Lecture 30

Chapter 33
EM Oscillations and AC
Review

• Characterized ideal LC circuit
  – Charge, current and voltage vary sinusoidally

• Added resistance to LC circuit
  – Oscillations become damped
  – Charge, current and voltage still vary sinusoidally but decay exponentially

• Added ac generator to circuits with just a
  – Resistor
  – Capacitor
  – Inductor
Review

<table>
<thead>
<tr>
<th>Element</th>
<th>Reactance/Resistance</th>
<th>Phase of Current</th>
<th>Phase angle $\phi$</th>
<th>Amplitude Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>In phase</td>
<td>0°</td>
<td>$V_R = I_R R$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$X_C = 1/\omega dC$</td>
<td>Leads $v_C$ (ICE)</td>
<td>-90°</td>
<td>$V_C = I_C X_C$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$X_L = \omega_d L$</td>
<td>Lags $v_L$ (ELI)</td>
<td>+90°</td>
<td>$V_L = I_L X_L$</td>
</tr>
</tbody>
</table>

• ELI the ICE man
  – Voltage or emf ($E$) before current ($I$) in an inductor ($L$)
  – Current ($I$) before voltage or emf ($E$) in capacitor ($C$)
EM Oscillations (41)

- RLC circuit – resistor, capacitor and inductor in series
- Apply alternating emf
  \[ E = E_m \sin \omega_d t \]
- Elements are in series so same current is driven through each
- From the loop rule, at any time \( t \), the sum of the voltages across the elements must equal the emf
  \[ E = V_R + V_C + V_L \]
EM Oscillations (42)

• Want to find amplitude $I$ and the phase constant $\phi$

• Using phasors, represent the current at time $t$
  – Length is amplitude $I$
  – Projection on vertical axis is current $i$ at time $t$
  – Angle of rotation is the phase at time $t$

\[ \omega_d t - \phi \]
EM Oscillations (43)

- Draw phasors for voltages of $R$, $C$ and $L$ at same time $t$
- Orient $V_R$, $V_L$, & $V_C$ phasors relative to current phasor
- Resistor – $V_R$ and $I$ are in phase
- Inductor – (ELI) $V_L$ is ahead of $I$ by 90°
- Capacitor – (ICE) $I$ is ahead of $V_C$ by 90°
- $v_R$, $v_C$, & $v_L$ are projections
EM Oscillations (44)

- Draw phasor for applied emf

\[ E = E_m \sin \omega_d t \]

- Length is amplitude \( E_m \)
- Projection is \( E \) at time \( t \)
- Angle is phase of emf \( \omega_d t \)
- From loop rule the projection \( E \) = the algebraic sum of projections \( v_R, v_L, v_C \)

\[ E = v_R + v_C + v_L \]
EM Oscillations (45)

\[ \mathbf{E} = V_R + V_C + V_L \]

- Phasors rotate together so equality always holds
- Phasor \( \mathbf{E}_m \) = vector sum of voltage phasors

\[ \mathbf{E}_m = \mathbf{V}_R + \mathbf{V}_C + \mathbf{V}_L \]

- Combine \( V_L \) & \( V_C \) to form single phasor

\[ V_L - V_C \]
EM Oscillations (46)

- Using Pythagorean theorem:
  \[ E_m^2 = V_R^2 + (V_L - V_C)^2 \]

- From amplitude relations:
  \[ V_R = IR \quad V_L = IX_L \quad V_C = IX_C \]
  \[ E_m^2 = (IR)^2 + (IX_L - IX_C)^2 \]

- Rearrange to find amplitude \( I \):
  \[ I = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}} \]
EM Oscillations (47)

- Define impedance, $Z$ to be
  \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

- Using reactances rewrite current as
  \[ X_L = \omega_d L \]
  \[ X_C = \frac{1}{\omega_d C} \]
  \[ I = \frac{E_m}{\sqrt{R^2 + \left(\frac{\omega_d L}{\omega_d C} - 1\right)^2}} \]
  \[ I = \frac{E_m}{Z} \]
EM Oscillations (48)

- Using trig find the phase constant $\phi$

\[
\tan \phi = \frac{V_L - V_C}{V_R}
\]

- Using amplitude relations

\[
\tan \phi = \frac{IX_L - IX_C}{IR}
\]
\[
\tan \phi = \frac{X_L - X_C}{R}
\]

- Examine 3 cases:
  - $X_L > X_C$
  - $X_L < X_C$
  - $X_L = X_C$
EM Oscillations (49)

\[ \tan \phi = \frac{X_L - X_C}{R} \]

- If \( X_L > X_C \) the circuit is more inductive than capacitive
  - \( \phi \) is positive
  - Emf is before current (ELI)
- If \( X_L < X_C \) the circuit is more capacitive than inductive
  - \( \phi \) is negative
  - Current is before emf (ICE)
EM Oscillations (50)

\[ \tan \phi = \frac{X_L - X_C}{R} \]

• If \( X_L = X_C \) the circuit is in resonance – emf and current are in phase

• Current amplitude \( I \) is max when impedance, \( Z \) is min

\[ X_L - X_C = 0 \]

\[ I = \frac{E_m}{Z} = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E_m}{R} \]
EM Oscillations (51)

- When $X_L = X_C$ the driving frequency is
  \[ \omega_d L = \frac{1}{\omega_d C} \quad \text{and} \quad \omega_d = \frac{1}{\sqrt{LC}}. \]

- This is the same as the natural frequency, $\omega$
  \[ \omega_d = \omega = \frac{1}{\sqrt{LC}}. \]

- For RLC circuit, resonance and the max current $I$ occurs when $\omega_d = \omega$
EM Oscillations (52)

- For small driving frequency, $\omega_d < \omega$
  - $X_L$ is small but $X_C$ is large
  - Circuit capacitive
- For large driving frequency, $\omega_d > \omega$
  - $X_C$ is small but $X_L$ is large
  - Circuit inductive
- For $\omega_d = \omega$, circuit is in resonance

\[
I = \frac{E_m}{\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}
\]
EM Oscillations (53)

- Instantaneous rate which energy is dissipated in resistor is \[ P = i^2 R \]

- But \[ i = I \sin(\omega_d t - \phi) \]

\[ P = I^2 R \sin^2(\omega_d t - \phi) \]

- Want average rate, \( P_{\text{avg}} \)
  - Average over complete cycle \( T \)

\[ \sin^2 \theta = 1/2 \]
EM Oscillations (53)

• For alternating current circuits define root-mean-square or rms values for $i$, $V$ and emf

\[
I_{rms} = \frac{I}{\sqrt{2}} \quad V_{rms} = \frac{V}{\sqrt{2}} \quad E_{rms} = \frac{E}{\sqrt{2}}
\]

• Ammeters, voltmeters - give rms values

• Write average power dissipated by resistor in an ac circuit is

\[
P_{avg} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R \quad P_{avg} = I_{rms}^2 R
\]
**EM Oscillations (54)**

- Write average power in another form using

\[
I_{\text{rms}} = \frac{E_{\text{rms}}}{Z}
\]

\[
P_{\text{avg}} = I_{\text{rms}}^2 R = \frac{E_{\text{rms}}}{Z} I_{\text{rms}} R = E_{\text{rms}} I_{\text{rms}} \frac{R}{Z}
\]

- Using phasor and amplitude relations

\[
\cos \phi = \frac{V_R}{E_m} = \frac{IR}{IZ} = \frac{R}{Z}
\]

- Rewrite average power as

\[
P_{\text{avg}} = E_{\text{rms}} I_{\text{rms}} \cos \phi
\]
EM Oscillations (55)

• If ac circuit has only resistive load $R/Z = 1$

$$P_{avg} = E_{rms} I_{rms} = I_{rms} V_{rms}$$

• Trade-off between current and voltage
  – For general use want low voltage
  – Means high current but

$$P_{avg} = I_{rms}^2 R$$

• General energy transmission rule:
  Transmit at the highest possible voltage and the lowest possible current
EM Oscillations (56)

- **Transformer** – device used to raise (for transmission) and lower (for use) the ac voltage in a circuit, keeping $iV$ constant
  - Has 2 coils (primary and secondary) wound on same iron core with different #s of turns
EM Oscillations (57)

- Alternating primary current induces alternating magnetic flux in iron core
- Same core in both coils so induced flux also goes through the secondary coil
- Using Faraday’s law

\[ V_P = -N_P \frac{d\Phi_B}{dt} \quad V_S = -N_S \frac{d\Phi_B}{dt} \]

\[ \frac{V_P}{N_P} = \frac{V_S}{N_S} \]
EM Oscillations (58)

- Transformation of voltage is
- If $N_S > N_P$ called a step-up transformer
- If $N_S < N_P$ called a step-down transformer
- Conservation of energy

\[ I_P V_P = I_S V_S \]

\[ I_S = I_P \frac{V_P}{V_S} = I_P \frac{N_P}{N_S} \]
EM Oscillations (59)

• The current $I_P$ appears in primary circuit due to $R$ in secondary circuit.

$$I_P V_P = I_S V_S \quad I_S = V_S / R$$

$$I_P = \frac{V_S}{R} \frac{V_S}{V_P} = \frac{1}{R} \frac{V_S^2}{V_P} V_P = \frac{1}{R} \left( \frac{N_S}{N_P} \right)^2 V_P$$

• Has for of $I_P = \frac{V_P}{R_{eq}}$ where

$$R_{eq} = \left( \frac{N_P}{N_S} \right)^2 R$$