Lecture 39

Chapter 37

Diffraction

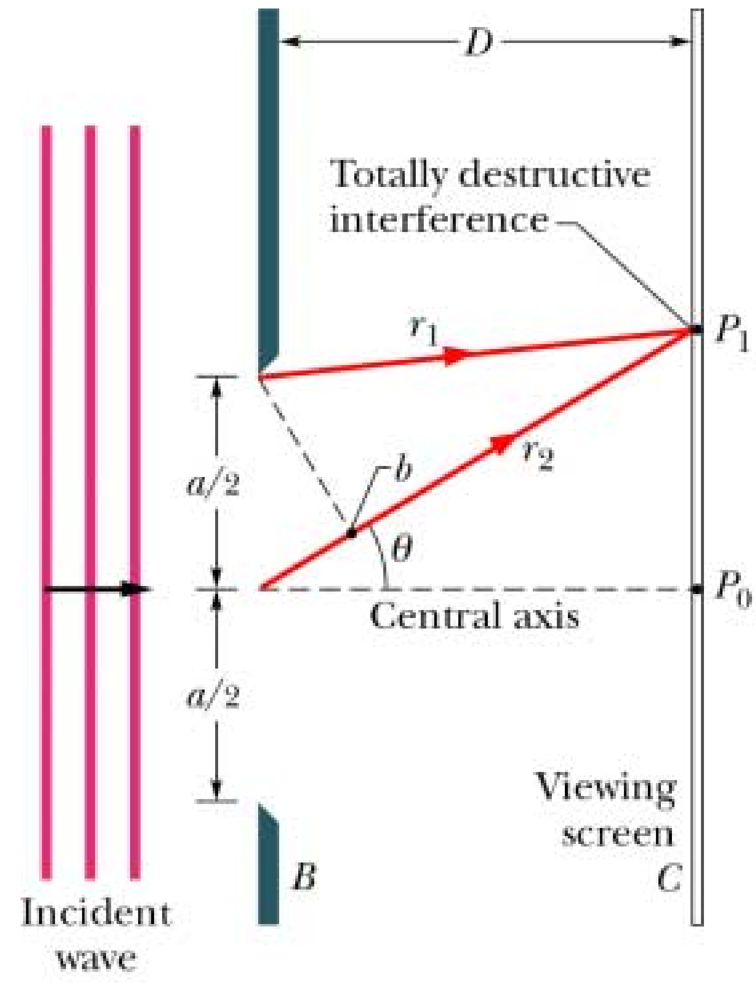
Review

- Interference –
  - Combining waves from small number of coherent sources – double-slit experiment with slit width much smaller than wavelength of the light
- Diffraction –
  - Combining of large number of wavelets from single wavefront – as in single slit experiment
- Diffraction and interference are both
  - the result of combining waves with different phases at a given point
  - Usually present simultaneously

Diffraction (4)

- Locate minima for single slit diffraction pattern
- Divide slit width in half
- Wavelet from bottom half travels further distance to point P_1 than from top half
- Wavelets will destructively interfere if path length difference ΔL is $\frac{1}{2}\lambda$

$$\Delta L = \frac{\lambda}{2}$$



Diffraction (5)

- Assume screen far away from slit so rays are ||
- Use trig to find ΔL

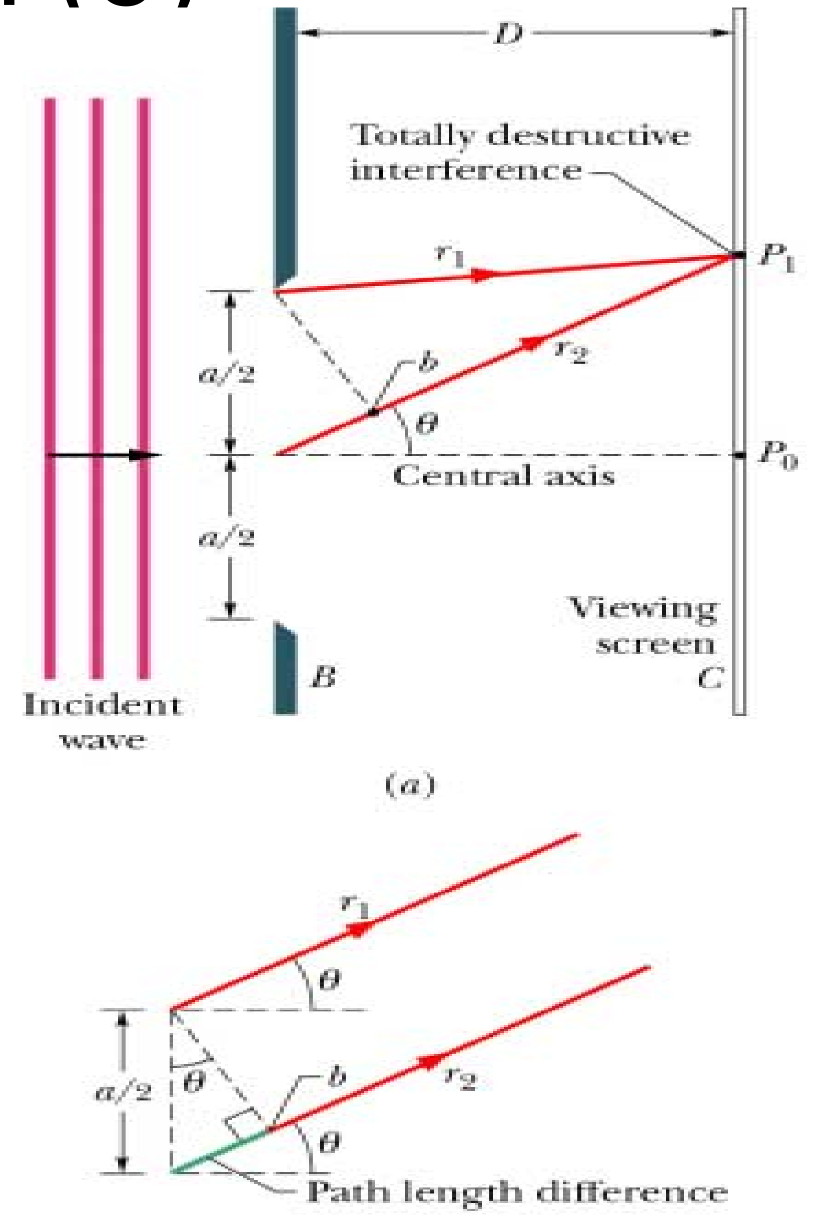
$$\sin\theta = \frac{\Delta L}{a/2}$$

- For first minima on screen

$$\Delta L = \frac{\lambda}{2} = \frac{a}{2}\sin\theta$$

- Rearrange to find

$$a\sin\theta = \lambda$$

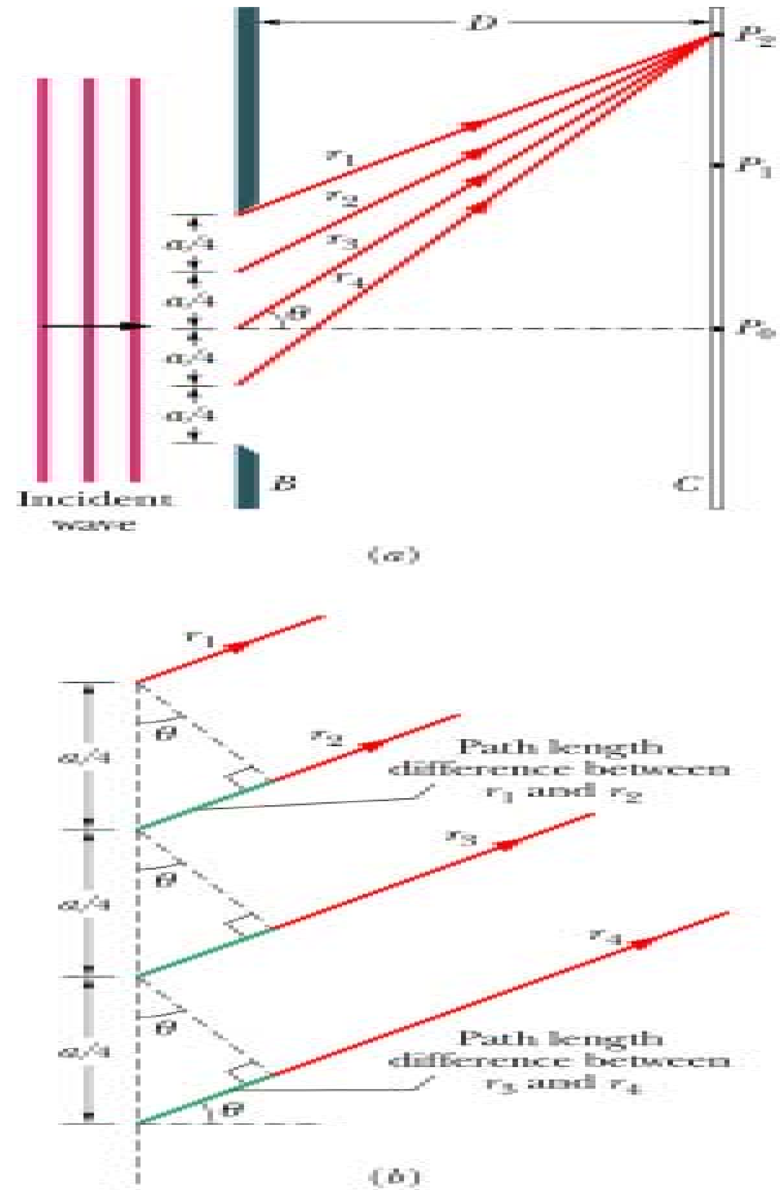


Diffraction (6)

- Find minima (dark fringes) from single-slit diffraction

$$a \sin\theta = m\lambda, \;\; m = 1,2,3..$$

- *a* is the width of slit
- *m* =0 is missing since that is where the bright central maximum is located
- Maxima are half-way between minima



Diffraction (7)

- Checkpoint #1 – Diffraction pattern from single narrow slit illuminated by blue light (450 nm). Do the minima shift away from or toward the bright central maximum if we A) switch to yellow light (575 nm) or B) decrease the slit width?

$$a \sin\theta = m\lambda$$

- A) If wavelength λ increases then $\sin\theta$ and θ increase so fringes move further away from center
- Location of minima from diffraction changes for different wavelengths λ

Diffraction (8)

- Checkpoint #1 – Diffraction pattern from single narrow slit illuminated by blue light (450 nm). Do the minima shift away from or toward the bright central maximum if we A) switch to yellow light (575 nm) or B) decrease the slit width?

$$\sin\theta = \frac{m\lambda}{a}$$

- B) Decrease slit width a, increases $\sin\theta$ and θ so fringes more further away from center
- If $a = \lambda$ then $\theta = 90°$ for first dark fringe, only see interference pattern since bright maxima from diffraction covers entire viewing area

Diffraction (9)

- Intensity of light as function of angle θ from central axis

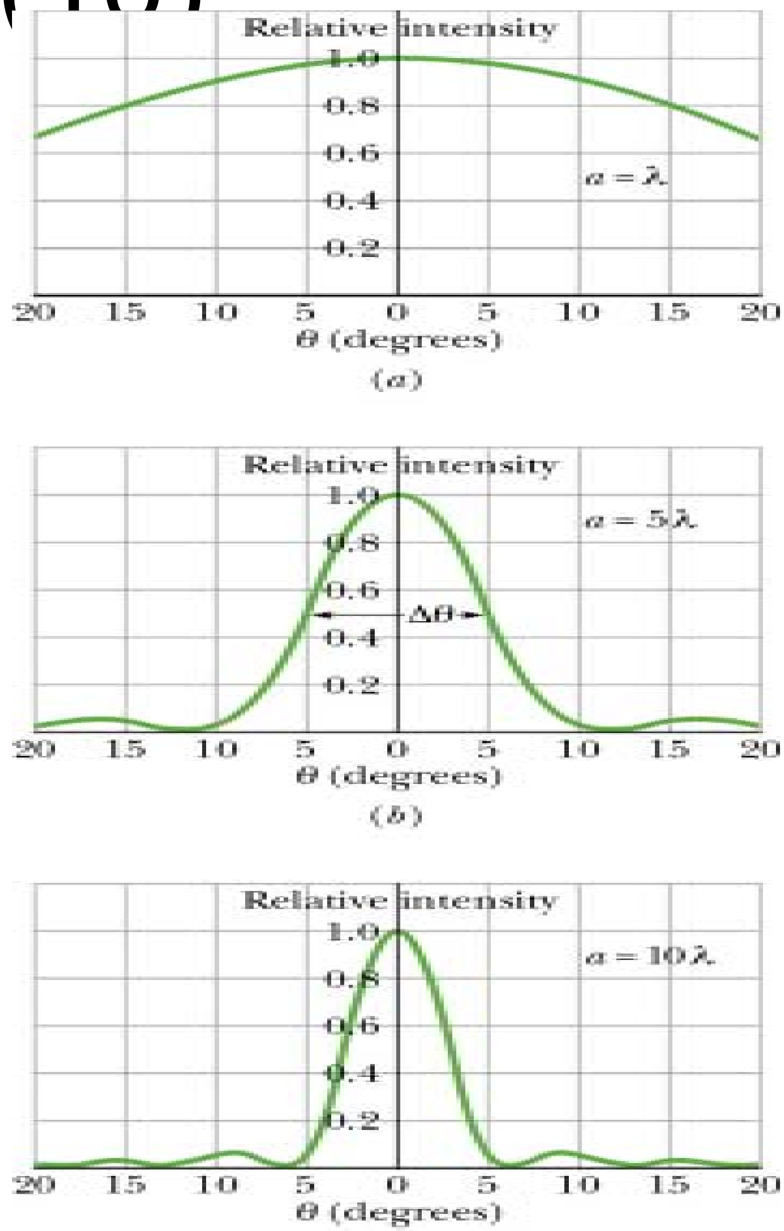
$$I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha} \right)^2$$

- I_m is intensity at central maximum
- α is ½ the phase difference ϕ between top and bottom rays from slit with width *a*

$$\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda} \sin\theta$$

Diffraction (10)

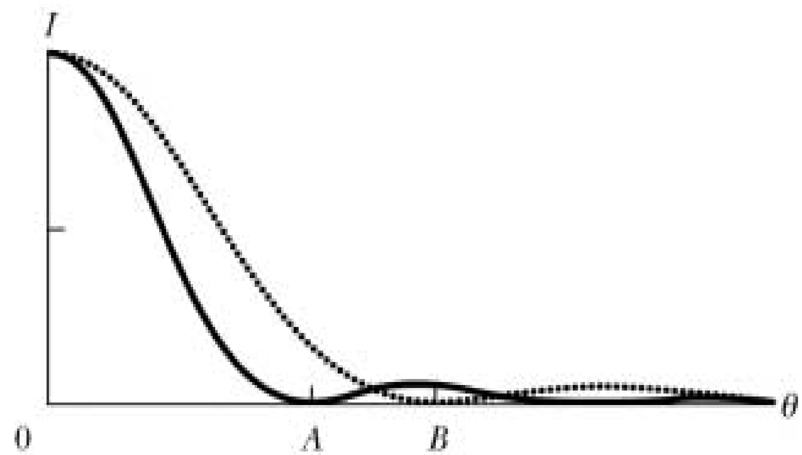
- Width of central diffraction maximum decreases as slit gets bigger (*a* increases)
  - Light undergoes less bending as slit gets bigger
- Secondary maxima much dimmer than central
- If slit is much wider than wavelength of light ($a >> \lambda$) the secondary maxima disappear, no longer have diffraction from single slit



Diffraction (11)

- Checkpoint #3 – The intensities are shown for red (650 nm) and blue (430 nm) which are used separately in a single-slit diffraction experiment. If both lights are used simultaneously what color will be seen in the combined diffraction pattern at angle A and B?

$$a \sin\theta = m\lambda$$

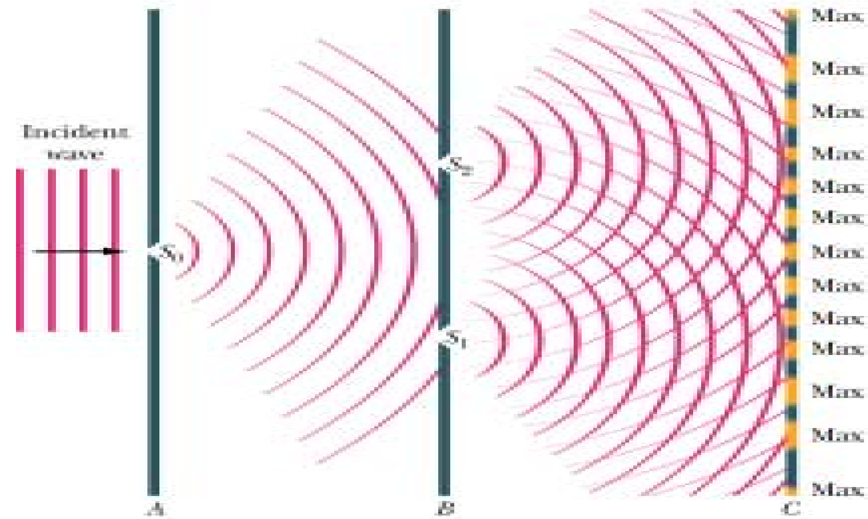


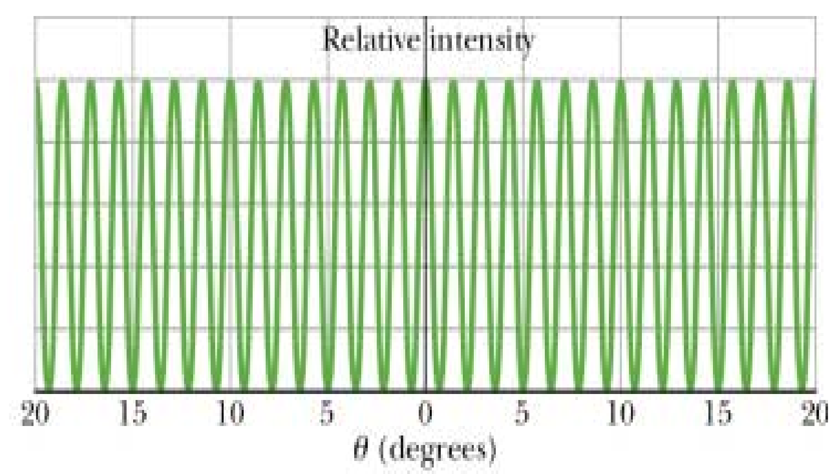
- Minima at greater θ for bigger λ
- Solid curve is for blue, dashed curve for red

Only red light at A and blue light at B

Diffraction (12)

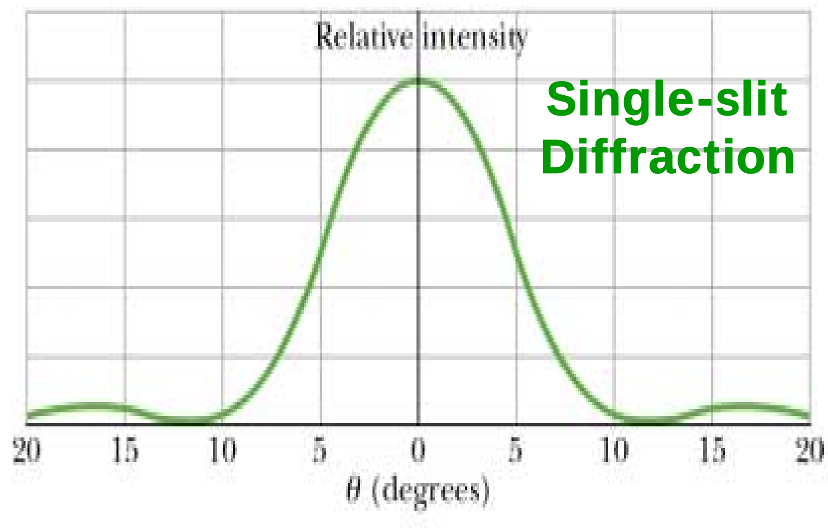
- Previous discussion of double-slit experiment assumed width of slit, a, was much smaller than λ
- Central maxima of diffraction pattern from each single slit covered entire screen so only saw interference pattern
- Bright fringes had same intensity and equal size

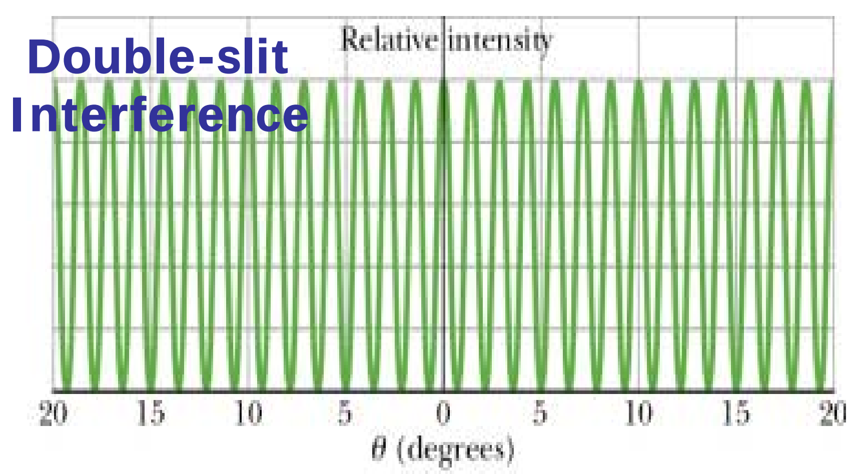


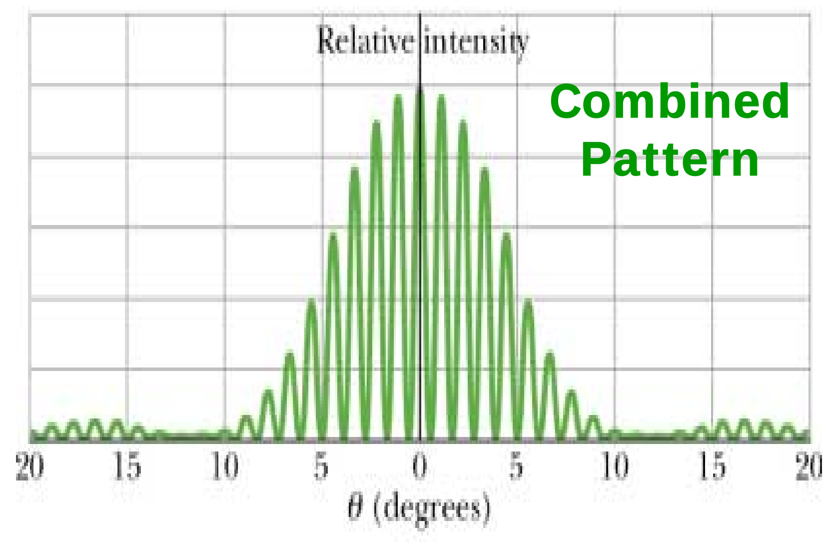


Diffraction (13)

- Double slit interference pattern is modified by diffraction pattern of light passing through each slit
- Position of fringes doesn't change but intensity is affected
  - See photo p.902







Diffraction (14)

- Intensity of double-slit interference pattern is given by

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2 (\cos^2 \beta)$$

- I_m is intensity of central maximum
- First term $(\sin\alpha/\alpha)^2$ is due to diffraction from slit of width *a*
- Second term of $\cos^2\beta$ is due to interference between 2 slits separated by distance *d*

$$\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta \qquad \beta = \frac{\pi d}{\lambda} \sin \theta$$

Diffraction (15)

- Problem 37- 4 – Consider interference and diffraction of light from a double slit experiment with slit width *a* =4.050 μm and slit separation *d* =19.44 μm and light of wavelength λ=405nm. How many bright interference fringes are within the central peak of the diffraction envelope?

- Single slit diffraction – limits of central maxima are first minima, *m* =1, in diffraction pattern due to either slit.

$$a \sin\theta = \lambda$$

Diffraction (16)

- Double-slit interference – Location of bright fringes of interference pattern given by

$$d \sin\theta = m\lambda, m = 0,1,2...$$

- From diffraction, position of first minima in radians is

$$\sin\theta = \frac{\lambda}{a}$$

- Want # of interference maxima, *m*

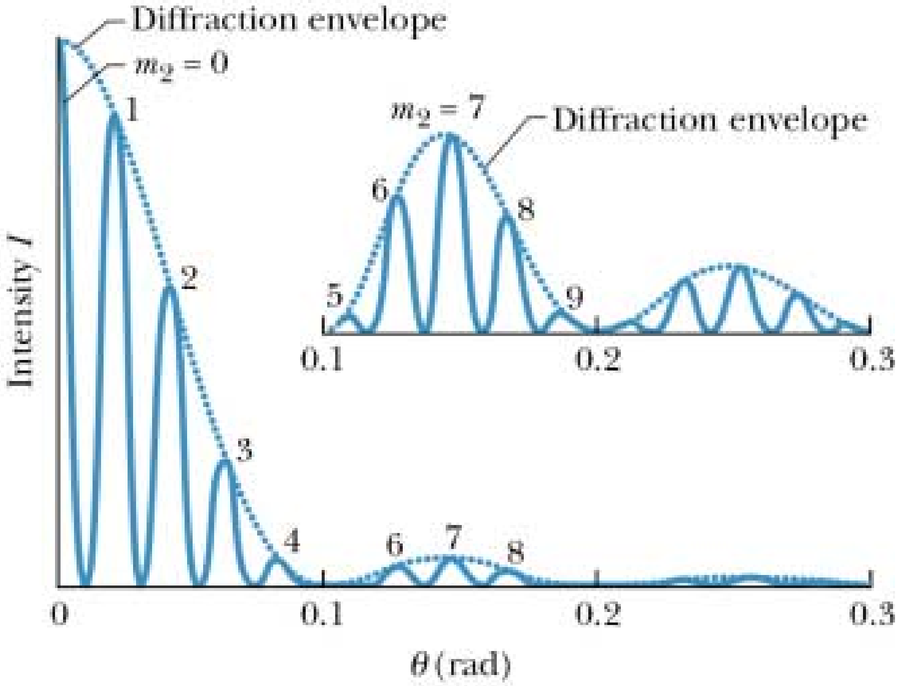
$$d\frac{\lambda}{a} = m\lambda \qquad m = \frac{d}{a}$$

Diffraction (17)

- Number of bright interference fringes depends on slit width *a* and slit separation *d*

$$m = \frac{d}{a} = \frac{19.44\,\mu\text{m}}{4.050\,\mu\text{m}} = 4.8$$

- Within central diffraction peak have 9 bright fringes from interference
  - Central maximum, *m* =0
  - Four maxima on each side of central maximum



Diffraction (18)

- Checkpoint #5 – If we increase wavelength of light in previous example to 550 nm does A) the width of the central diffraction peak and B) the # of bright interference fringes within the central diffraction peak increase, decrease or remain the same?

$$a \sin \theta = \lambda$$

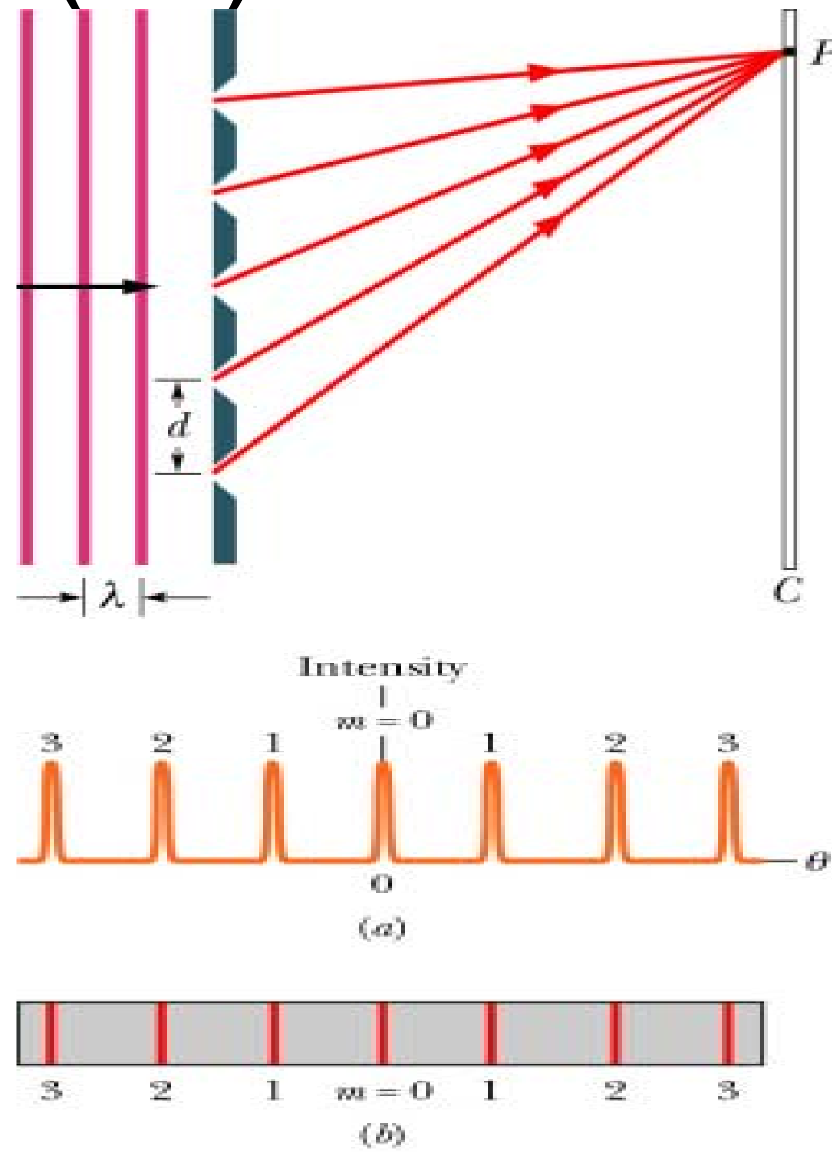
- For diffraction, bigger λ means bigger θ so
- A) Width increases
- Found # of bright interference fringes

$$m = \frac{d}{a}$$

- Slit width *a* and distance *d* did not change so
- B) # of interference fringes remains the same

Diffraction (19)

- Increase # of slits from 2 to a large number
- Bright fringes in intensity plot are now very narrow (called lines) and separated by wide dark regions
- A mask that contains a large number of || slits at equal separation distances, d, is called a diffraction grating



Diffraction (20)

- Waves from 2 adjacent slits are in phase and have a maxima (or a line) when

$$d \sin\theta = m\lambda, m = 0,1,2...$$

- Angle θ from central axis to any line depends on wavelength of light λ
- Light from a given source can be split into its emission lines (below are lines for hydrogen)
  - Use this to determine types of gases in stars