Lecture 30

Chapter 33

EM Oscillations and AC

Review

- Characterized ideal LC circuit
  - Charge, current and voltage vary sinusoidally
- Added resistance to LC circuit
  - Oscillations become damped
  - Charge, current and voltage still vary sinusoidally but decay exponentially
- Added ac generator to circuits with just a
  - Resistor
  - Capacitor
  - Inductor

Review

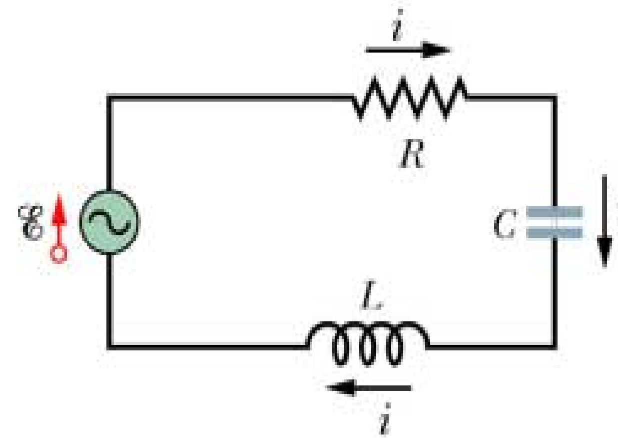
Element	Reactance / Resistance	Phase of Current	Phase angle ϕ	Amplitude Relation
Resistor	R	In phase	0°	$V_R=I_RR$
Capacitor	$X_C= 1/\omega_dC$	Leads v_C (ICE)	-90°	$V_C=I_CX_C$
Inductor	$X_L=\omega_dL$	Lags v_L (ELI)	+90°	$V_L=I_LX_L$

- ELI the ICE man
  - Voltage or emf (E) before current (I) in an inductor (L)
  - Current (I) before voltage or emf (E) in capacitor (C)

EM Oscillations (41)

- RLC circuit – resistor, capacitor and inductor in series
- Apply alternating emf

$$E = E_m \sin \omega_d t$$



- Elements are in series so same current is driven through each

$$i = I \sin(\omega_d t - \phi)$$

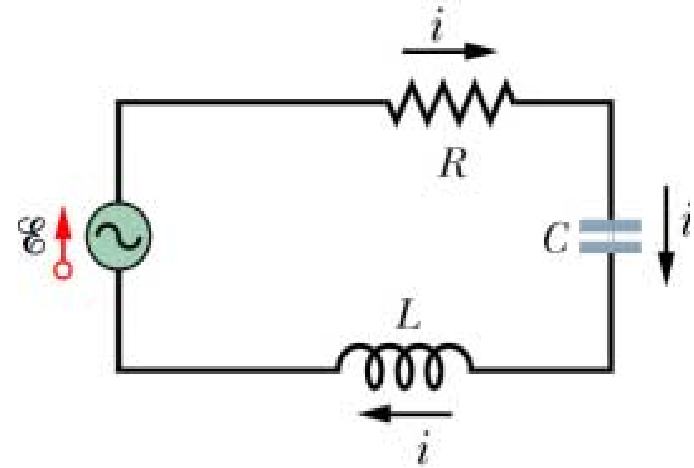
- From the loop rule, at any time *t*, the sum of the voltages across the elements must

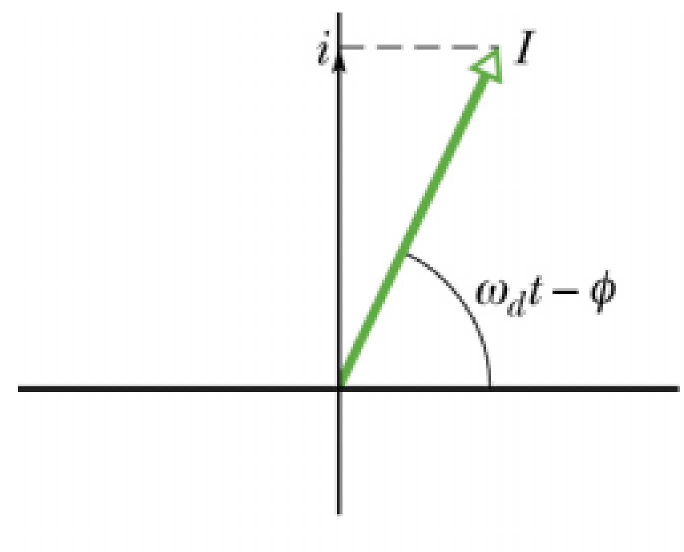
$$E = v_R + v_C + v_L$$

EM Oscillations (42)

- Want to find amplitude *I* and the phase constant ϕ
- Using phasors, represent the current at time *t*
  - Length is amplitude *I*
  - Projection on vertical axis is current *i* at time *t*
  - Angle of rotation is the phase at time *t*

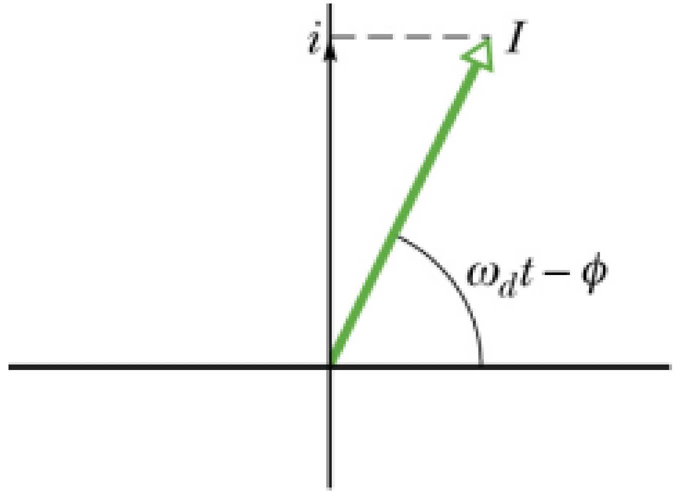
$$\omega_d t - \phi$$

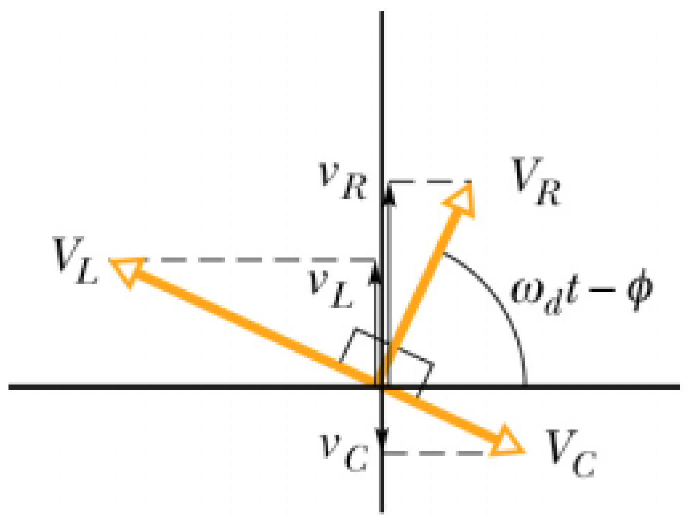




EM Oscillations (43)

- Draw phasors for voltages of *R, C* and *L* at same time *t*
- Orient V_R, V_L, & V_C phasors relative to current phasor
- Resistor – V_R and *I* are in phase
- Inductor – (ELI) V_L is ahead of *I* by 90°
- Capacitor – (ICE) *I* is ahead of V_C by 90°
- v_R, v_C, & v_L are projections





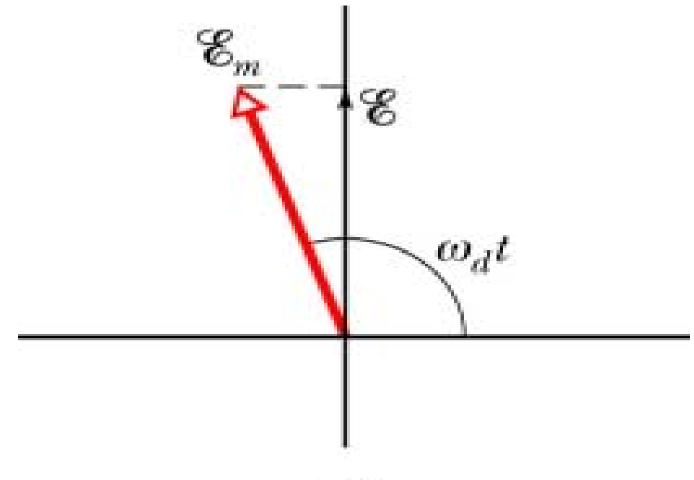
EM Oscillations (44)

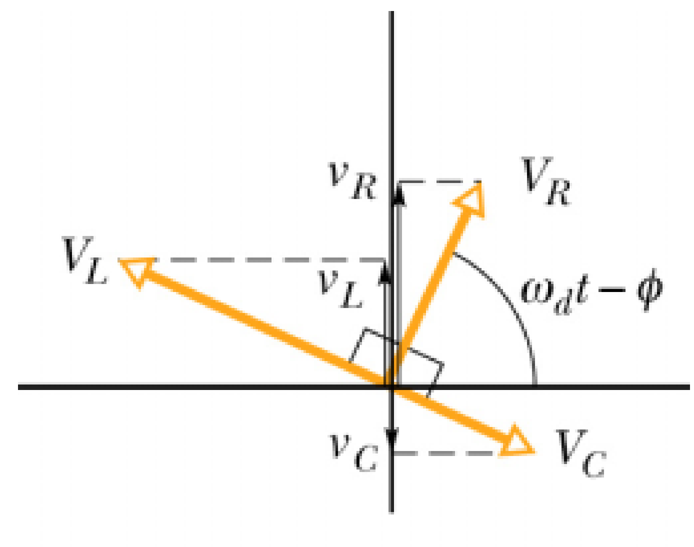
- Draw phasor for applied emf

$$E = E_m \sin \omega_d t$$

- Length is amplitude E_m
- Projection is E at time t
- Angle is phase of emf $\omega_d t$
- From loop rule the projection E = the algebraic sum of projections v_R, v_L & v_C

$$E = v_R + v_C + v_L$$





EM Oscillations (45)

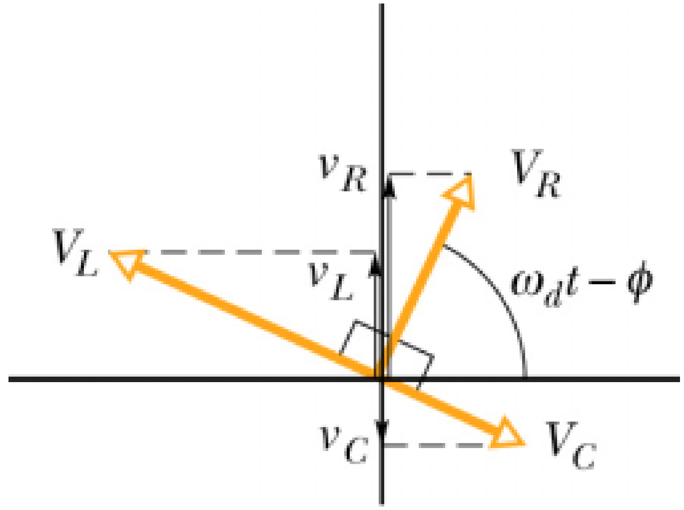
$$E = v_R + v_C + v_L$$

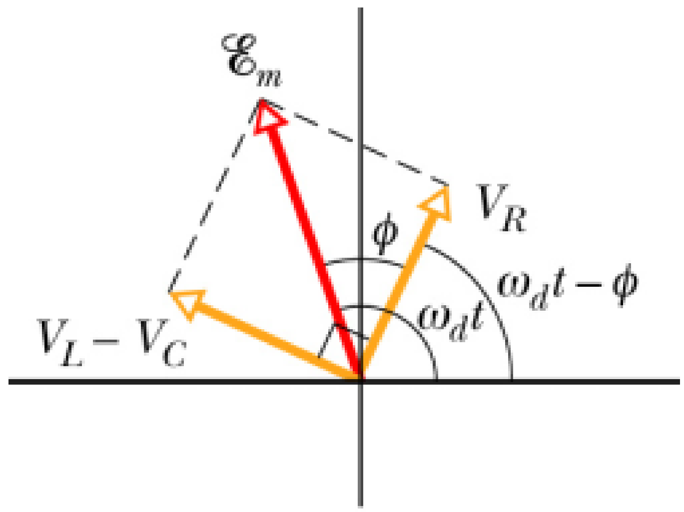
- Phasors rotate together so equality always holds
- Phasor E_m = vector sum of voltage phasors

$$\vec{E}_m = \vec{V}_R + \vec{V}_C + \vec{V}_L$$

- Combine V_L & V_C to form single phasor

$$V_L - V_C$$

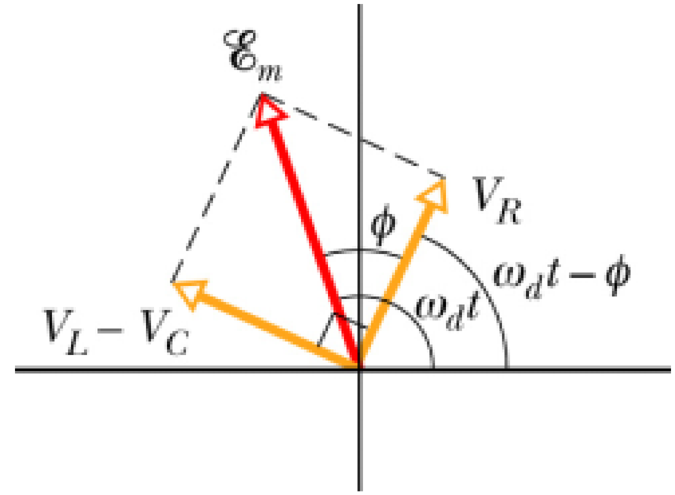




EM Oscillations (46)

- Using Pythagorean theorem

$$E_m^2 = V_R^2 + (V_L - V_C)^2$$



- From amplitude relations re… olt…

$$V_R = IR \qquad V_L = IX_L \qquad V_C = IX_C$$

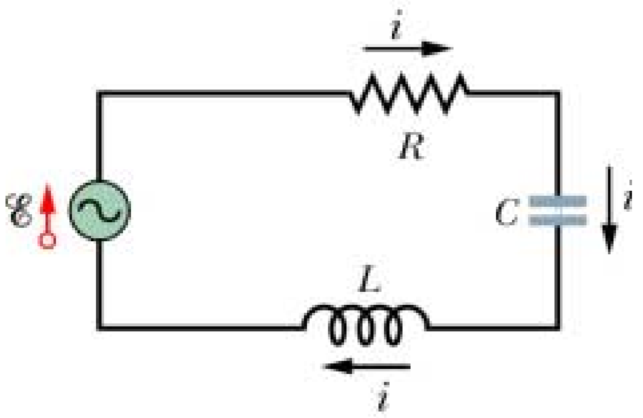
$$E_m^2 = (IR)^2 + (IX_L - IX_C)^2$$

- Rearrange to find amplitude *I*

$$I = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

EM Oscillations (47)

$$I = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$



- Define impedance, Z to be

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Using reactances rewrite current as

$$X_L = \omega_d L$$

$$X_C = \frac{1}{\omega_d C}$$

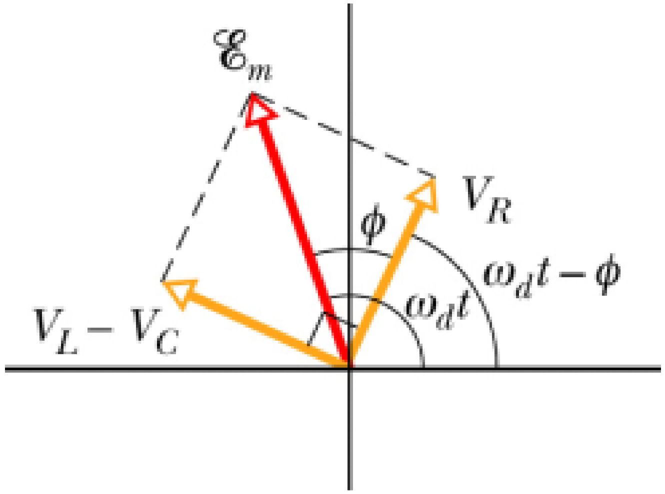
$$I = \frac{E_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

$$I = \frac{E_m}{Z}$$

EM Oscillations (48)

- Using trig find the phase constant ϕ

$$\tan\phi = \frac{V_L - V_C}{V_R}$$



- Using amplitude relations

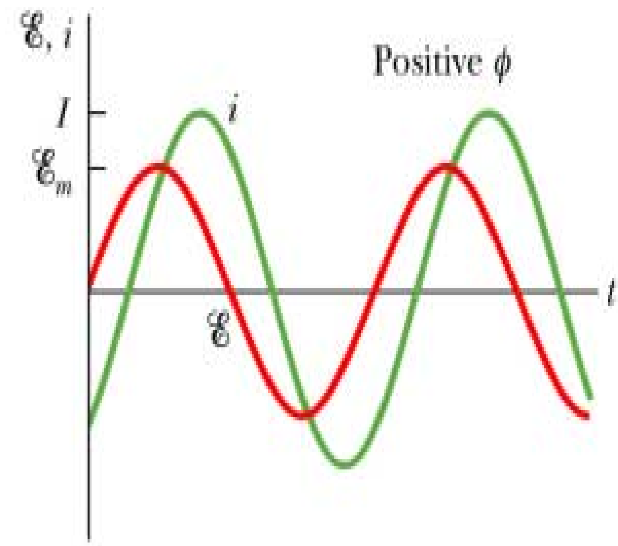
$$\tan\phi = \frac{IX_L - IX_C}{IR}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

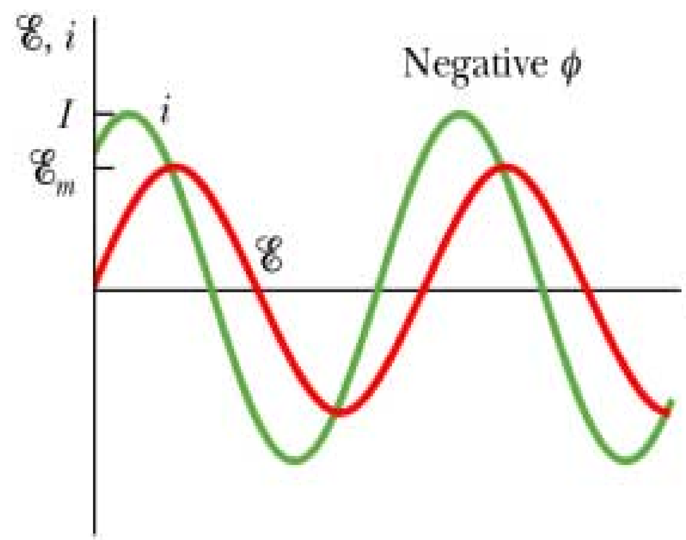
- Examine 3 cases:
  - $X_L > X_C$
  - $X_L < X_C$
  - $X_L = X_C$

EM Oscillations (49)

$$\tan\phi = \frac{X_L - X_C}{R}$$

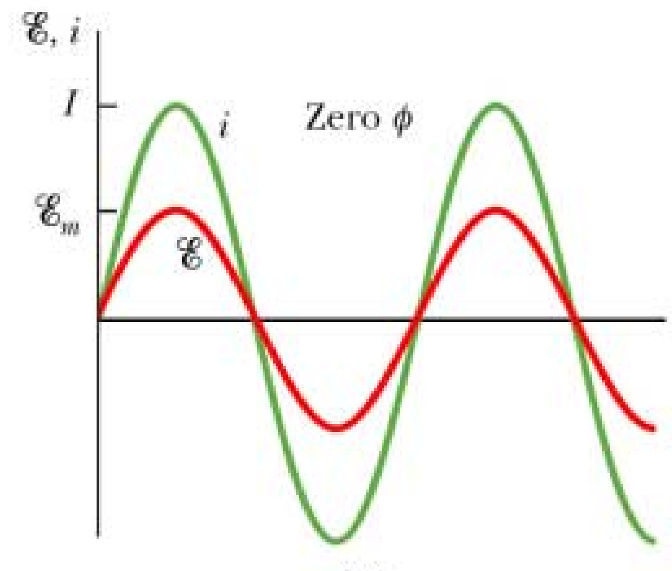


- If $X_L > X_C$ the circuit is more inductive than capacitive
  - ϕ is positive
  - Emf is before current (ELI)
- If $X_L < X_C$ the circuit is more capacitive than inductive
  - ϕ is negative
  - Current is before emf (ICE)



EM Oscillations (50)

$$\tan\phi = \frac{X_L - X_C}{R}$$



- If $X_L = X_C$ the circuit is in resonance – emf and current are in phase
- Current amplitude I is max when impedance, Z is min

$$X_L - X_C = 0$$

$$Z = R$$

$$I = \frac{E_m}{Z} = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E_m}{R}$$

EM Oscillations (51)

- When $X_L = X_C$ the driving frequency is

$$\omega_d L = \frac{1}{\omega_d C}$$

$$\omega_d = \frac{1}{\sqrt{LC}}$$

- This is the same as the natural frequency, ω

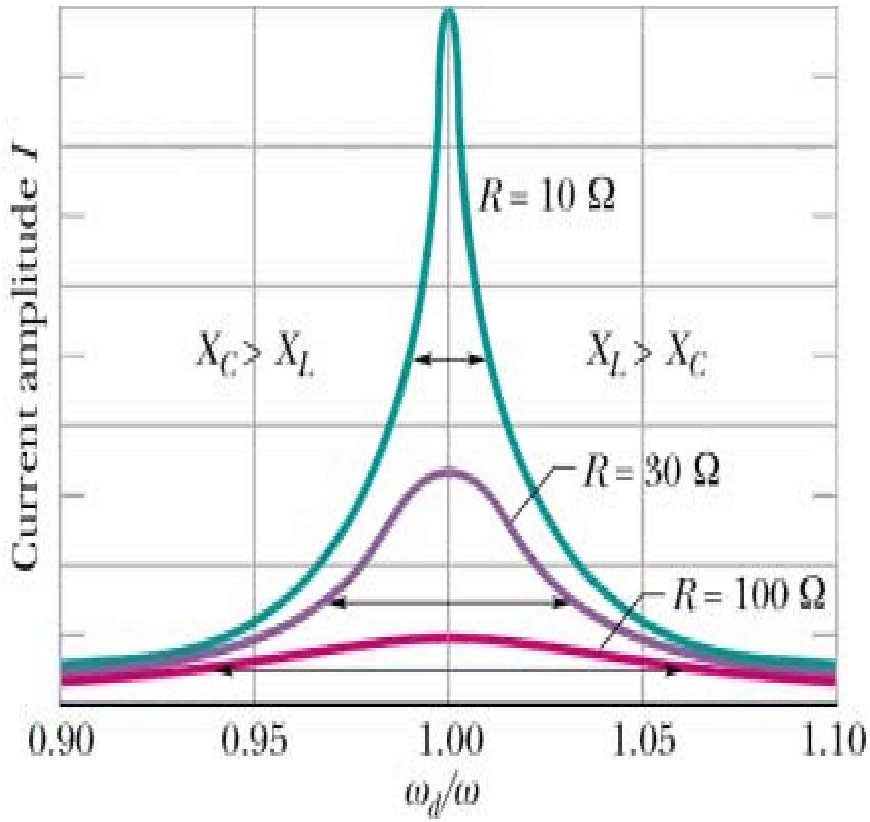
$$\omega_d = \omega = \frac{1}{\sqrt{LC}}$$

- For RLC circuit, resonance and the max current I occurs when $\omega_d = \omega$

EM Oscillations (52)

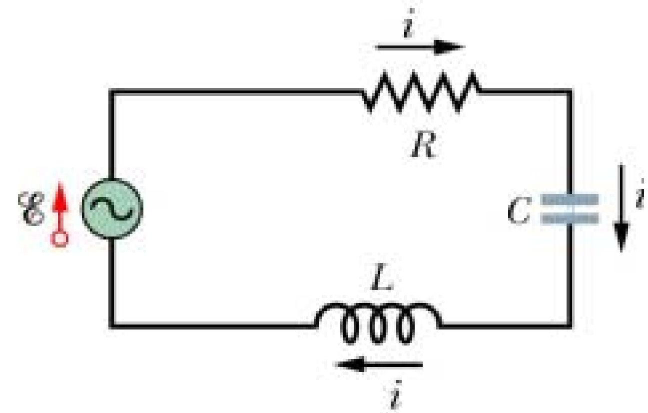
$$I = \frac{E_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

- For small driving frequency, $\omega_d < \omega$
  - X_L is small but X_C is large
  - Circuit capacitive
- For large driving frequency, $\omega_d > \omega$
  - X_C is small but X_L is large
  - Circuit inductive
- For $\omega_d = \omega$, circuit is in resonance



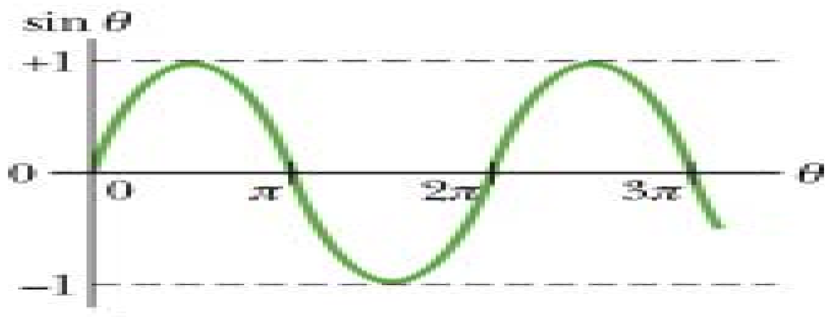
EM Oscillations (53)

- Instantaneous rate which energy is dissipated in resistor is $P = i^2 R$



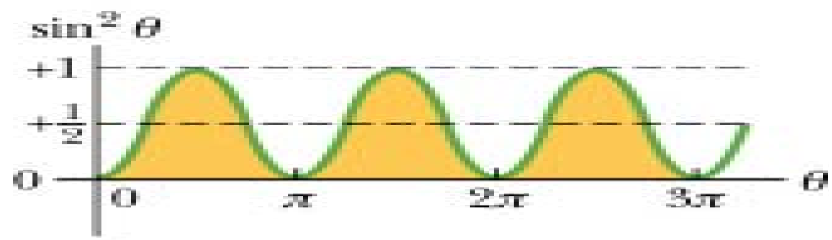
- But $i = I \sin(\omega_d t - \phi)$

$$P = I^2 R \sin^2(\omega_d t - \phi)$$



(a)

- Want average rate, P_{avg}
  - Average over complete cycle T $\sin^2 \theta = 1/2$



EM Oscillations (53)

- For alternating current circuits define root-mean-square or rms values for *i, V* and emf

$$I_{rms} = \frac{I}{\sqrt{2}} \qquad V_{rms} = \frac{V}{\sqrt{2}} \qquad E_{rms} = \frac{E}{\sqrt{2}}$$

- Ammeters, voltmeters - give rms values
- Write average power dissipated by resistor in an ac circuit is

$$P_{avg} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R \qquad P_{avg} = I_{rms}^2 R$$

EM Oscillations (54)

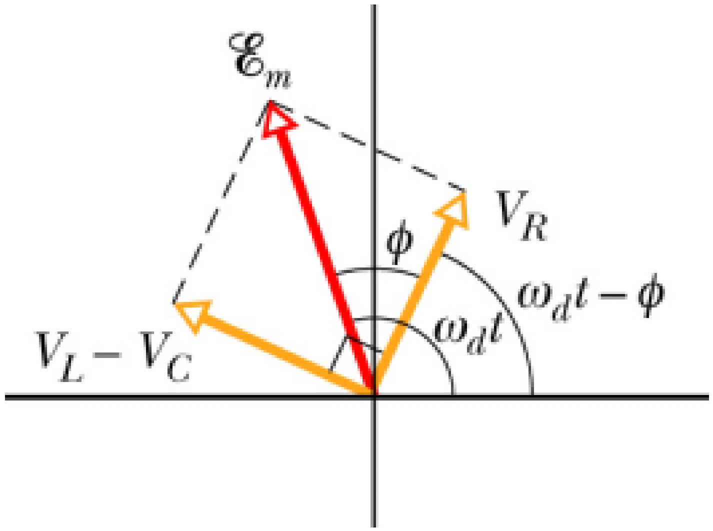
- Write average power in another form using

$$I_{rms} = \frac{E_{rms}}{Z}$$

$$P_{avg} = I_{rms}^2 R = \frac{E_{rms}}{Z} I_{rms} R = E_{rms} I_{rms} \frac{R}{Z}$$

- Using phasor and amplitude relations

$$\cos\phi = \frac{V_R}{E_m} = \frac{IR}{IZ} = \frac{R}{Z}$$



- Rewrite average power as

$$P_{avg} = E_{rms} I_{rms} \cos\phi$$

EM Oscillations (55)

- If ac circuit has only resistive load $R/Z = 1$

$$P_{avg} = E_{rms} I_{rms} = I_{rms} V_{rms}$$

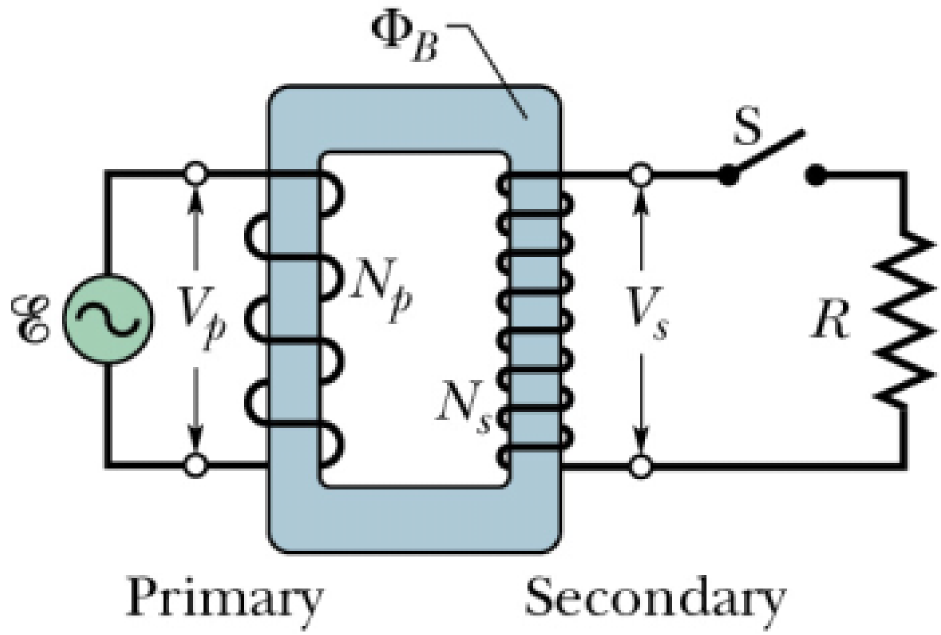
- Trade-off between current and voltage
  - For general use want low voltage
  - Means high current but

$$P_{avg} = I_{rms}^2 R$$

- General energy transmission rule: Transmit at the highest possible voltage and the lowest possible current

EM Oscillations (56)

- Transformer – device used to raise (for transmission) and lower (for use) the ac voltage in a circuit, keeping *iV* constant
  - Has 2 coils (primary and secondary) wound on same iron core with different #s of turns



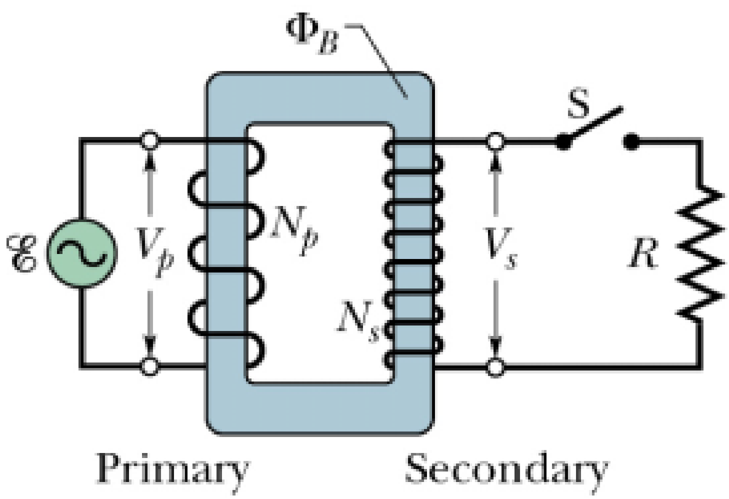
EM Oscillations (57)

- Alternating primary current induces alternating magnetic flux in iron core
- Same core in both coils so induced flux also goes through the secondary coil
- Using Faraday's law

$$V_P = -N_P \frac{d\Phi B}{dt}$$

$$V_S = -N_S \frac{d\Phi B}{dt}$$

$$\frac{V_P}{N_P} = \frac{V_S}{N_S}$$



EM Oscillations (58)

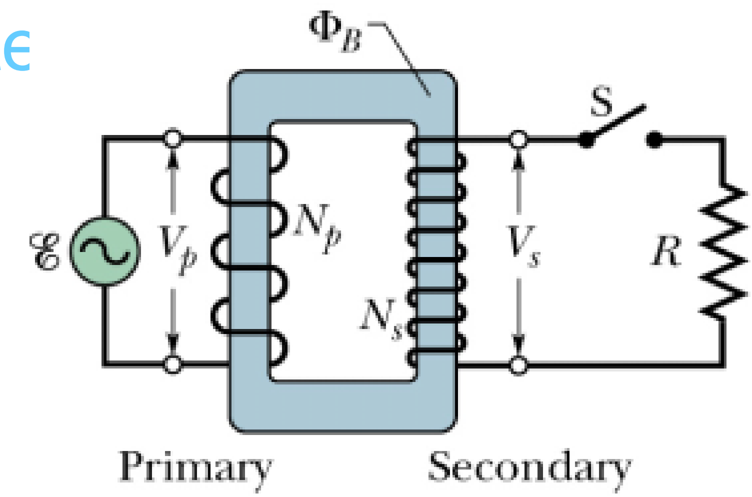
- Transformation of voltage is

$$V_S = V_P \frac{N_S}{N_P}$$

- If $N_S > N_P$ called a step-up transformer
- If $N_S < N_P$ called a ste down transformer
- Conservation of energy

$$I_P V_P = I_S V_S$$

$$I_S = I_P \frac{V_P}{V_S} = I_P \frac{N_P}{N_S}$$



EM Oscillations (59)

- The current I_P appears in primary circuit due to R in secondary circuit.

$$I_P V_P = I_S V_S \qquad I_S = V_S / R$$

$$I_P = \frac{V_S}{R}\frac{V_S}{V_P} = \frac{1}{R}\frac{V_S^2}{V_P^2}V_P = \frac{1}{R}\left(\frac{N_S}{N_P}\right)^2 V_P$$

- Has for of IP = VP/Req where

$$R_{eq} = \left(\frac{N_P}{N_S}\right)^2 R$$