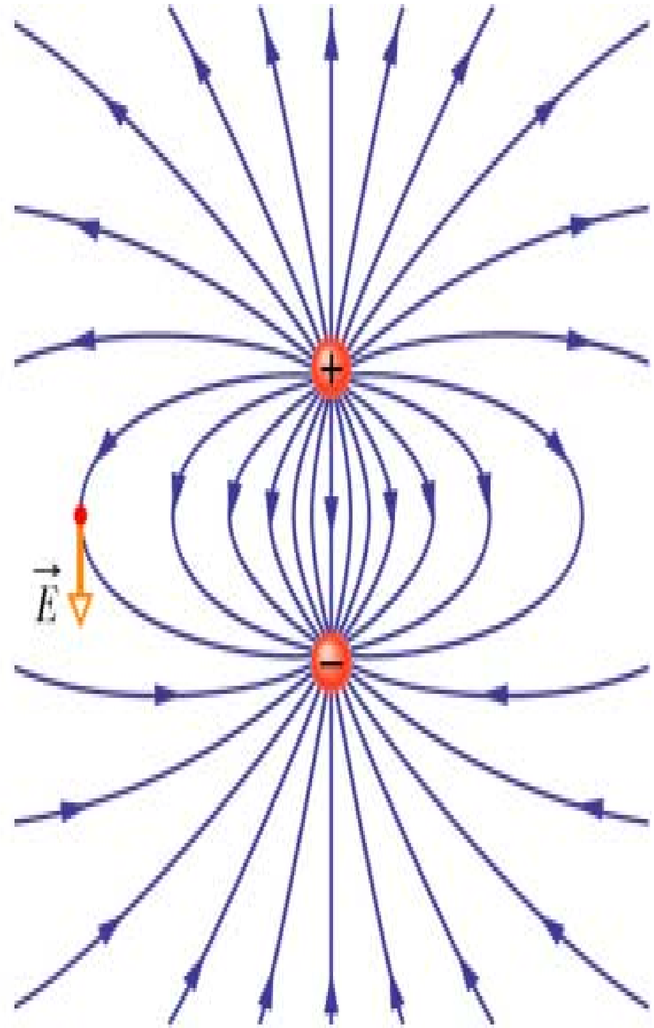
Lecture 4

Electric Field – Chapter 23

Electric Field (8)

- Electric field lines:
  - Point away from positive and towards negative
  - Tangent to the field line is the direction of the *E* field at that point
  - # lines is proportional to magnitude of the charge



Electric Field (9)

- Electric field, *E*, is the force per unit test charge in *N/C*

$$\vec{E} = \frac{\vec{F}}{q_0}$$

- For a point charge

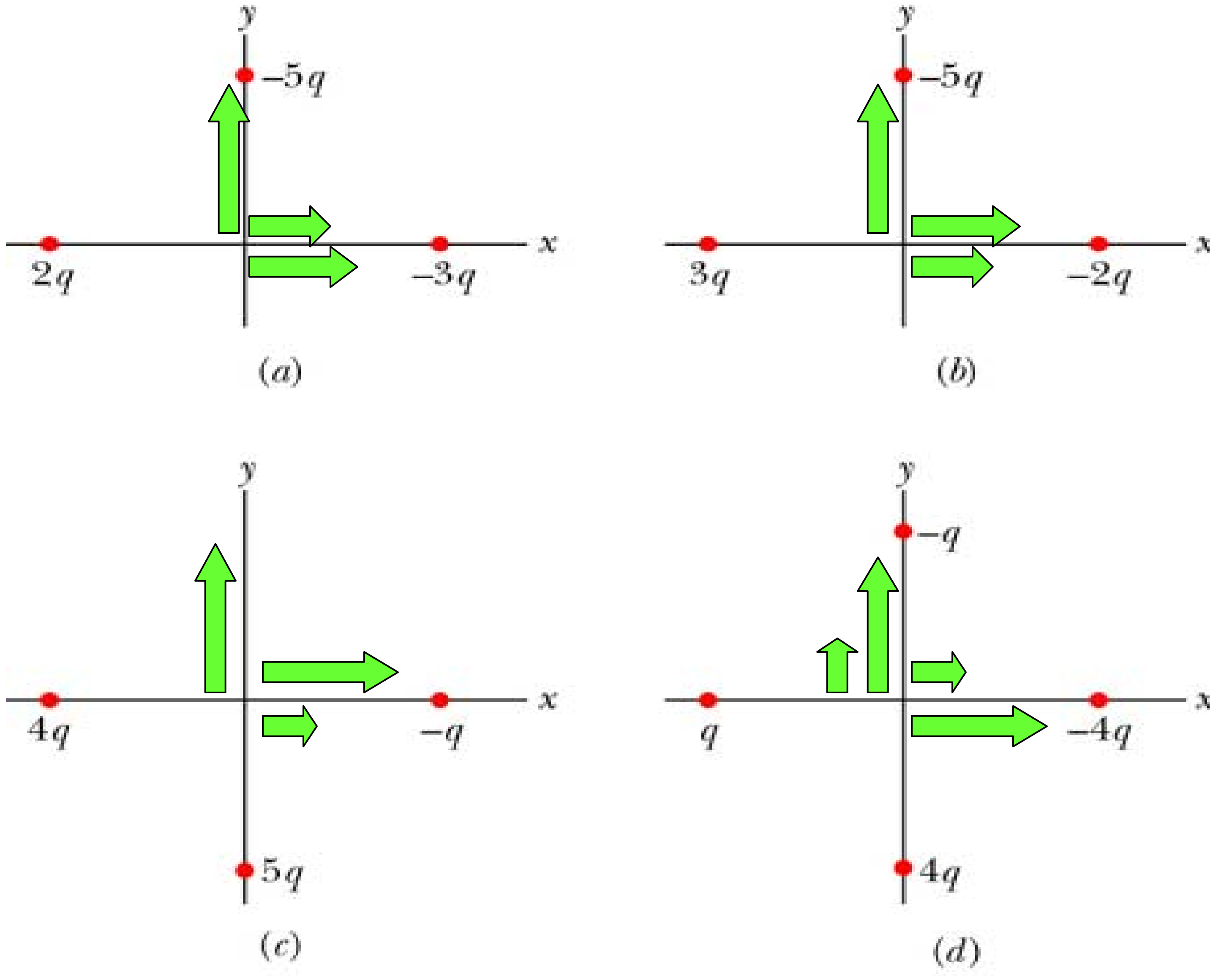
$$\vec{F} = k\frac{|q_0||q|}{r^2}$$ so $$\vec{E} = k\frac{|q|}{r^2}$$

Electric Field (10)

- Direction of *E* = direction of *F*
- *E* points towards (away from) a negative (positive) point charge
- Superposition of electric fields

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

Checkpoint #2 – Rank magnitude of net E



Checkpoint #2 – Rank magnitude of net E

- a) $E_x = k\frac{2q}{d^2} + k\frac{3q}{d^2} = k\frac{5q}{d^2}\hat{i}$ $E_y = k\frac{5q}{d^2}\hat{j}$

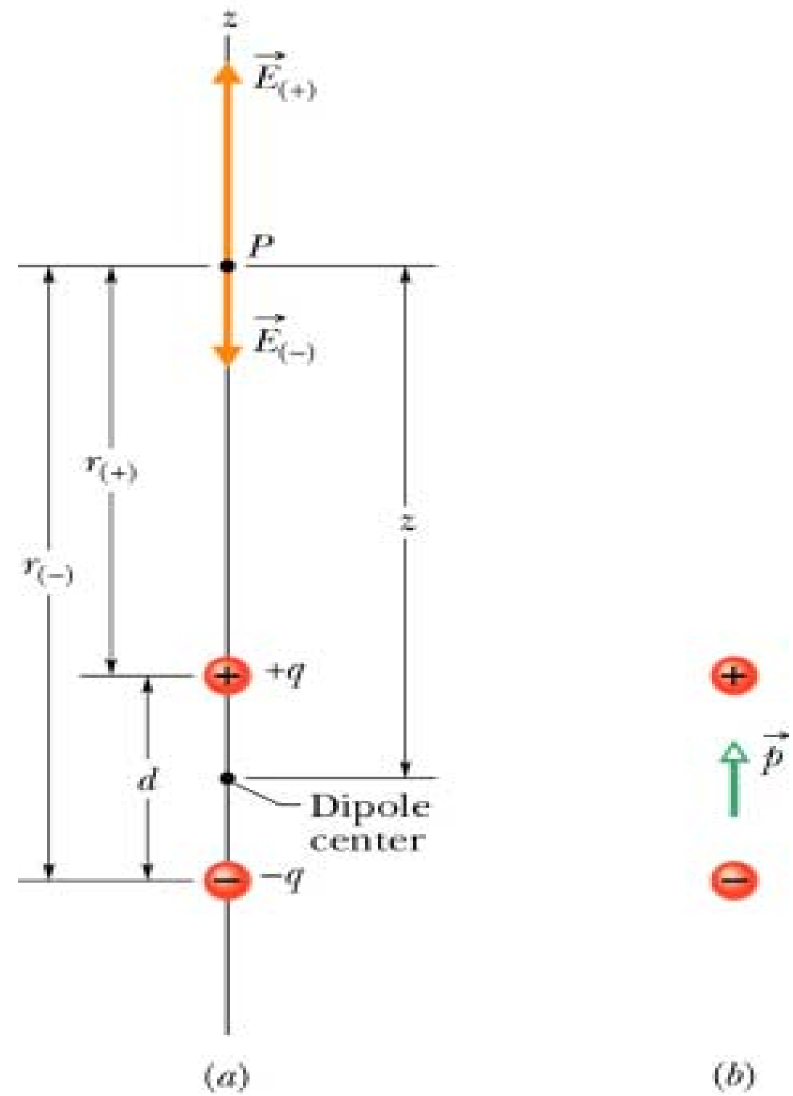
$$E = \sqrt{E_x^2 + E_y^2} = k\frac{\sqrt{50}q}{d^2}$$

- Do this for the rest and find

All Equal

Electric Field (11)

- Electric dipole – 2 equal magnitude, opposite charged particles separated by distance d
- What's the electric field at point P due to the dipole?



Electric Field (12)

- E is on z-axis so

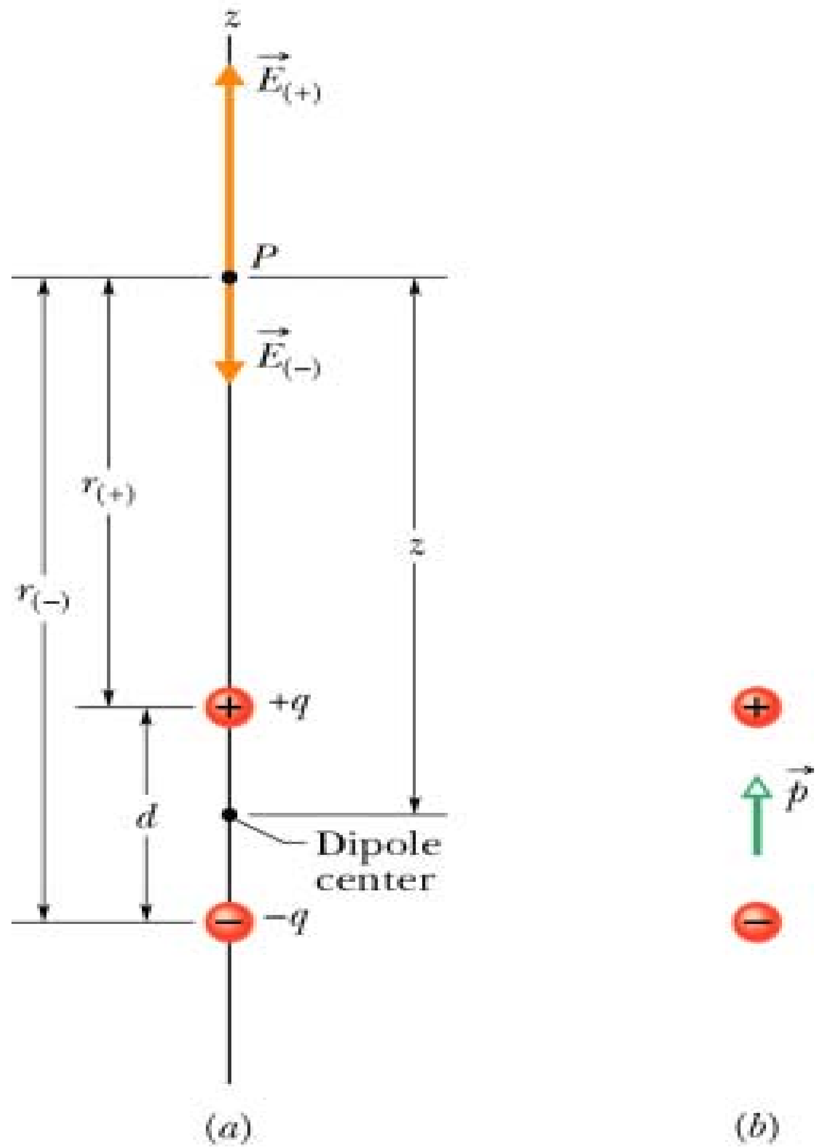
$$E = E_z = E_+ - E_-$$

- giving

$$E = k\frac{q}{r_+^2} - k\frac{q}{r_-^2}$$

- where

$$r_+ = z - \frac{d}{2} \qquad r_- = z + \frac{d}{2}$$



Electric Field (13)

- Substituting and rearranging gives

$$E = \frac{kq}{z^2}\left[\left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2}\right]$$

- Assuming z>>d then expand using binomial theorem ignoring higher order terms d/z<<1

$$E = \frac{kq}{z^2}\left[\left(1 + \frac{d}{z} + \ldots\right) - \left(1 - \frac{d}{z} + \ldots\right)\right]$$

Electric Field (14)

- Approximate *E* field for a dipole is

$$E = \frac{2\,kqd}{z^{3}}$$

- Define electric dipole moment, *p*, as

$$\vec{p} = qd$$

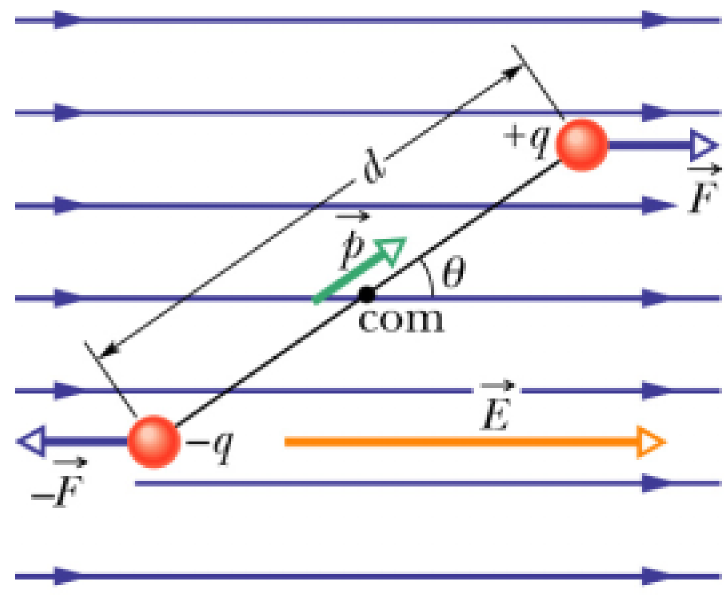
where direction of *p* is from the negative to positive end

- E field along dipole axis at large distances (z>>d) is

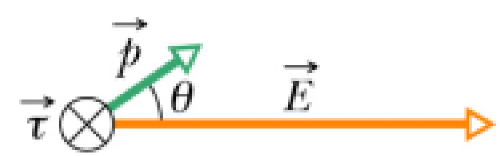
$$E = \frac{2kp}{z^{3}}$$

Electric Field (15)

- What happens when a dipole is put in an electric field?
- Net force, from uniform *E*, is zero
- But force on charged ends produces a net torque about its center of mass



(a)



(b)

Electric Force (16)

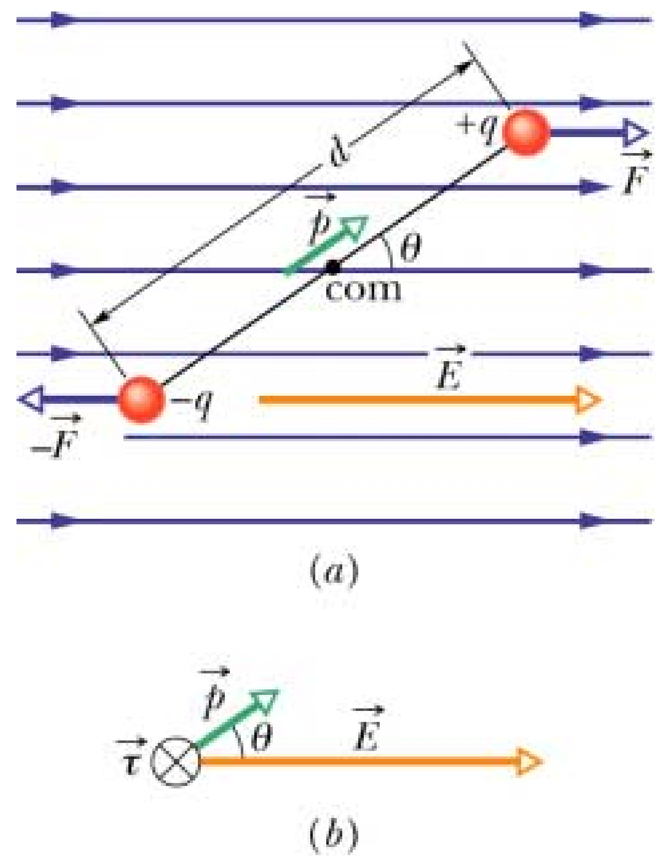
- Definition of torque $\vec{\tau} = \vec{r} \times \vec{F} = rF \sin \phi$

- For dipole rewrite it as

$$\tau = xF\sin\theta + (d - x)F\sin\theta$$

- Substitute *F* and *d* to get

$$\vec{\tau} = \vec{p} \times \vec{E}$$



Electric Field (17)

- Torque acting on a dipole tends to rotate *p* into the direction of *E*

- Associate potential energy, *U*, with the orientation of an electric dipole in an *E* field
- Dipole has least *U* when *p* is lined up with *E*

Electric Field (18)

- Remember $$U = -W = -\int_{90}^{\theta} \tau \, d\theta = \int_{90}^{\theta} pE \sin\theta \, d\theta$$

- Potential energy of a dipole

$$U = -pE\cos\theta = -\vec{p} \bullet \vec{E}$$

- *U* is least (greatest) when p and E are in same (opposite) directions

Checkpoint #5

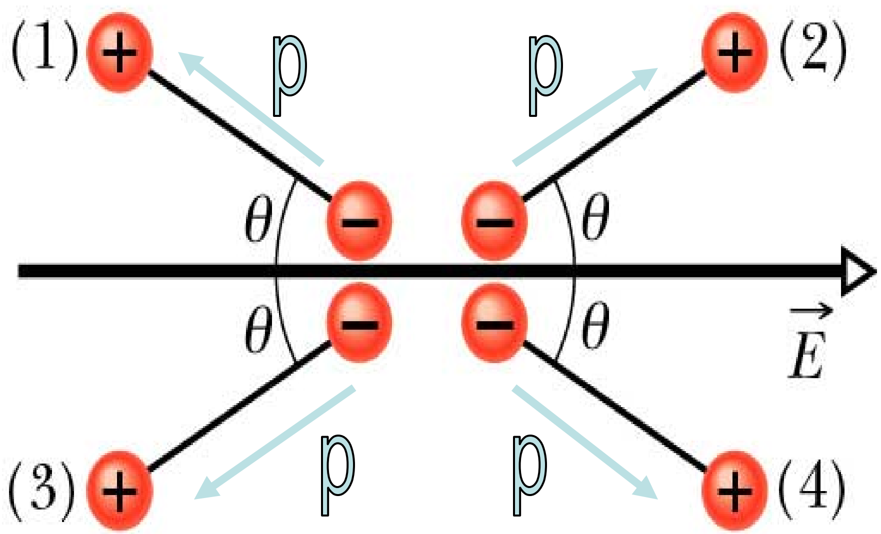
- Rank a) magnitude of torque and b) *U* , greatest to least

$$\tau = pE \sin\theta$$

- a) Magnitudes are same

$$U = -pE\cos\theta$$

- *U* greatest at θ=180
- b) 1 & 3 tie, then 2 &4



Electric Field (19)

- *E* field from a continuous line of charge
- Use calculus and a charge density instead of total charge, *Q*
- Linear charge density $\lambda = Q / Length$
- Surface charge density $\sigma = Q / Area$
- Volume charge density $\rho = Q / Volume$

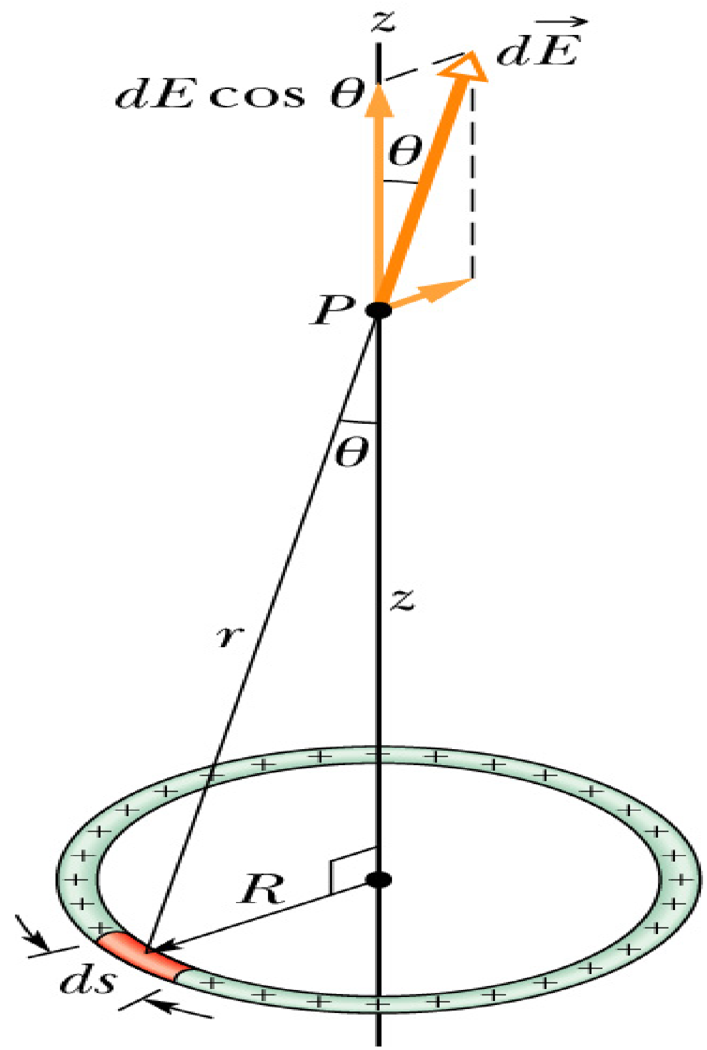
Electric Field (20)

- Ring of radius *R* and positive charge density λ
- Use

$$E = k\frac{q}{r^2}$$

- Divide ring into diff. elements of charge so

$$dq = \lambda ds$$



Electric Field (21)

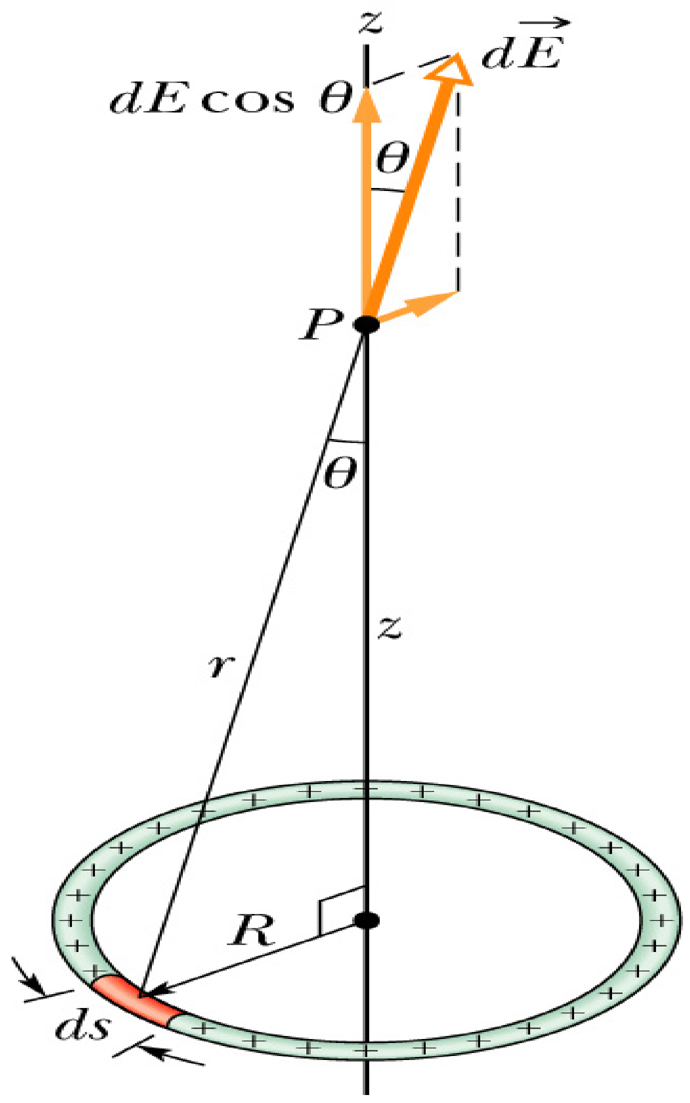
- Differential *dE* at P is

$$dE = k\frac{dq}{r^2} = k\frac{\lambda ds}{r^2}$$

- From trig

$$r^2 = z^2 + R^2$$

- Look for symmetry
  - All $\perp$ cancel and $||$ point upward



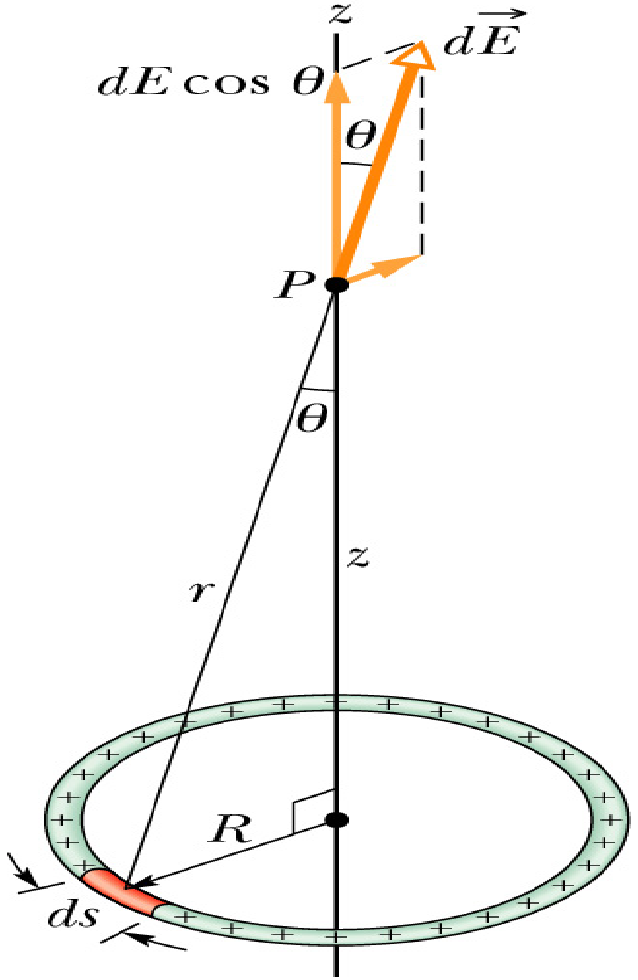
Electric Field (22)

- Parallel component *dE* is

$$dE\cos\theta = k\frac{\lambda ds}{\left(z^2 + R^2\right)}\cos\theta$$

- Use trig to rewrite cos θ

$$\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2 + R^2\right)^{1/2}}$$



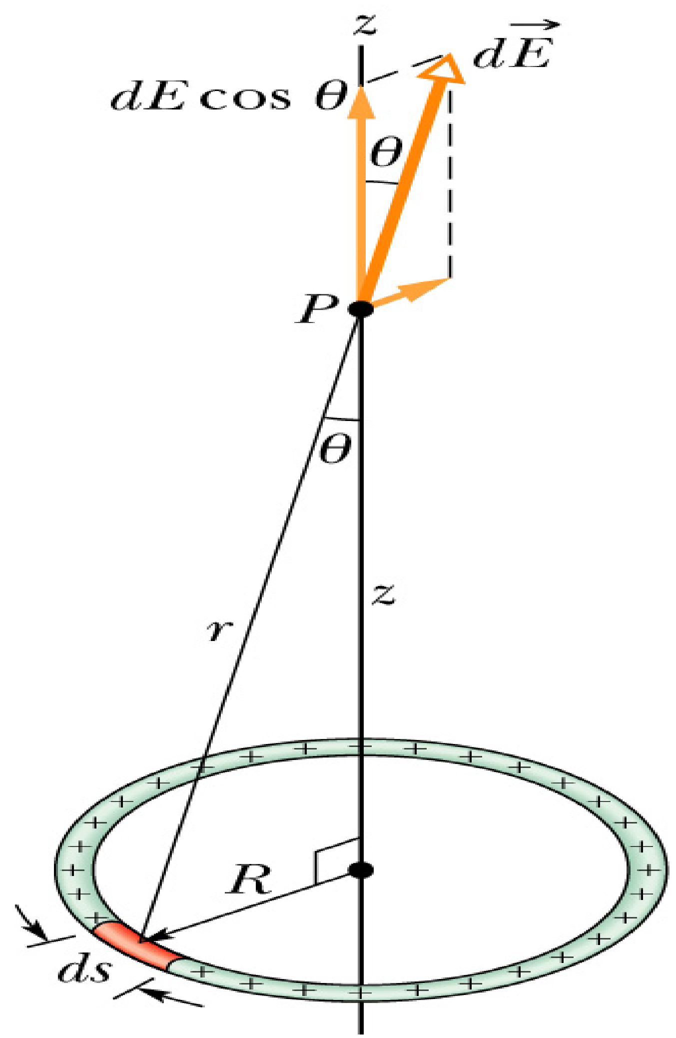
Electric Field (23)

- Substituting

$$dE\cos\theta = k\frac{z\lambda}{\left(z^2+R^2\right)^{3/2}}ds$$

- Integrate around the ring

$$E=\int dE\cos\theta=\frac{kz\lambda}{\left(z^2+R^2\right)^{3/2}}\int_0^{2\pi r}ds$$



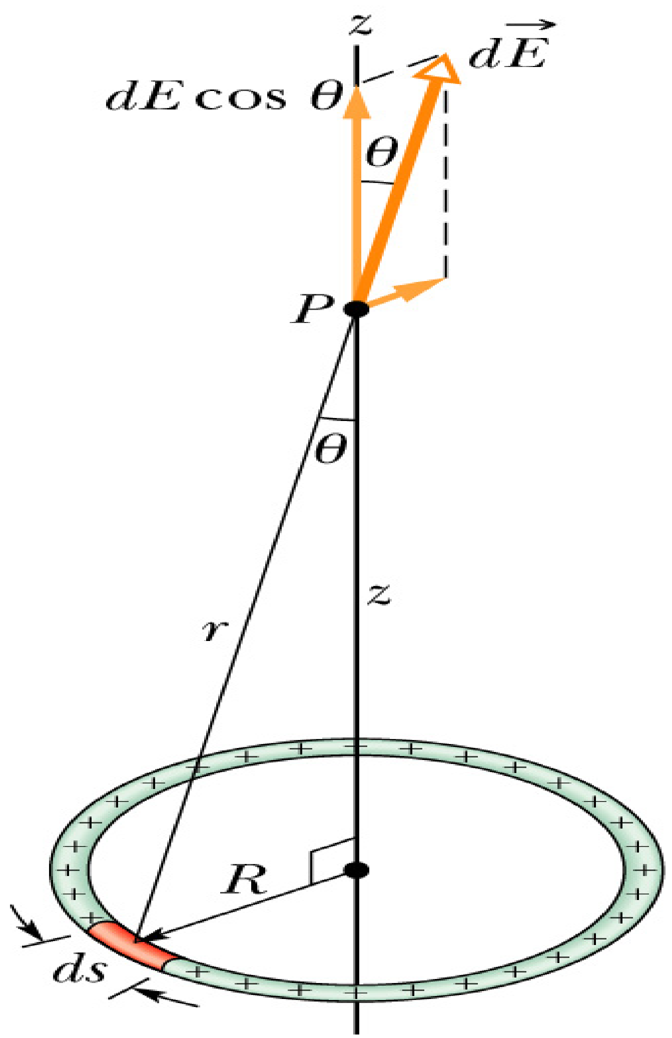
Electric Field (24)

- Finally get $$E = \frac{kz\lambda(2\pi r)}{\left(z^2 + R^2\right)^{3/2}}$$

- Replace λ with $$\lambda = q/(2\pi r)$$

- Charge ring has *E* of

$$E = \frac{kqz}{\left(z^2 + R^2\right)^{3/2}}$$



Electric Field (25)

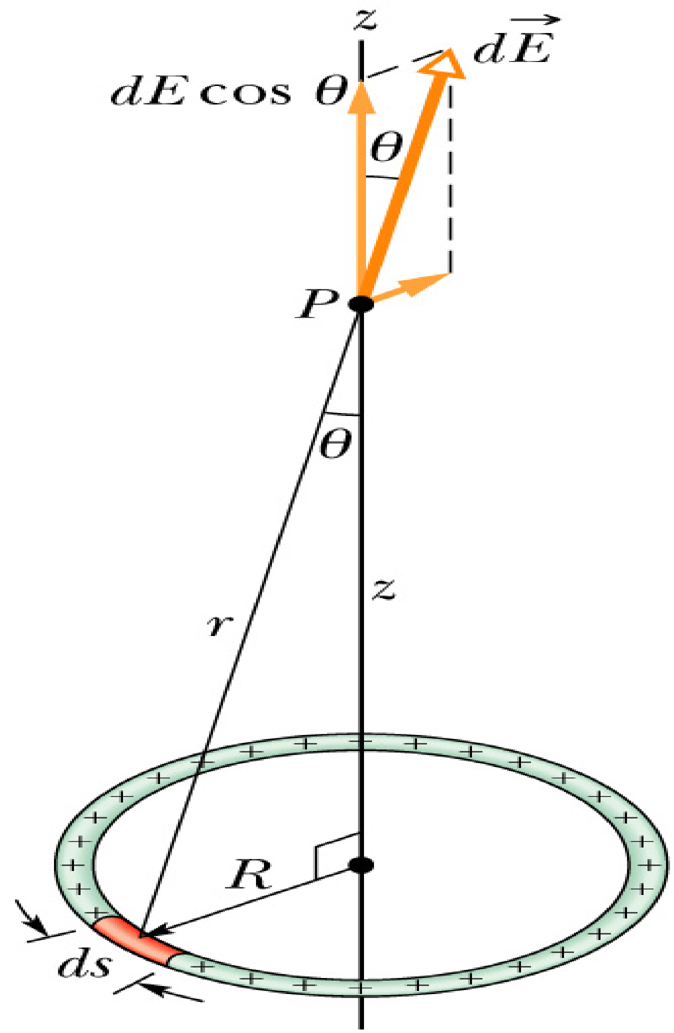
- Charge ring has *E* of

$$E = \frac{kqz}{\left(z^2 + R^2\right)^{3/2}}$$

- Check *z* >>*R* then

$$E = \frac{kq}{z^2}$$

- From far away ring looks like point charge



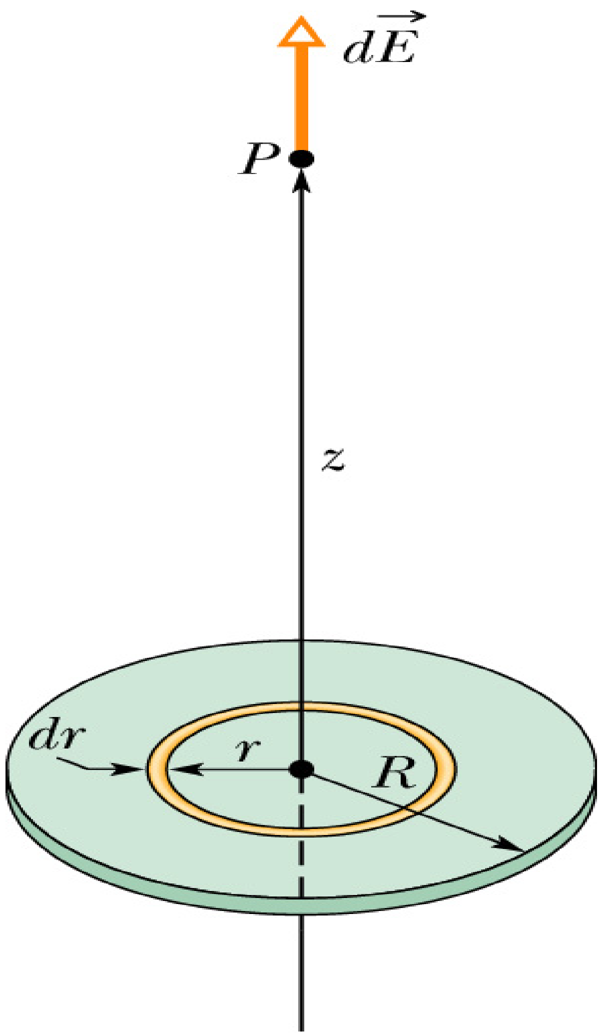
Electric Field (26)

- Also do this for charged disk
- But now surface charge

$$dq = \sigma A = \sigma\left(2\pi r dr\right)$$

- Integrate to get

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$



Electric Field (27)

- Charge disk of radius *R*

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- Let R→∞ then get

$$E = \frac{\sigma}{2\varepsilon_0}$$

- Acts as infinite sheet of a non-conductor with uniform charge