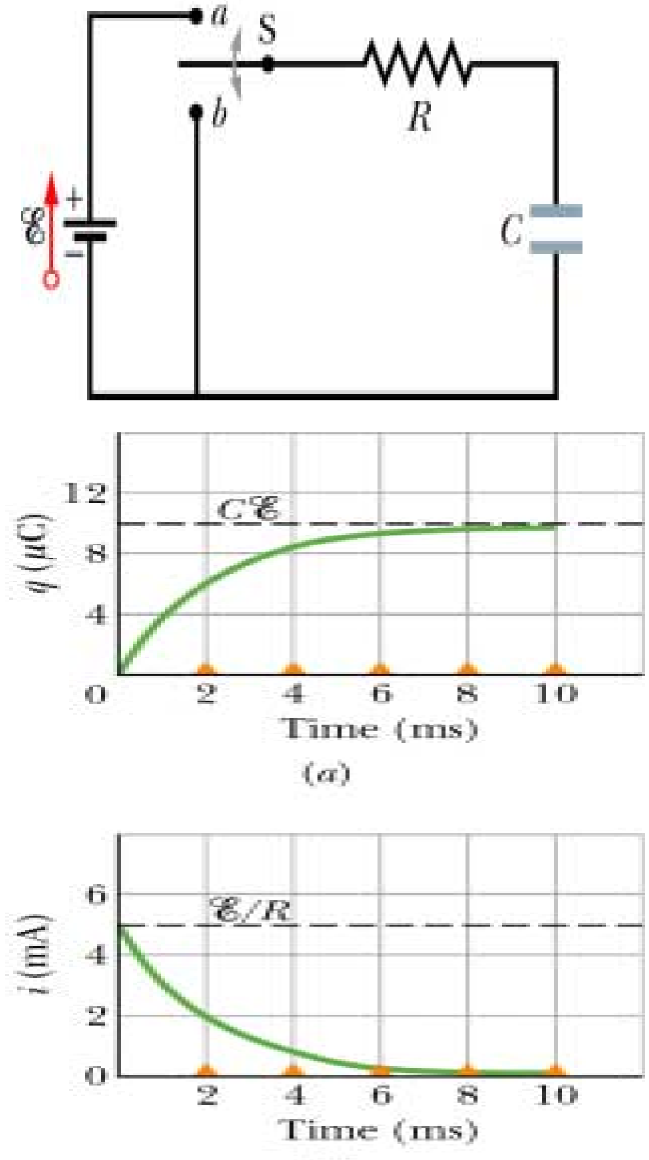
$$q = CE(1 - e^{-t/\tau_c})$$

  - Discharging capacitor

$$q = q_0 e^{-t/\tau_c}$$

  - where

$$\tau_C = RC$$



EM Oscillations (3)

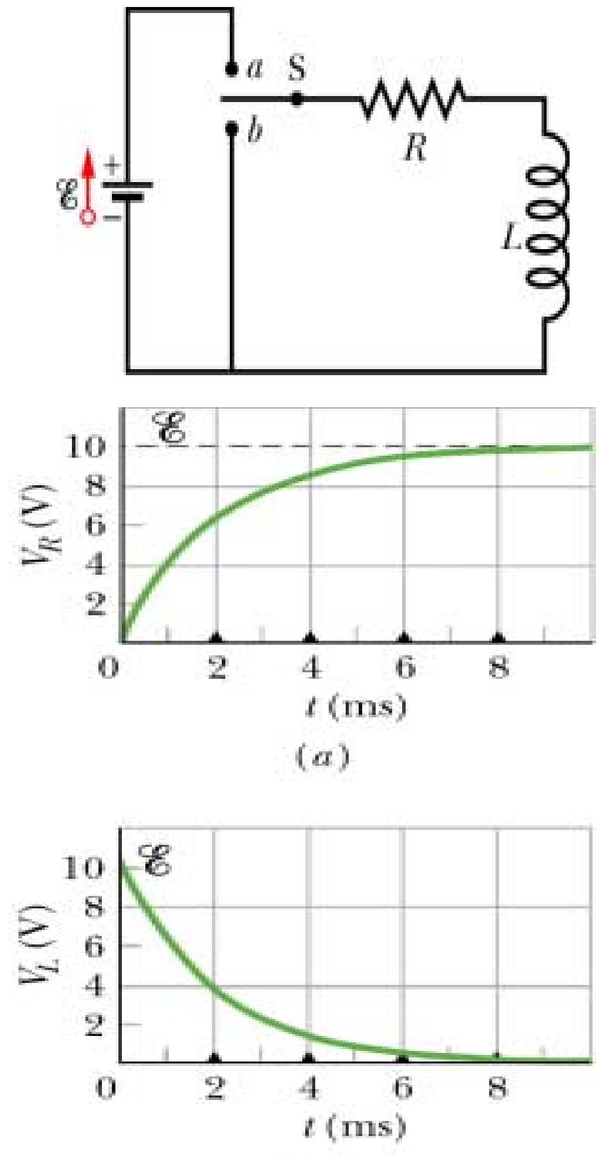
- RL Circuit – resistor & inductor in series
  - Rise of current

$$i = \frac{E}{R}\left(1 - e^{-t/\tau_L}\right)$$

  - Decay of current

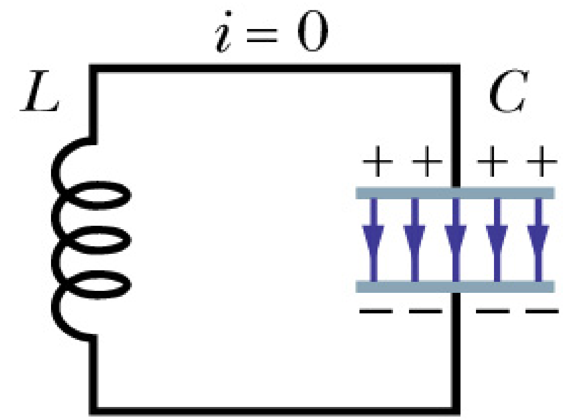
$$i = i_0 e^{-t/\tau_L}$$

  - where $\tau_L = \frac{L}{R}$



EM Oscillations (4)

- LC Circuit – inductor & capacitor in series
- Find *q*, *i* and *V* vary sinusoidally with period *T* and angular frequency ω
- *E* field of capacitor and *B* field of inductor oscillate

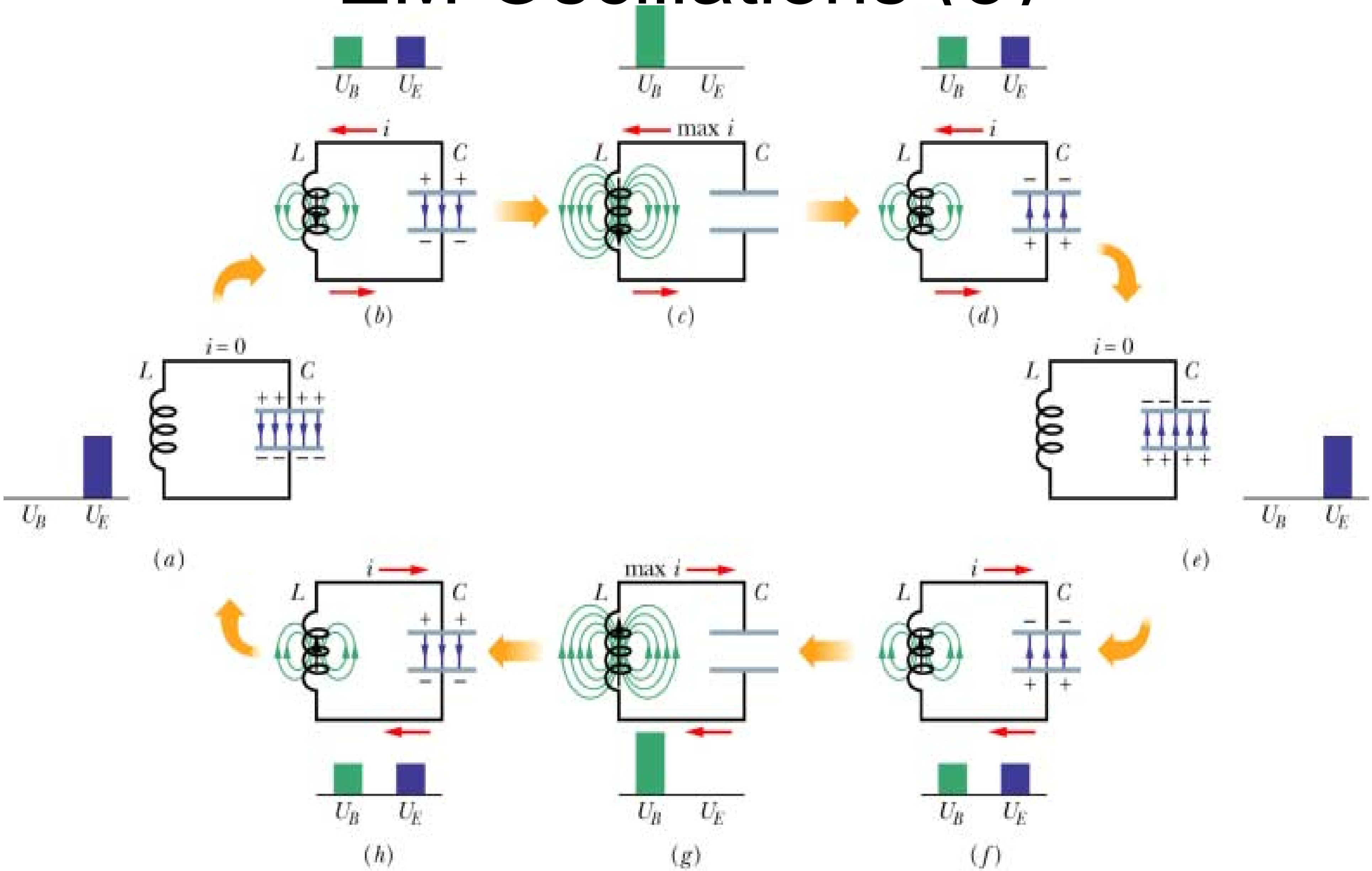


- Called Electromagnetic oscillations
- Energy stored in *E* of capacitor and *B* of inductor

$$U_E = \frac{q^2}{2C}$$

$$U_B = \frac{Li^2}{2}$$

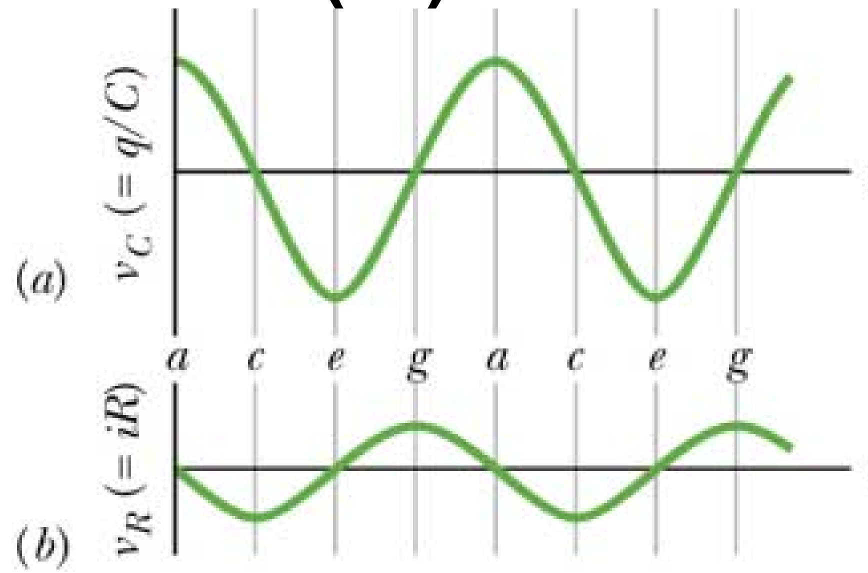
EM Oscillations (5)



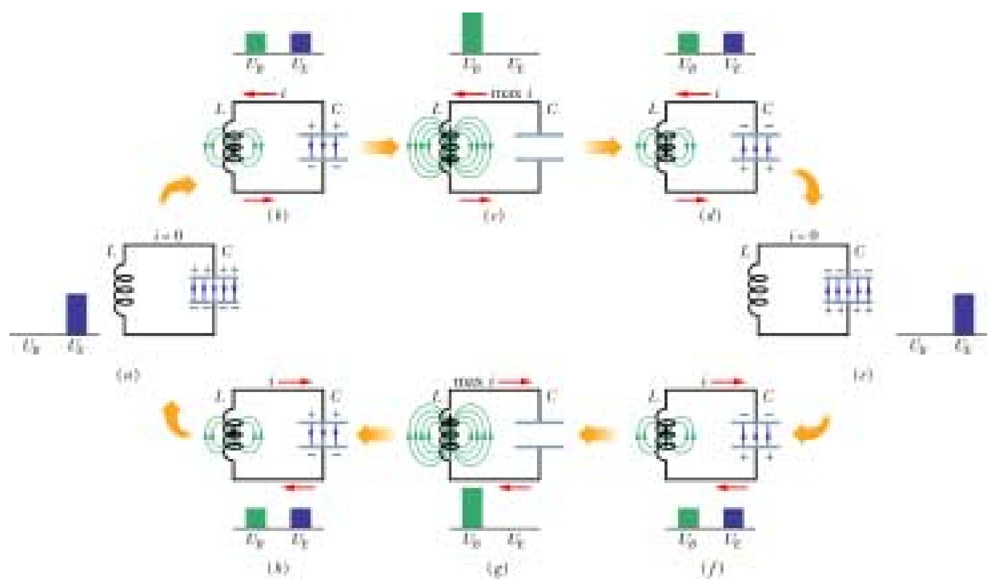
EM Oscillations (6)

- Cycle repeats at some frequency, *f* and thus angular frequency, ω

$$\omega = 2\pi f$$



- Ideal LC circuit, no *R* so oscillations continue indefinitely
- Real LC, oscillations die away as energy goes into heat in *R*



EM Oscillations (7)

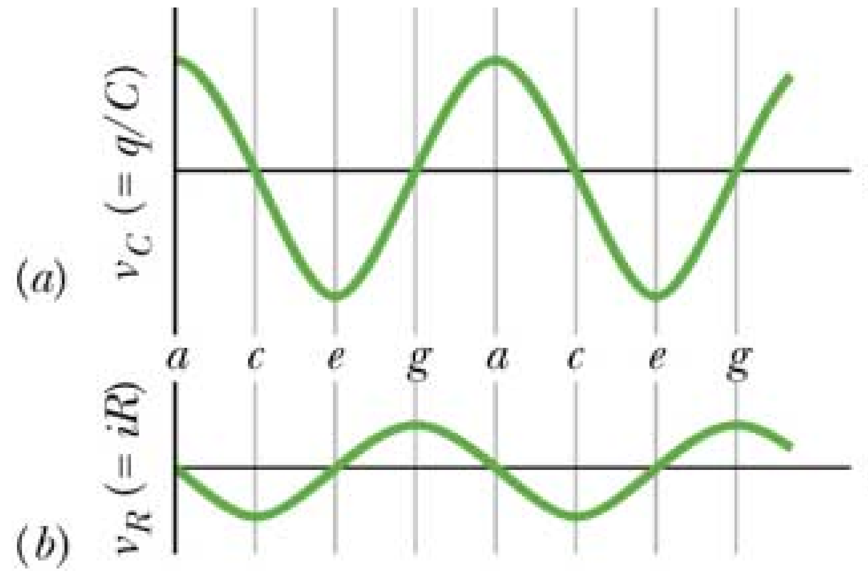
- Checkpoint #1 – A charged capacitor & inductor are connected in series at time t=0. In terms of period, T, how much later will the following reach their maximums:
  - *q* of capacitor

    T/2
  - V_C with original polarity

    T
  - Energy stored in *E* field

    T/2
  - The current

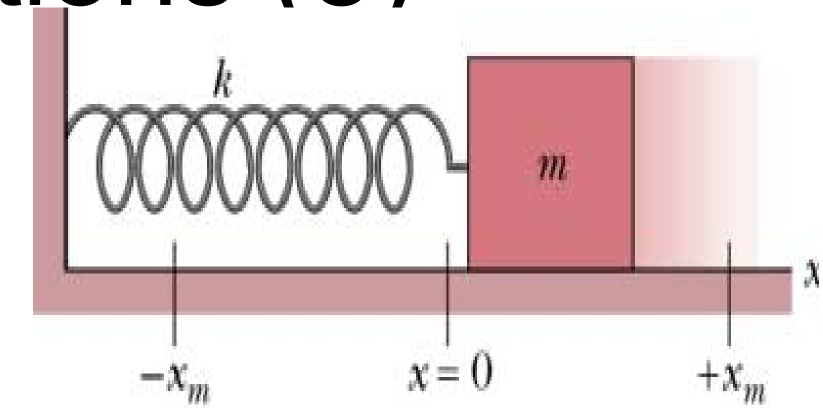
    T/4



EM Oscillations (8)

- LC circuits analogous to block-spring system
- Total energy of block

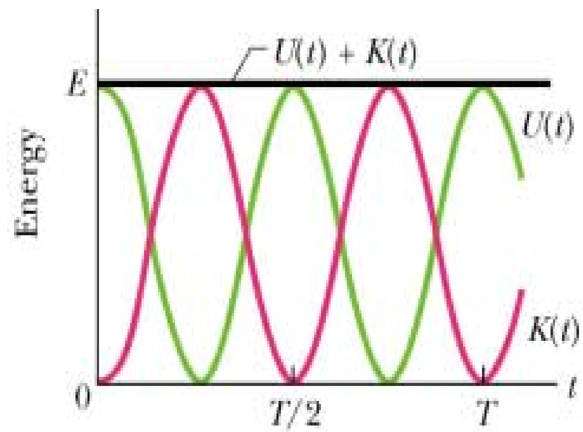
$$U = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$$



- Energy is conserved

$$\frac{dU}{dt} = 0$$

- Differentiating gives



$$\frac{dU}{dt} = \frac{d}{dt}\left(\tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2\right) = m v \frac{dv}{dt} + k x \frac{dx}{dt} = 0$$

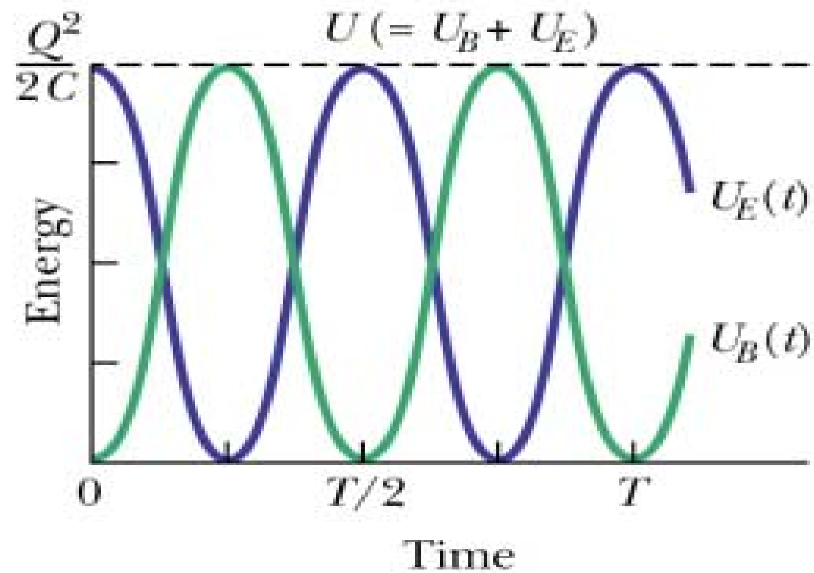
EM Oscillations (9)

- Using $$v = \frac{dx}{dt} \qquad \frac{dv}{dt} = \frac{d^2x}{dt^2}$$
- Substitute $$mv\frac{dv}{dt} + kx\frac{dx}{dt} = mv\frac{d^2x}{dt^2} + kxv = 0$$
- Gives $$m\frac{d^2x}{dt^2} + kx = 0$$
- Solution is $$x = X\cos(\omega t + \phi)$$
- X is the amplitude
- ω is the angular frequency $$\omega = \sqrt{\frac{k}{m}}$$
- φ is the phase constant

EM Oscillations (10)

- Total energy of LC circuit

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$



- Total energy is constant

$$\frac{dU}{dt} = 0$$

- Differentiating gives

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0$$

EM Oscillations (11)

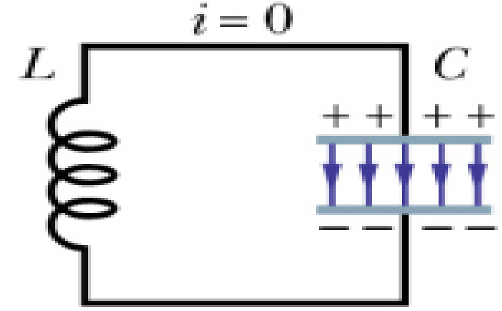
- Using $i = \frac{dq}{dt}$ $\frac{di}{dt} = \frac{d^2q}{dt^2}$
- Substitute $$Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = Li\frac{d^2q}{dt^2} + \frac{q}{C}i = 0$$
- Equation same form as block and spr
$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$
- Solution is
$$q = Q\cos(\omega t + \phi)$$

- Q is the amplitude
- ω is the angular frequency
- ϕ is the phase constant

EM Oscillations (12)

- Charge of LC circuit

$$q = Q\cos(\omega t + \phi)$$



- Find current by $i = \dfrac{dq}{dt}$

$$i = \frac{d}{dt}\left[Q\cos(\omega t + \phi)\right] = -Q\omega\sin(\omega t + \phi)$$

- Amplitude I is

$$I = \omega Q$$

$$i = -I\sin(\omega t + \phi)$$

EM Oscillations (13)

- What is ω for an LC circuit?

$$q = Q\cos(\omega t + \phi)$$

$$\frac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi)$$

- Substitute into

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

$$-L\omega^2 Q\cos(\omega t + \phi) + \frac{1}{C}Q\cos(\omega t + \phi) = 0$$

- Find ω for LC circuit is

$$\omega = \sqrt{\frac{1}{LC}}$$

EM Oscillations (14)

- The phase constant, ϕ, is determined by conditions at any certain time, t

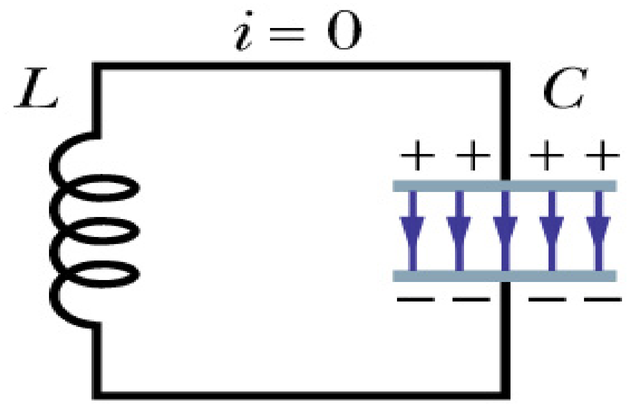
$$q = Q\cos(\omega t + \phi)$$

$$i = -I\sin(\omega t + \phi)$$

- If $\phi = 0$ at $t = 0$ then

  $q = Q$

  $i = 0$



EM Oscillations (15)

- The energy stored in an LC circuit at any time, *t*

$$U = U_B + U_E$$

- Substitute $q = Q\cos(\omega t + \phi)$ $i = -I\sin(\omega t + \phi)$

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{Li^2}{2} = \frac{L}{2}\omega^2 Q^2 \sin^2(\omega t + \phi)$$

- Using $\omega = \sqrt{\frac{1}{LC}}$

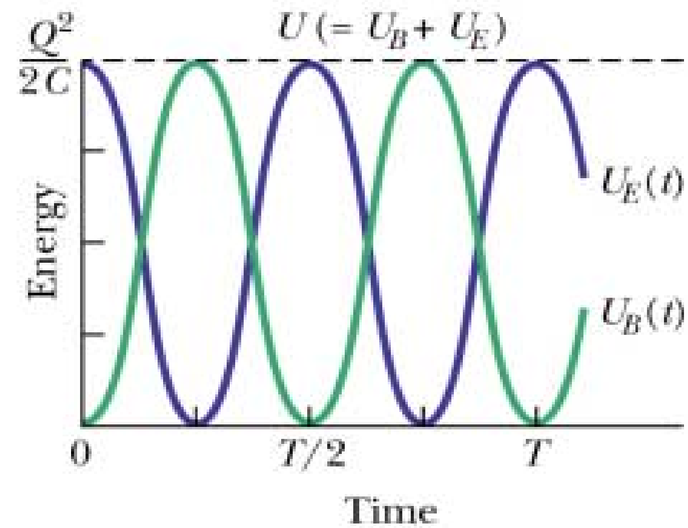
$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$$

EM Oscillations (16)

- For the case where $\phi = 0$

$$U_E = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$$

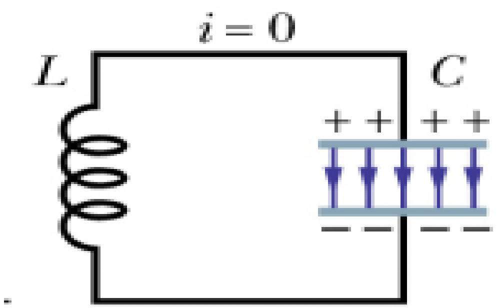


- Maximum value for both $U_{E,\max} = U_{B,\max} = Q^2/2C$

- At any instant, sum is $U = U_B + U_E = Q^2/2C$
- When U_E = max, U_B = 0, and conversely, when U_B = max, U_E = 0

EM Oscillations (17)

- Checkpoint #2 – Capacitor in LC circuit has $V_{C,max}$ = *17V* and $U_{E,max}$ = *160μJ*. When capacitor has V_C = *5V* and U_E = *10μJ* , what are the a) emf across the inductor and b) the energy stored in the *B* field, U_B ?



- Can apply the loop rule
  - Net potential difference around the circuit must be zero

$$v_L(t) = v_c(t)$$

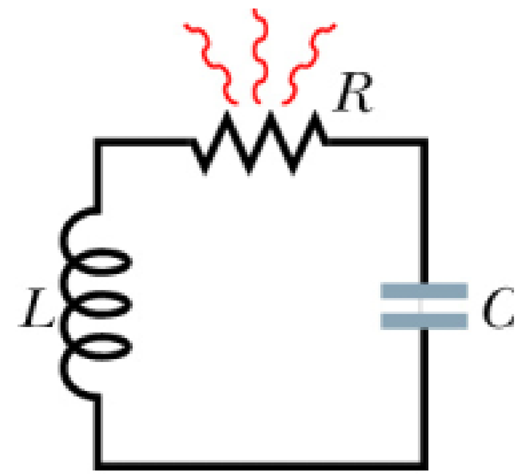
A) vL = 5V

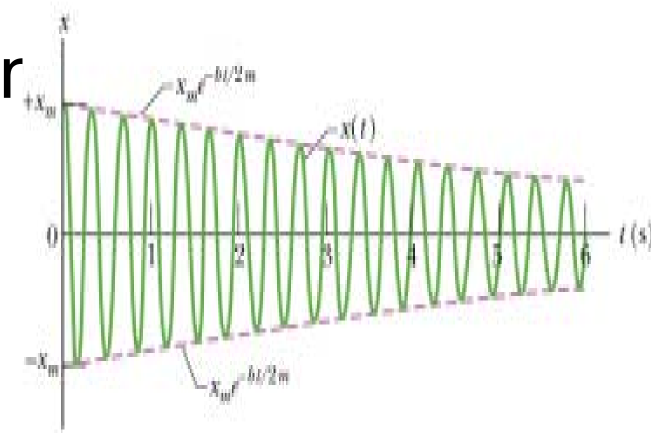
$$U_{E,\max} = U_E(t) + U_B(t)$$

B) UB = 160-10=150μJ

EM Oscillations (18)

- Consider a RLC circuit – resistor, inductor and capacitor in series
- Total electromagnetic energy, $U = U_E + U_B$, is no longer constant
- Energy decreases with time as it is transferred to thermal energy in the resistor
- Oscillations in *q*, *i* and *V* are damped
  - Same as damped block and spring

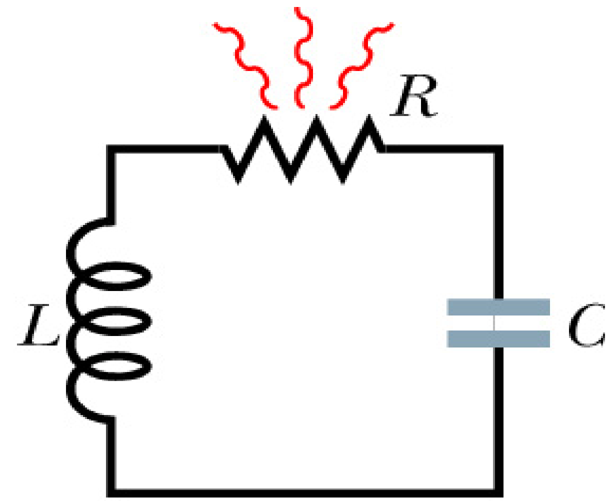




EM Oscillations (19)

- Resistor does not store electromagnetic energy so total energy at any time is

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$



- Rate of transfer to thermal energy is (minus sign means U is decreasing)

$$\frac{dU}{dt} = -i^2 R$$

- Differentiating gives

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2 R$$

EM Oscillations (20)

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2R$$

- Use relations $i = \frac{dq}{dt}$ $\frac{di}{dt} = \frac{d^2q}{dt^2}$

- Differential equation for damped RLC circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

- Solution $q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$

EM Oscillations (21)

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

- Where $\omega' = \sqrt{\omega^2 - (R/2L)^2}$ $\omega = \sqrt{\frac{1}{LC}}$
- Charge in RLC circuit is sinusoidal but with an exponentially decaying amplitude $Qe^{-Rt/2L}$
- Damped angular frequency, ω´, is always less than ω of the undamped oscillations

EM Oscillations (22)

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

- Find U_E as function of time

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}e^{-Rt/L}\cos^2(\omega' t + \phi)$$

- Total energy decreases as $U_{tot} = \frac{Q^2}{2C}e^{-Rt/L}$