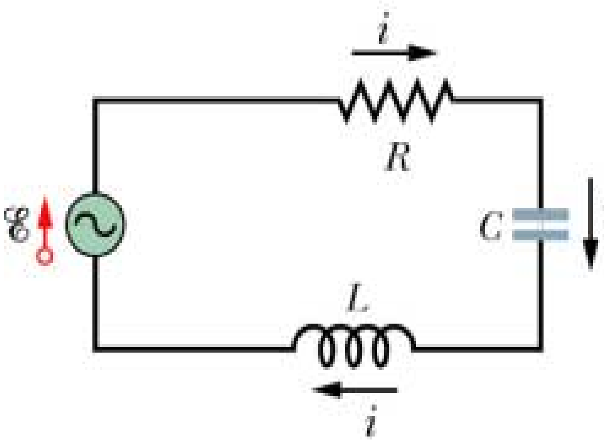
Lecture 31

Chapter 34

Electromagnetic Waves

Review



- For an RLC circuit
  - Voltages add up to emf

$$E = v_R + v_C + v_L$$

  - Maximum current given by $I = \frac{E_m}{Z}$
  - Impedance defined as

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad X_L = \omega_d L \qquad X_C = \frac{1}{\omega_d C}$$

  - Phase constant defined as

$$\tan \phi = \frac{X_L - X_C}{R}$$

Review

- For RLC circuit, resonance and the max current *I* occurs when $\omega_d = \omega$

$$\omega_d = \omega = \frac{1}{\sqrt{LC}}$$

- For an ac circuit, define rms values

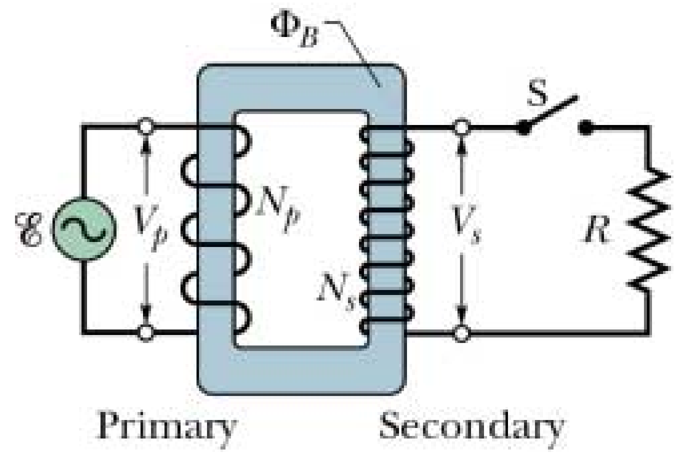
$$I_{rms} = \frac{I}{\sqrt{2}} \qquad V_{rms} = \frac{V}{\sqrt{2}} \qquad E_{rms} = \frac{E}{\sqrt{2}}$$

- Average power dissipated to thermal energy

$$P_{avg} = I_{rms}^2 R \qquad P_{avg} = E_{rms} I_{rms} \cos\phi$$

Review

- Transformer –
  - 2 coils (primary and secondary) wound around same iron core
  - Transformation voltage and current are related to ratio of the number of turns in the coils



$$V_S = V_P \frac{N_S}{N_P}$$

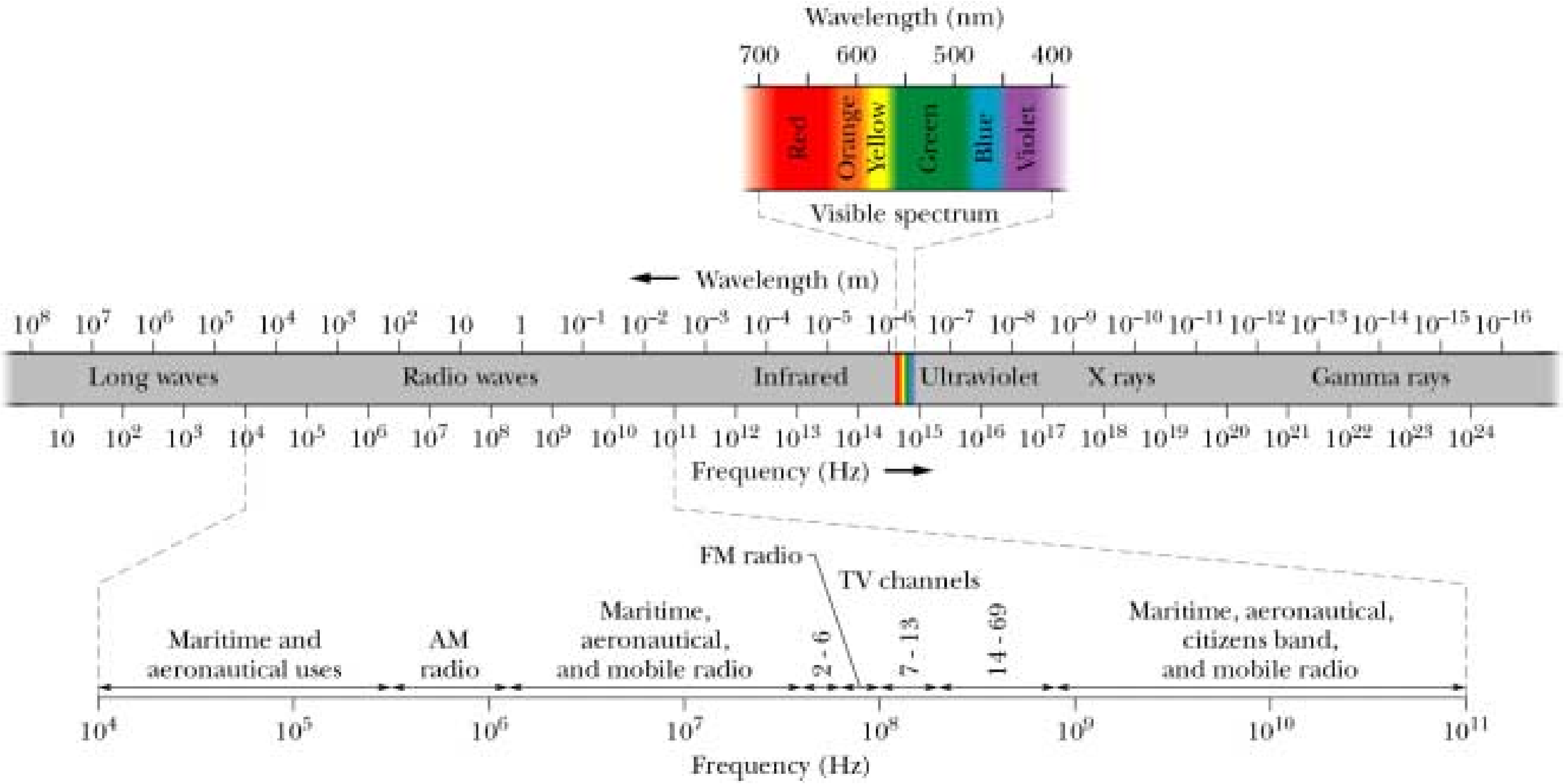
$$I_S = I_P \frac{N_P}{N_S}$$

  - Equivalent resistance seen by generator

$$R_{eq} = \left(\frac{N_P}{N_S}\right)^2 R$$

EM Waves (1)

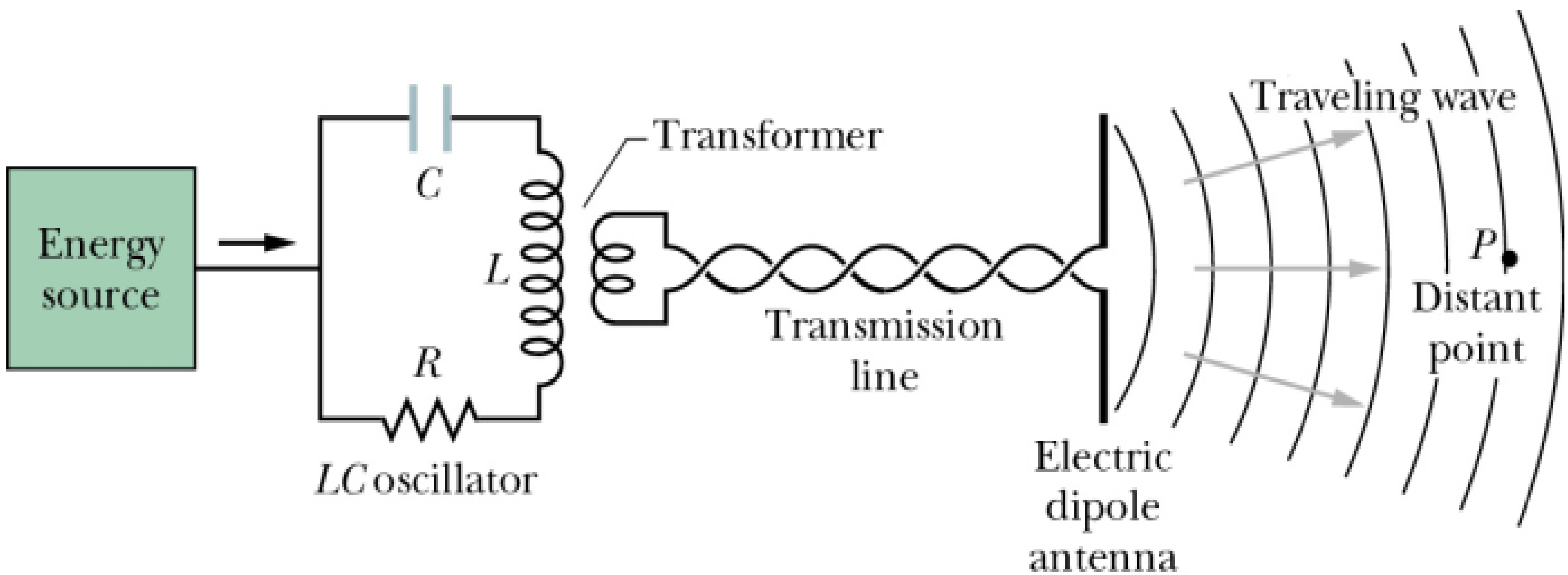
- Electromagnetic waves –
  - Beam of light is a traveling wave of E and B fields
  - All waves travel through free space with same speed



EM Waves (2)

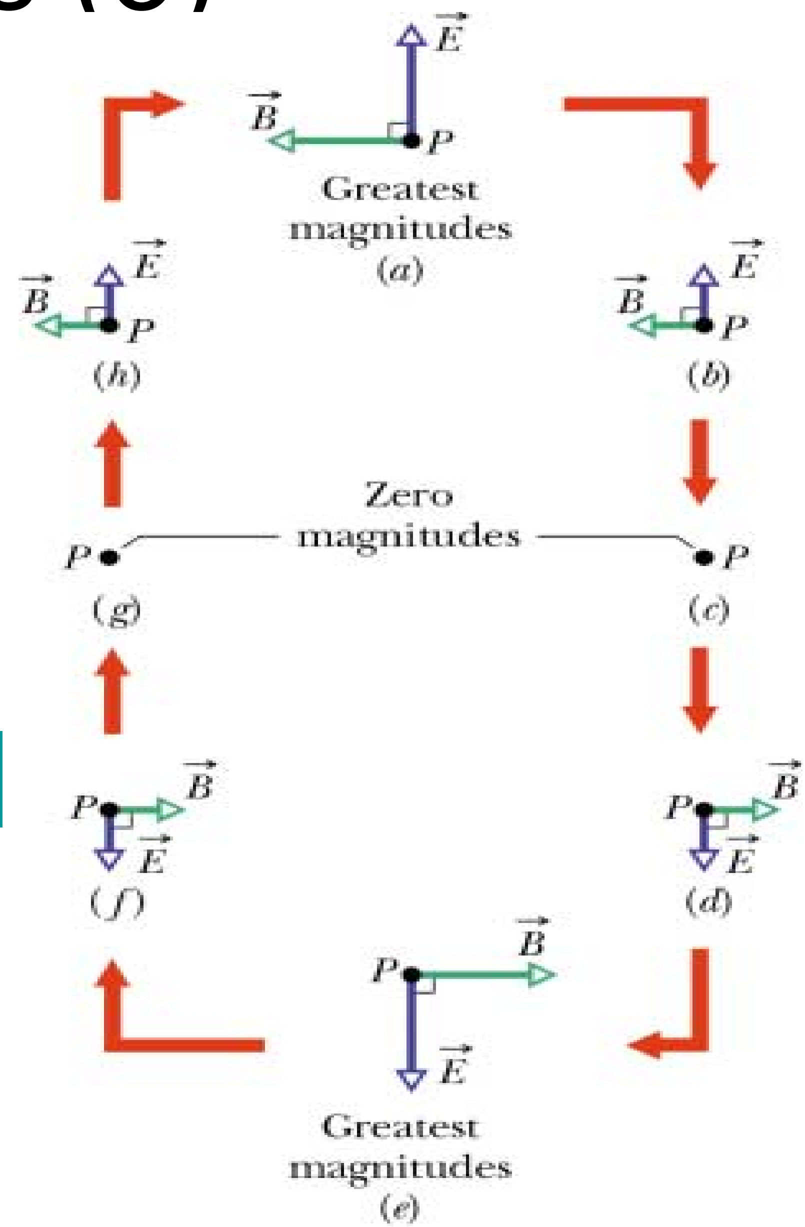
- Generate electromagnetic (EM) waves –
  - Sinusoidal current in RLC causes charge and current to oscillate along rods of antenna with angular frequency ω
  - Changing E and B fields form EM wave that travels away from antenna at speed of light, c

$$\omega = \frac{1}{\sqrt{LC}}$$



EM Waves (3)

- E and B fields change with time and have features:
  - *E* and *B* fields ⊥ to direction of wave's travel – transverse wave
  - *E* field is ⊥ *B* field
  - Direction of wave's travel is given by cross product $\vec{E} \times \vec{B}$
  - *E* and *B* fields vary
    - Sinusodially
    - With same frequency and in phase

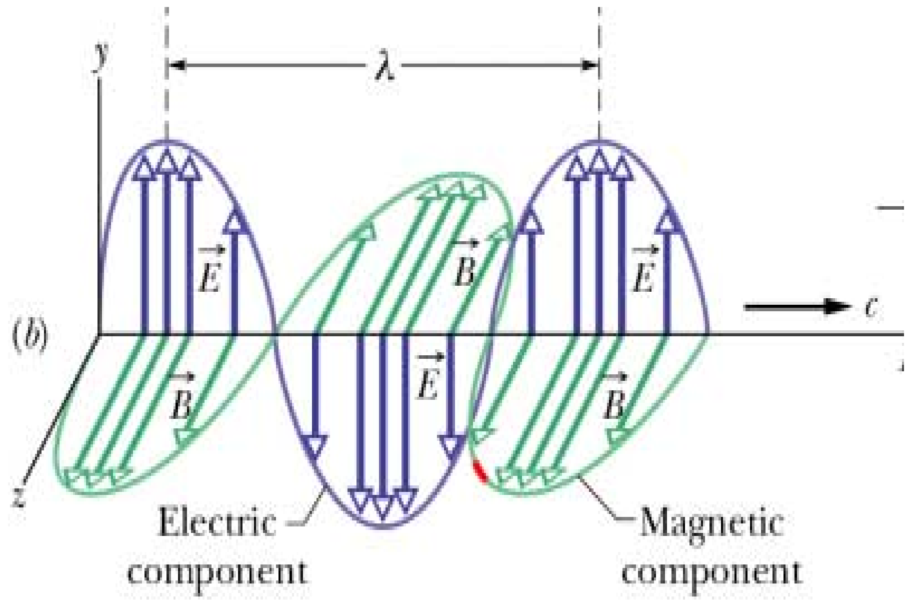


EM Waves (4)

- Write *E* and *B* fields as sinusoidal functions of position *x* (along path of wave) and time *t*

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$



- Angular frequency *ω* and angular wave number *k*

$$\omega = 2\pi f$$

$$k = \frac{2\pi}{\lambda}$$

- *E* and *B* components cannot exist independently

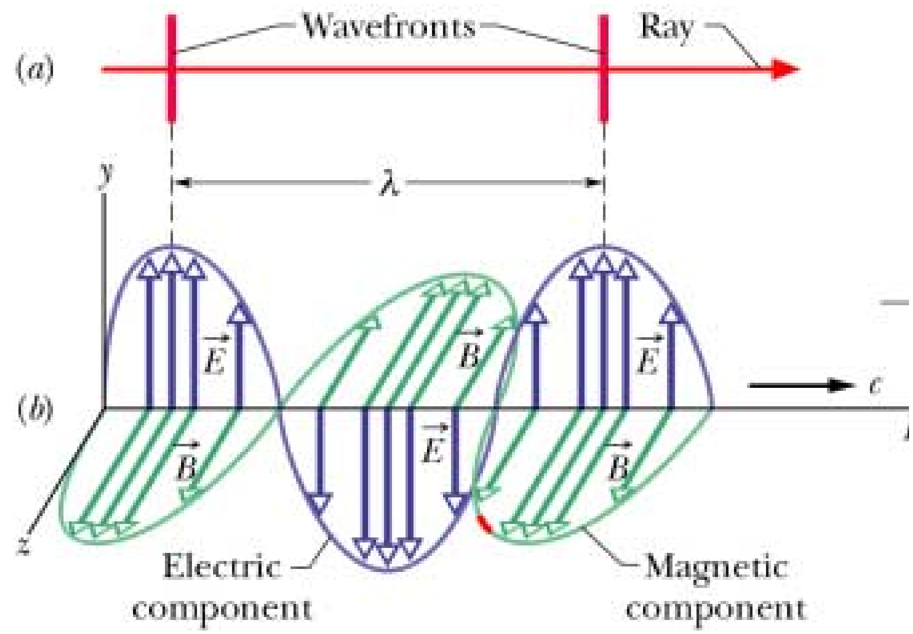
EM Waves (5)

- Speed of wave is $v = \dfrac{\omega}{k}$
- Using definition of ω and k, velocity is

$$\omega = 2\pi f \qquad k = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} = \frac{2\pi f}{2\pi / \lambda} = f\lambda$$

- In vacuum EM waves move at speed of light



$$v = c = f\lambda$$

$$c = 3\times 10^{8}\, m/s$$

EM Waves (6)

- Use Faraday's and Maxwell's laws of indu

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Can prove that speed of light c is given by (proof do tion

$$c = \frac{E_m}{B_m}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c = 3 \times 10^8 m/s$$

- Light travels at same speed regardless of what reference frame its measured in

EM Waves (7)

- EM waves can transport energy and deliver it to an object it falls on
- Rate of energy transported per unit area is given by Poynting vector, *S*, and defined as

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- SI unit is W/m2
- Direction of *S* gives wave's direction of travel

EM Waves (8)

- Magnitude of *S* is given by $$S = \frac{1}{\mu_0} EB$$
- Found relation $$c = \frac{E_m}{B_m}$$
- Rewrite *S* in terms of *E* since most instruments measure *E* component rather than *B* $$S = \frac{1}{\mu_0} E \frac{E}{c}$$
- Instantaneous energy flow rate is $$S = \frac{1}{c\mu_0} E^2$$

EM Waves (9)

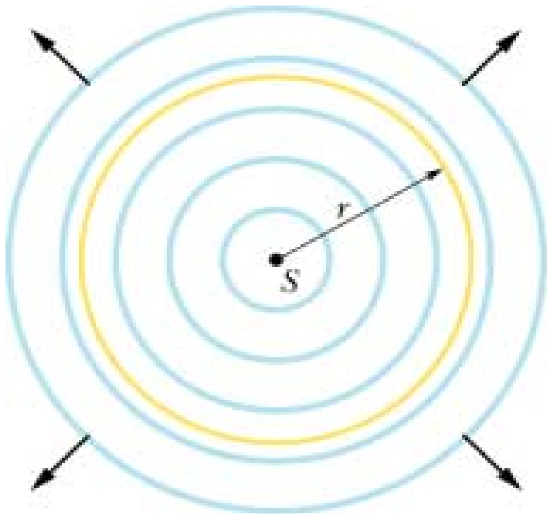
- Usually want time-averaged value of *S* also called intensity I $I = S_{avg} = \left(\frac{energy\ / time}{area}\right)_{ave} = \left(\frac{power}{area}\right)_{ave}$

$$I = \frac{1}{\mu_0 c}\left[E^2\right]_{avg} = \frac{1}{\mu_0 c}\left[E_m^2 \sin^2(kx - \omega t)\right]_{avg}$$

- Average value over full cycle of $\sin^2\theta = 1/2$
- Use the rms value $E_{rms} = \frac{E_m}{\sqrt{2}}$
- Rewrite average S or intensity as $I = \frac{1}{\mu_0 c} E_{rms}^2$

EM Waves (10)

- Find intensity, *I* , of point source which emits light isotropically – equal in all directions



$$I = S_{avg} = \left(\frac{energy\ /\ time}{area}\right)_{ave} = \left(\frac{power}{area}\right)_{ave}$$

- Find *I* at distance *r* from source
- Imagine sphere of radius *r* and area

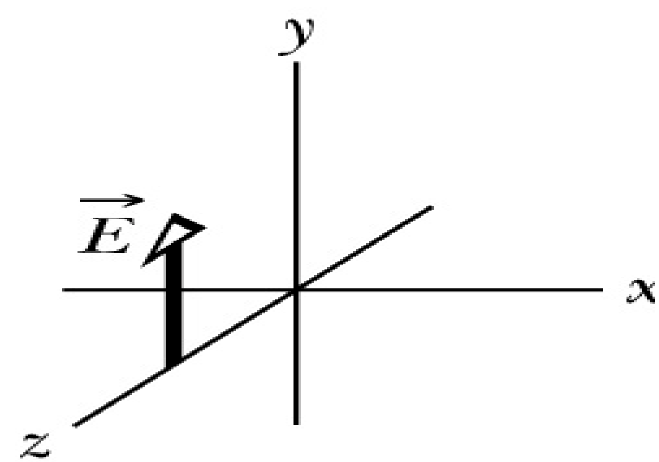
$$A = 4\pi r^2$$

$$I = \frac{Power}{Area} = \frac{P_S}{4\pi r^2}$$

- *I* decreases with square of distance

EM Waves (11)

- Checkpoint #2 – Have an *E* field shown in picture. A wave is transporting energy in the negative *z* direction. What is the direction of the *B* field of the wave?



- Poynting vector gives

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- Use right-hand rule to find *B* field

Positive *x* direction