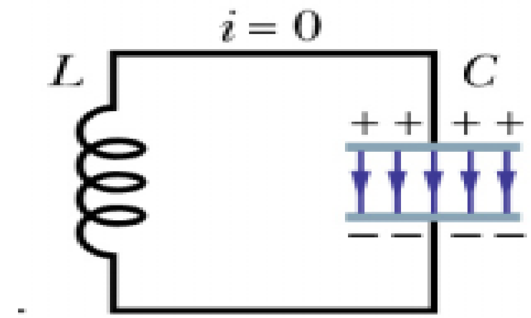
Lecture 29

Chapter 33

EM Oscillations and AC

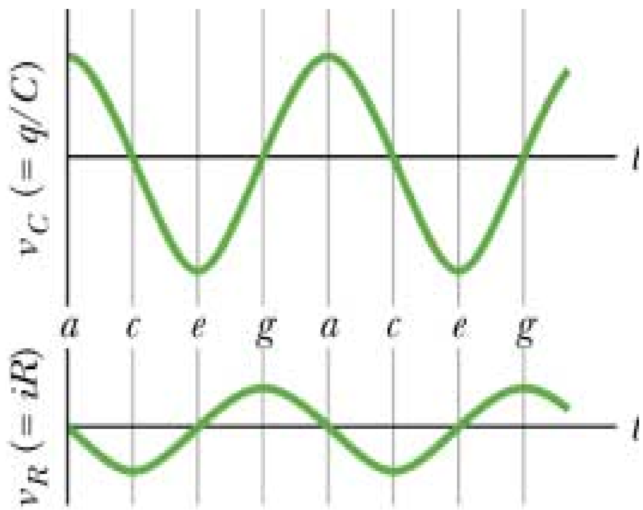
Review

- RL and RC circuits
  - Charge, current, and potential grow and decay exponentially



- LC circuit
  - Charge, current, and potential grow and decay sinusoidally – electromagnetic oscillations
  - Total electromagnetic energy is

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$



Review

- Ideal LC circuit
  - Total energy conserved $\dfrac{dU}{dt} = 0$
  - Solved differential equation to find

$$q = Q\cos(\omega t + \phi)$$

$$i = -I\sin(\omega t + \phi)$$

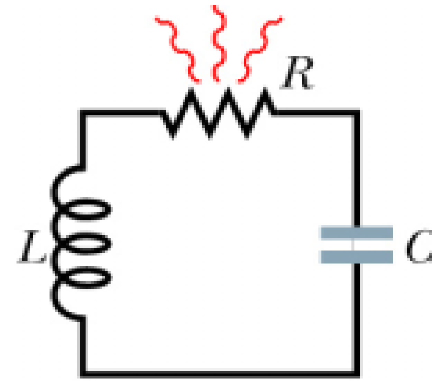
$$\omega = \sqrt{\frac{1}{LC}}$$

  - Substituting q and i into energy equations

$$U_E = \frac{Q^2}{2C}\cos^2(\omega t + \phi) \qquad U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$$

$$U = U_B + U_E = Q^2/2C$$

Review



- RLC circuit
  - Energy is no longer conserved, becomes thermal energy in resistor
  - Oscillations are damped
  - Solved differential equation to find

$$\frac{dU}{dt} = -i^2 R$$

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}$$

If R is very small $\omega' = \omega$

  - Energy decreases as

$$U_E = \frac{Q^2}{2C} e^{-Rt/L}\cos^2(\omega' t + \phi)$$

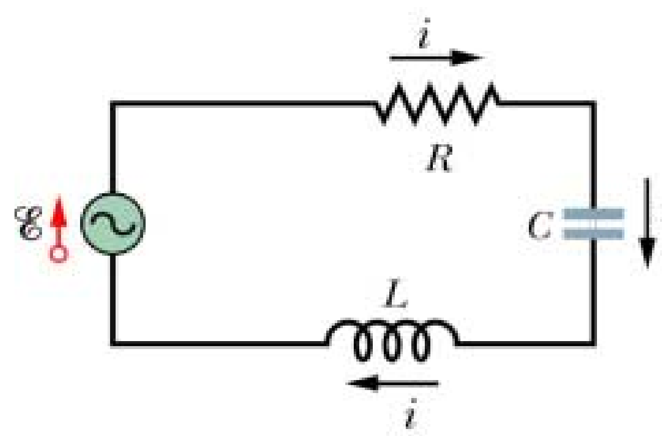
$$U_{tot} = \frac{Q^2}{2C} e^{-Rt/L}$$

EM Oscillations (23)

- LC and RLC circuits with no external emf
  - Free oscillations with natural frequency, ω

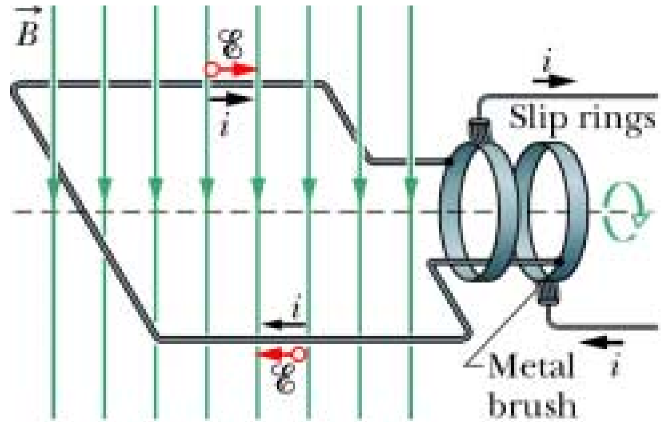
$$\omega = \sqrt{\frac{1}{LC}}$$

- Add external emf (e.g. ac to RLC circuit
  - Oscillations said to be or forced
  - Oscillations occur at driving frequency, ω_d
  - When $\omega_d = \omega$, called resonance, the current amplitude, I, is maximum



EM Oscillations (24)

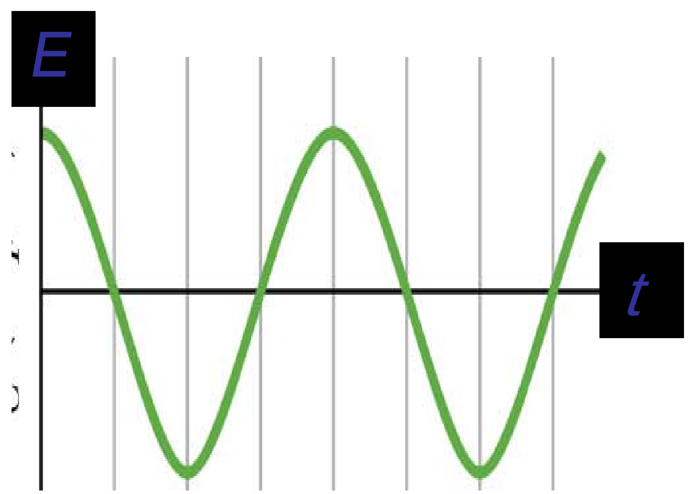
- ac generator – mechanically turn loop in *B* field, induces a current and therefore an emf
- Used Faraday's law to find emf



$$E = -N\frac{d\Phi_B}{dt} = NBA\,\omega \sin \omega t$$

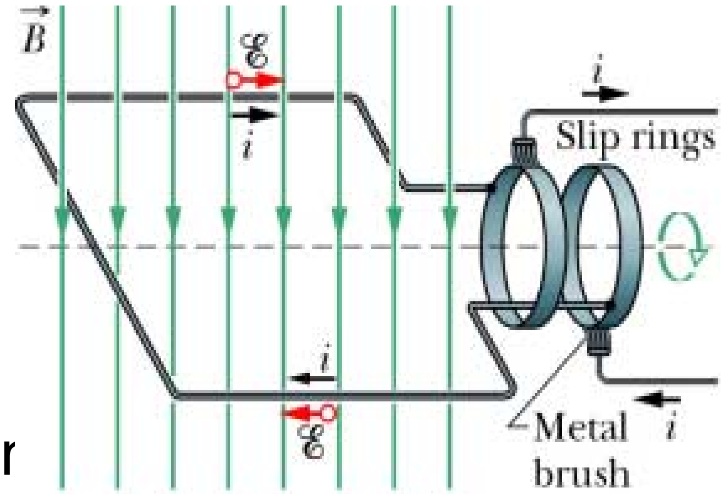
$$E_{\max} = NBA\,\omega$$

$$E = E_{\max} \sin \omega t$$



EM Oscillations (25)

$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$



- Driving angular frequency, ω_d is equal to angular speed that loop rotates in B field
- The phase of the emf is $\omega_d t$
- Amplitude is $\mathcal{E}_m$ where m star for maximum

$$\omega_d = 2\pi f_d$$

- Current of ac generator where ϕ corresponds to phase difference between the current and emf

$$i = I \sin(\omega_d t - \phi)$$

- I is amplitude
- Minus sign historical

EM Oscillations (26)

- Purely resistive load
- Apply loop rule

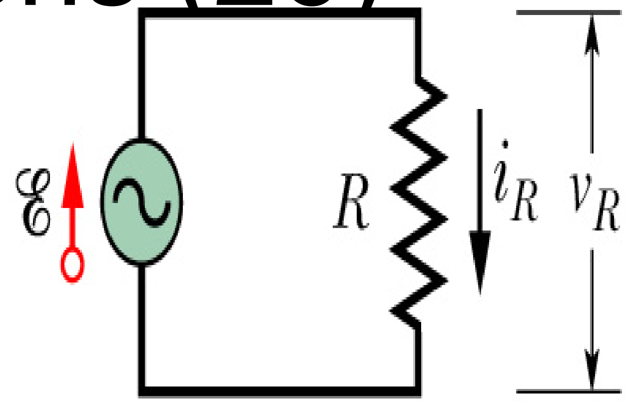
$$E - v_R = 0$$

- Using

$$E = E_m \sin \omega_d t$$

- Substitute

$$v_R = E_m \sin \omega_d t$$



- Amplitude across resistor is same as across emf

$$E_m = V_R$$

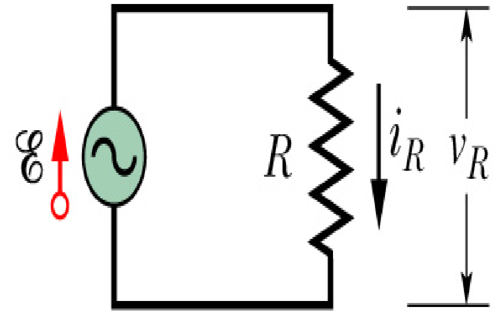
- Rewrite v_R as

$$v_R = V_R \sin \omega_d t$$

EM Oscillations (27)

- Use definition of resistance to find i_R

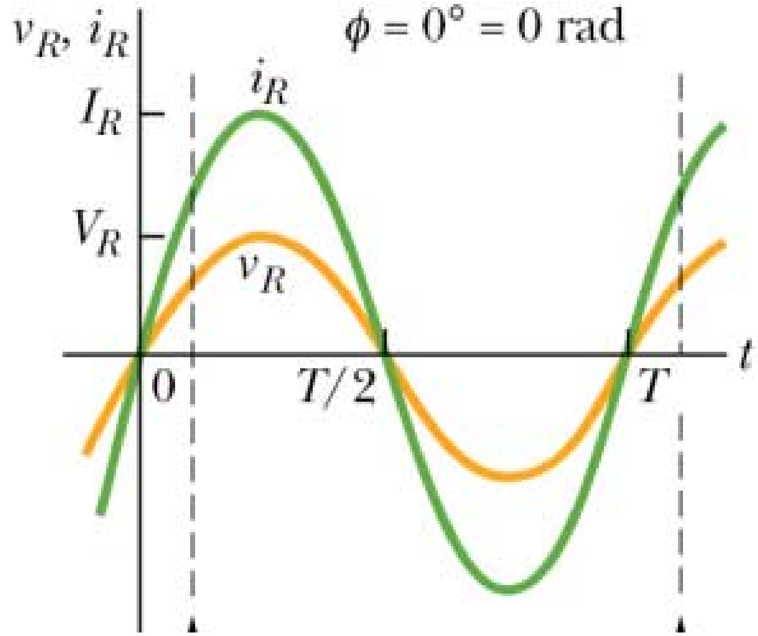
$$R = \frac{V_R}{I_R}$$



$$v_R = V_R \sin \omega_d t$$

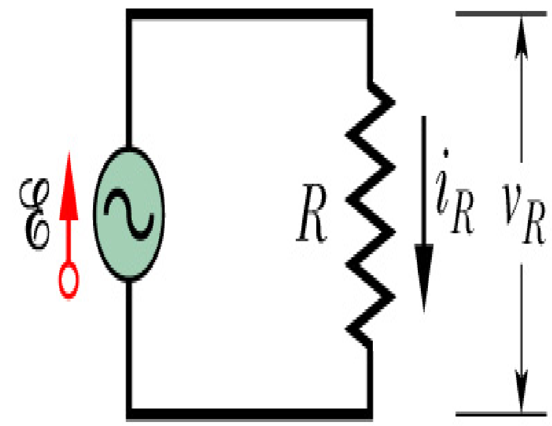
$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t$$

- Voltage and current are functions of *sin(ω$_d$t)* with $\phi = 0$ so are in phase
- No damping of v_R and i_R, generator supplies energy



EM Oscillations (28)

- Compare i_R to i for emf



$$i_R = \frac{v_R}{R} = \frac{V_R}{R}\sin\omega_d t$$

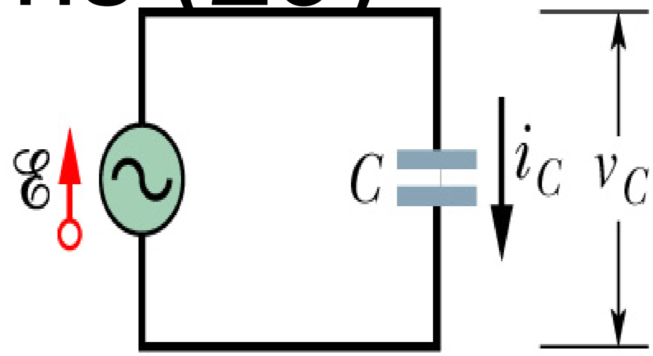
$$i_{emf} = i_R = I_R \sin(\omega_d t - \phi)$$

$$I_R = \frac{V_R}{R}$$

- For purely resistive load the phase constant $\phi = 0$
- Voltage amplitude is related to current amplitude

$$V_R = I_R R$$

EM Oscillations (29)



- Purely capacitive load
- Apply loop rule

$$E - v_C = 0$$

- Using

$$E = E_m \sin \omega_d t$$

- Substitute

$$v_C = E_m \sin \omega_d t$$

- Amplitude across capacitor is same as across emf

$$E_m = V_C$$

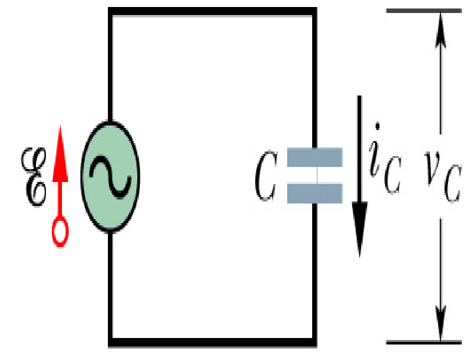
- Rewrite v_C as

$$v_C = V_C \sin \omega_d t$$

EM Oscillations (30)

- Use definition of capacitance

$$q_C = Cv_C$$



$$q_C = Cv_C = CV_C \sin \omega_d t$$

- Use definition of current and differentiate

$$i = \frac{dq}{dt}$$

$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos \omega_d t$$

- Replace cosine term with a phase-shifted sine term

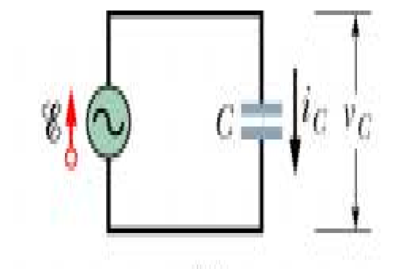
$$\cos \omega_d t = \sin(\omega_d t + 90°)$$

EM Oscillations (31)

- Compare v_C and i_C of capacitor

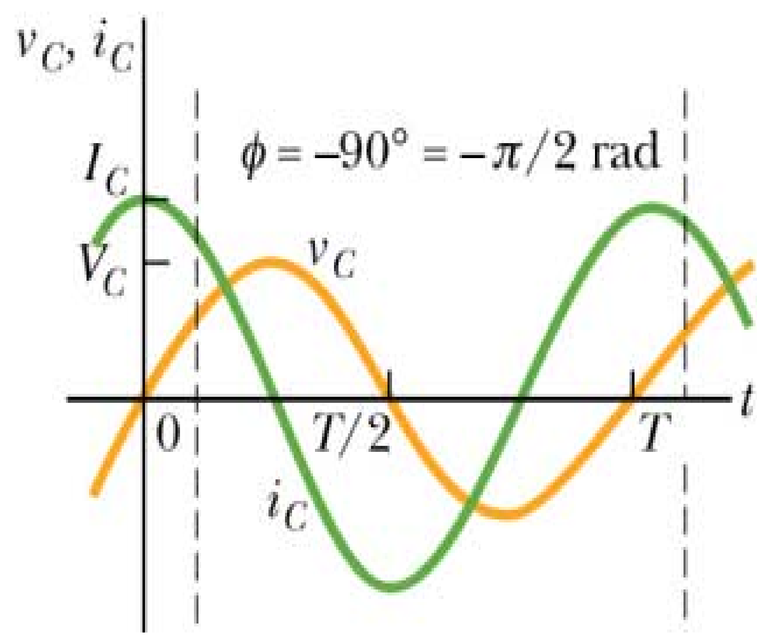
$$v_C = V_C \sin \omega_d t$$

$$i_C = \omega_d C V_C \sin(\omega_d t + 90°)$$

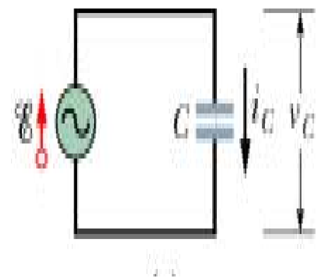


- Voltage and current are out of phase by 90°

- Current leads voltage
  - Current reaches its max before voltage does by a quarter cycle or T/4



EM Oscillations (32)



- Now compare currents

$$i_C = \omega_d C V_C \sin(\omega_d t + 90°)$$ $$i_{emf} = i_C = I_C \sin(\omega_d t - \varphi)$$

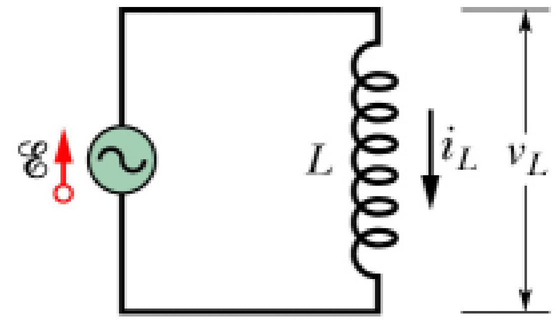
- For purely capacitive load phase ϕ = -90°
- V_C amplitude is related to I_C amplitude

$$I_C = \omega_d C V_C$$ $$V_C = I_C \frac{1}{\omega_d C} = I_C X_C$$

- X_C is the capacitive reactance and has SI unit of ohm, Ω just like resistance

$$V_C = I_C X_C$$ $$X_C = \frac{1}{\omega_d C}$$

EM Oscillations (33)



- Purely inductive load
- Apply loop rule

$$E - v_L = 0$$

- Using

$$E = E_m \sin \omega_d t$$

- Substitute

$$v_L = E_m \sin \omega_d t$$

- Amplitude across inductor is same as across emf

$$E_m = V_L$$

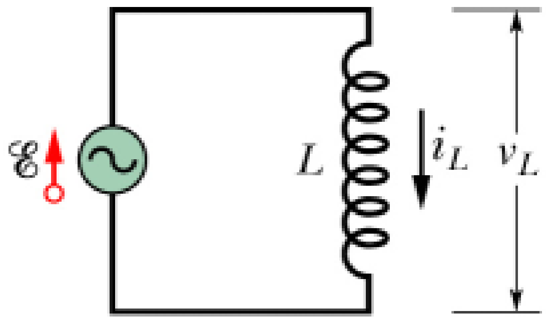
- Rewrite v_L as

$$v_L = V_L \sin \omega_d t$$

EM Oscillations (34)

- Self-induced emf across an inductor is

$$E_L = v_L = L\frac{di}{dt}$$



- Relate

$$v_L = V_L \sin\omega_d t = L\frac{di}{dt}$$

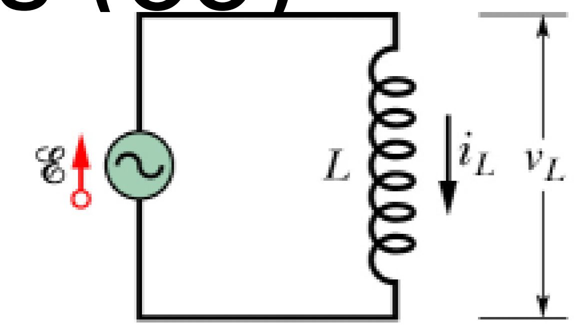
$$\frac{di}{dt} = \frac{V_L}{L}\sin\ \omega_d t$$

- Want current so integrate

$$i_L = \int di_L = \frac{V_L}{L}\int \sin\omega_d t dt = -\left(\frac{V_L}{\omega_d L}\right)\cos\omega_d t$$

EM Oscillations (35)

- Replace -cosine with a phase-shifted sine term

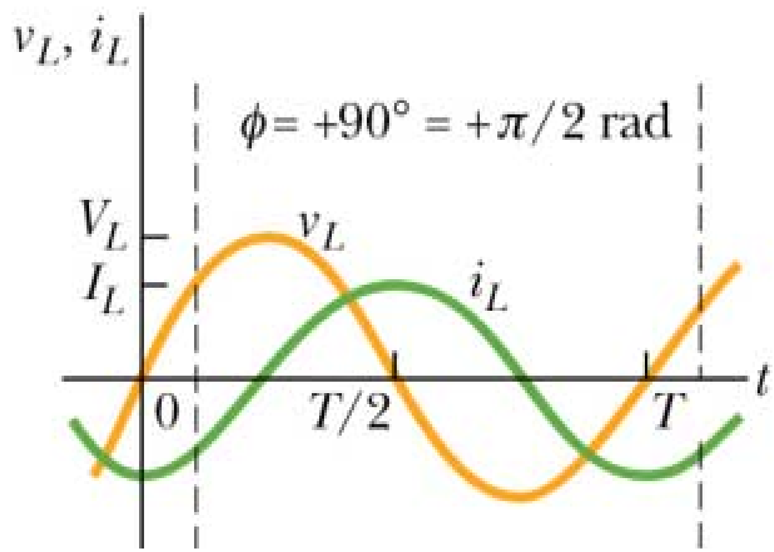


$$-\cos\omega_d t = \sin(\omega_d t - 90°)$$

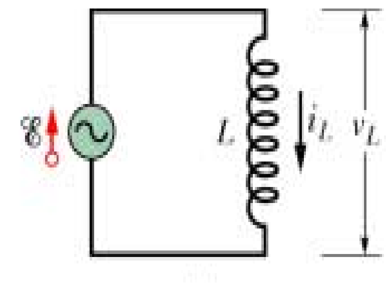
$$i_L = -\left(\frac{V_L}{\omega_d L}\right)\cos\omega_d t = \frac{V_L}{\omega_d L}\sin(\omega_d t - 90°)$$

$$v_L = V_L \sin\omega_d t$$

- Compare i_L to v_L
- i_L and v_L are 90° out of phase
- Current lags voltage
  - i_L reaches max after v_L by a quarter cycle or T/4



EM Oscillations (36)



- Now compare currents

$$i_L = \frac{V_L}{\omega_d L}\sin(\omega_d t - 90°)$$

$$i_{emf} = i_L = I_L \sin\left(\omega_d t - \varphi\right)$$

- For purely inductive load phase $\phi = +90°$
- V_L amplitude is related to I_L amplitude

$$I_L = \frac{V_L}{\omega_d L}$$

$$V_L = I_L \omega_d L = I_L X_L$$

- X_L is the inductive reactance and has SI unit of ohm, Ω just like resistance R

$$V_L = I_L X_L$$

$$X_L = \omega_d L$$

EM Oscillations (37)

Element	Reactance / Resistance	Phase of Current	Phase angle ϕ	Amplitude Relation
Resistor	R	In phase	0°	$V_R=I_R R$
Capacitor	$X_C= 1/\omega_d C$	Leads v_C (ICE)	-90°	$V_C=I_C X_C$
Inductor	$X_L=\omega_d L$	Lags v_L (ELI)	+90°	$V_L=I_L X_L$

- ELI the ICE man
  - Voltage or emf (E) before current (I) in an inductor (L)
  - Current (I) before voltage or emf (E) in capacitor (C)

EM Oscillations (38)

- Checkpoints #3, 5, & 6 – If the driving frequency, ω_d, in a circuit is increased does the amplitude voltage and amplitude current increase, decrease or remain the same?

- For purely resistive circuit
- From loop rule $V_R = E_m$
- So amplitude voltage, V_L stays the same
- I_R also stays the same - only depends on R

$$I_R = \frac{V_R}{R}$$

I_R

EM Oscillations (39)

- Checkpoints #3, 5, & 6 – If the driving frequency, ω_d, in a circuit is increased does the amplitude voltage and amplitude current increase, decrease or remain the same?

- For purely capacitive circuit
- From loop rule $V_c = E_m$
- So amplitude voltage, V_C stays the same
- I_C depends on X_C which depends on ω_d by
- So I_C increases

$$I_C = \frac{V_C}{X_C} = \omega_d C V_C$$

EM Oscillations (40)

- Checkpoints #3, 5, & 6 – If the driving frequency, ω_d, in a circuit is increased does the amplitude voltage and amplitude current increase, decrease or remain the same?

- For purely inductive circuit
- From loop rule $V_L = E_m$
- So amplitude voltage, V_L stays the same
- I_L depends on X_L which depends on ω_d by
- So I_L decreases

$$I_L = \frac{V_L}{X_L} = \frac{V_L}{\omega_d L}$$