Lecture 5

Gauss' Law – Chapter 24

Gauss' Law (1)

- Gauss' law - new form of Coulomb's law
  - Easier to use in symmetry situations
  - Electrostatics fully equivalent

- Gaussian surface – hypothetical closed surface

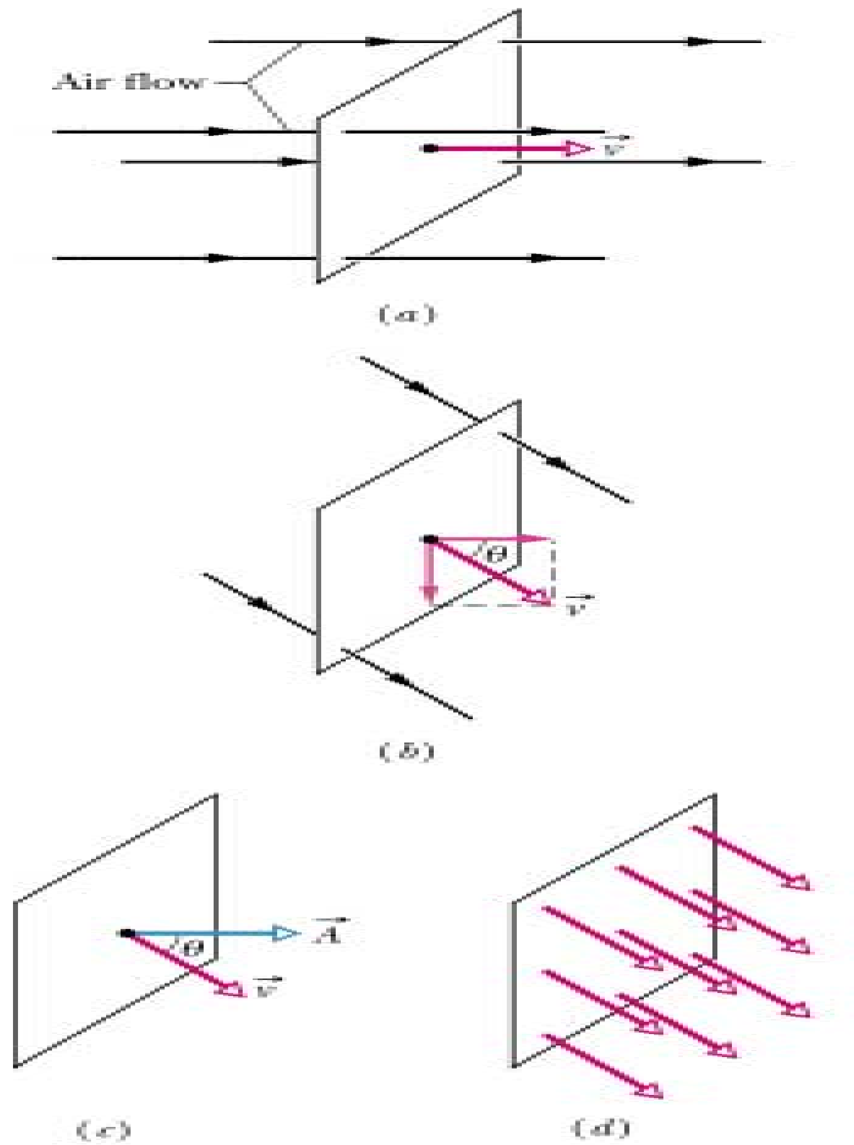
- Gauss' law relates E fields at points on a Gaussian surface to the net Q enclosed by surface

Gauss' Law (2)

- Flux, Φ, is rate of flow through an area
- Create area vector
  - mag. is A, dir. is normal (⊥) to area
- Relate velocity and area by

$$\Phi = \left(v\cos\theta\right)A = \vec{v}\bullet\vec{A}$$

- Flux of a velocity field through an area

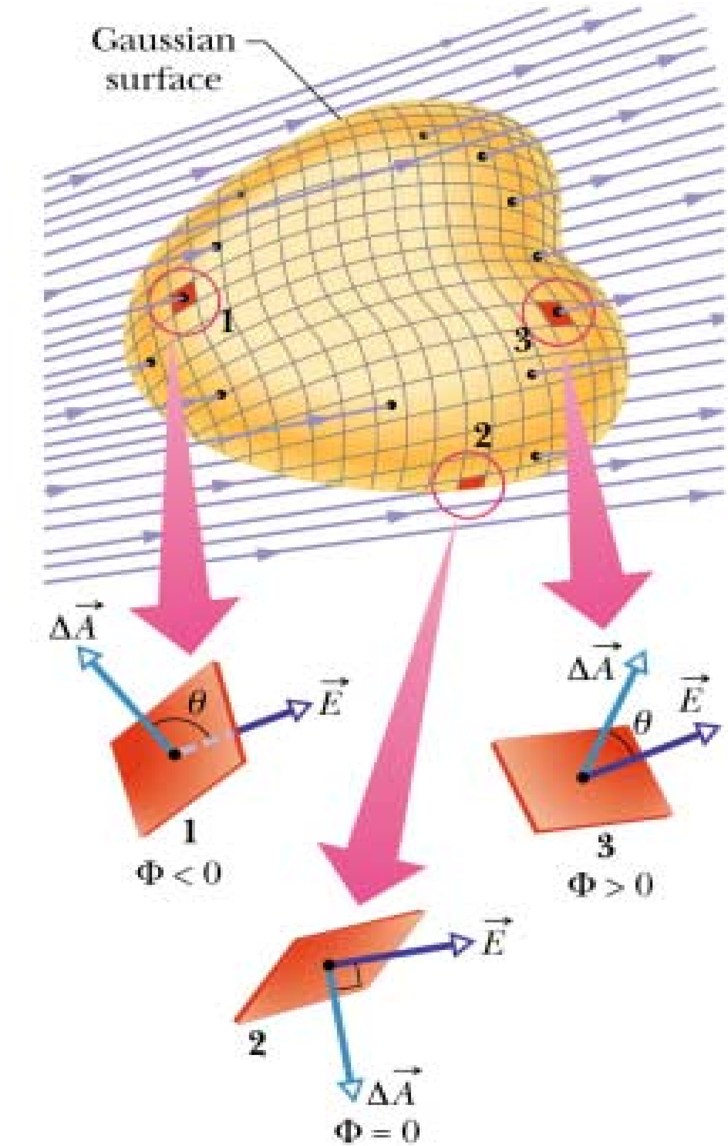


Gauss' Law (3)

- Gaussian surface in non-uniform E field
- Divide Gaussian surface into squares of area ΔA

- Flux of E field is

$$\Phi = \sum \vec{E} \bullet \Delta\vec{A}$$



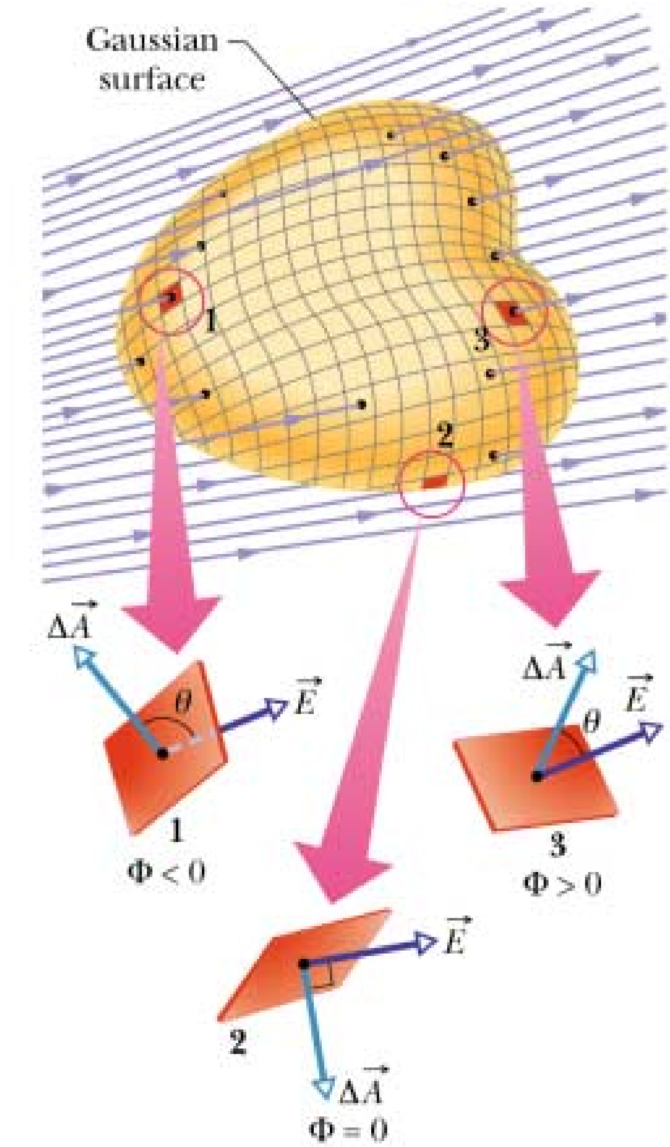
Gauss' Law (4)

$$\Phi = \sum \vec{E} \bullet \Delta\vec{A}$$

- Let ΔA become small so flux becomes integral over Gaussian surface

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

- Flux is proportional to net # of E field lines passing through surface

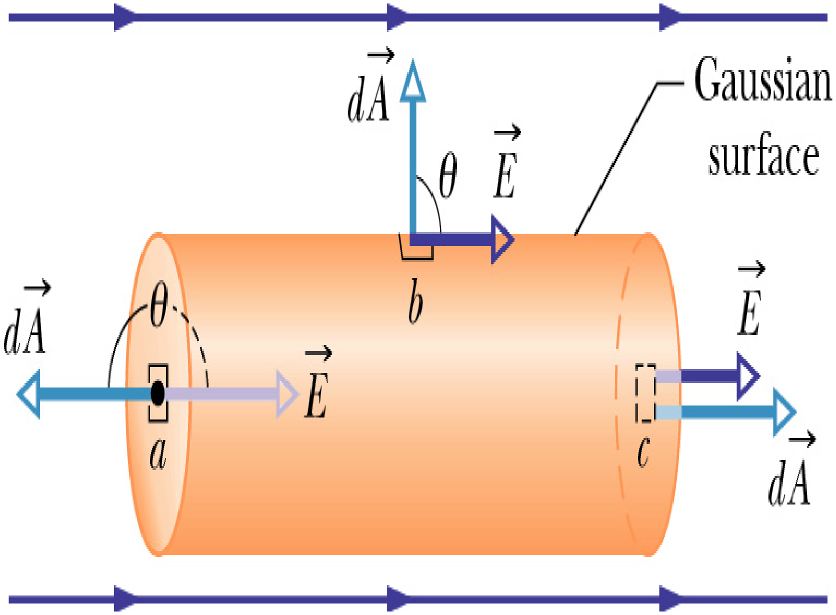


Gauss' Law (5)

- Calculate flux of uniform *E* through cylinder

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

- 3 surfaces - a, b, and c



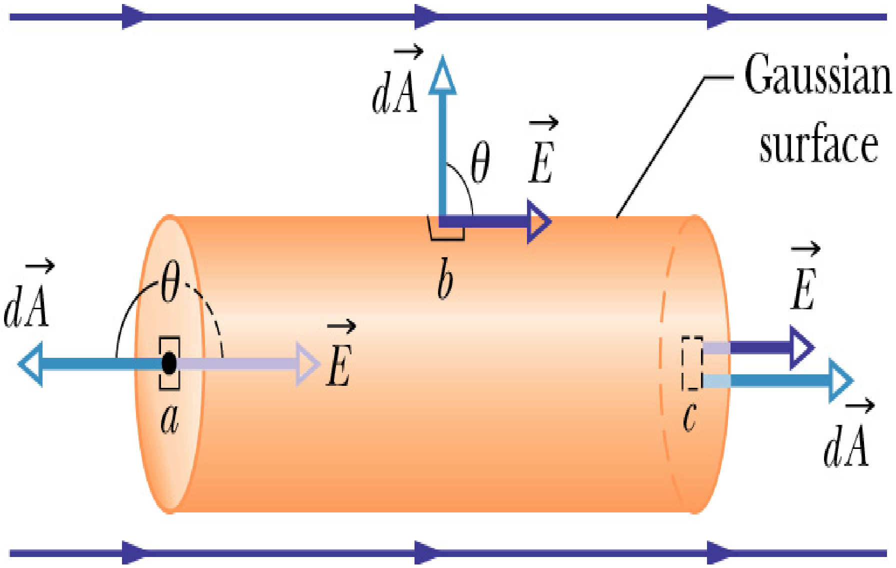
$$\Phi = \int_a \vec{E} \bullet d\vec{A} + \int_b \vec{E} \bullet d\vec{A} + \int_c \vec{E} \bullet d\vec{A}$$

Gauss' Law (6)

$$\int_a E(\cos 180)dA = -EA$$

$$\int_b E(\cos 90)dA = 0$$

$$\int_c E(\cos 0)dA = EA$$



$$\Phi = \oint \vec{E} \bullet d\vec{A} = -EA + 0 + EA = 0$$

Gauss' Law (7)

- If *E* field points inward at surface, Φ is –
- If *E* field points outward at surface, Φ is +
- If *E* field is along surface, Φ is zero
- If equal # of field lines enter as leave closed surface the net Φ is zero

Gauss' Law (8)

- Gauss' Law

$$\varepsilon_0 \Phi = q_{enc}$$

- Also write it as

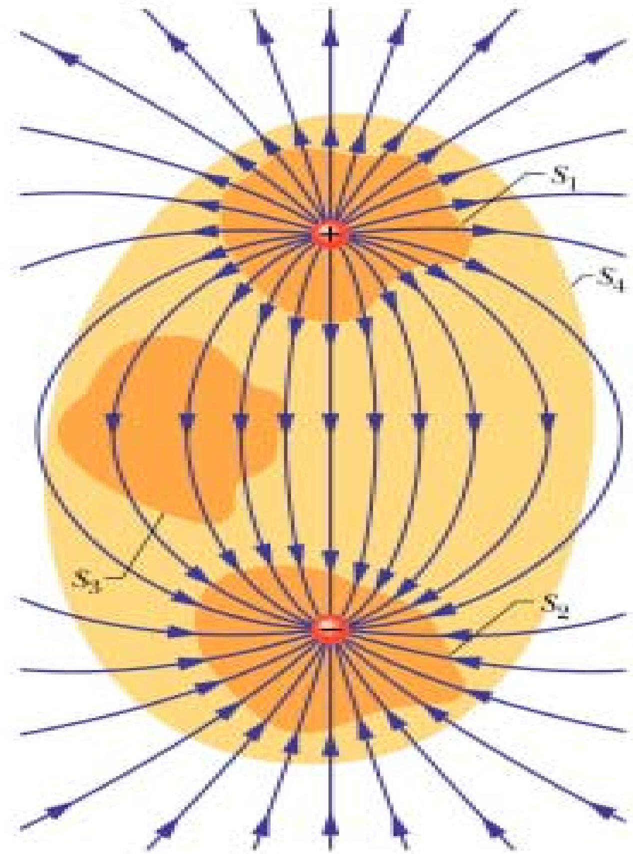
$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- Net charge q_{enc} is sum of all enclosed charges and may be +, -, or zero

Gauss' Law (9)

- What is the flux for each surface?

- S_1 - q_{enc} is +, net Φ is outward and +
- S_2 - q_{enc} is -, net Φ is inward and –
- S_3 – q_{enc} is 0, Φ is 0
- S_4 – total q_{enc} is 0, Φ is 0



Gauss' Law (10)

- What happens to the flux if have a charge, Q, outside a Gaussian surface?

NOTHING

- q_{enc} didn't change
- E field does change but charge outside surface contributes zero net Φ through surface

Gauss' Law (11)

- Theorem for charged isolated conductor with a net charge Q
  - Charge is always on the surface
  - No charge inside the conductor

- At the surface of a charged conductor the E field is $\perp$ to the surface

Gauss' Law (12)

- From Coulomb's law can calculate *E* for continous charge by integrating over *dE* and using density

$$dE = k\frac{dq}{r^2}$$

- Coulomb's law works best with symmetric distributions of charge where several components cancel

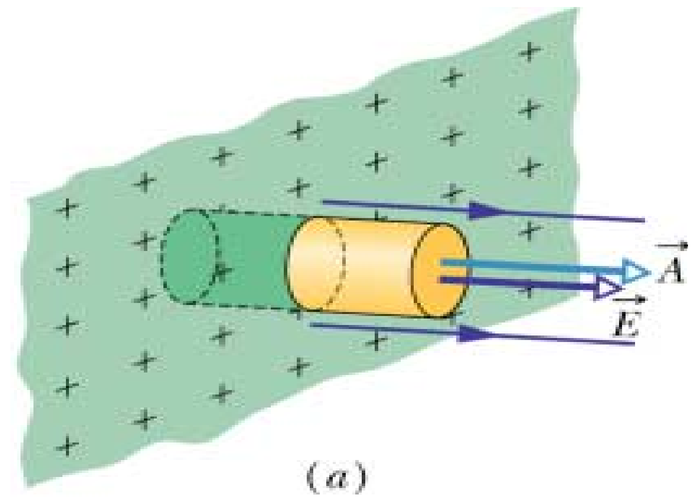
- Still mathematically intensive

Gauss' Law (13)

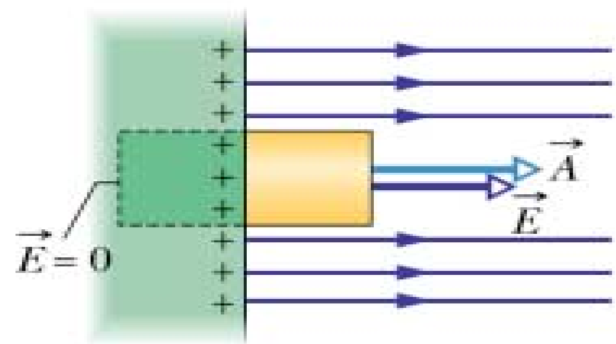
- Usually charge on conductor is not uniform (except for a sphere)
- Charge will accumulate more at sharp points on an irregularly shaped conductor
- How do we find E for for a conducting surface?

Gauss' Law (13)

- Pick a cylindrical Gaussian surface embedded in the conductor
- Sum the flux through suface
- Inside conductor E = 0 so Φ = 0
- Along walls of the cylinder outside the conductor E is ⊥ to A so Φ = 0
- Outer endcap Φ = EA



(a)



(b)

Gauss' Law (15)

- Using Gauss' law and $\Phi = EA$

$$\boxed{\varepsilon_0 \Phi = \varepsilon_0 EA = q_{enc}}$$

- If σ is charge per unit area, then

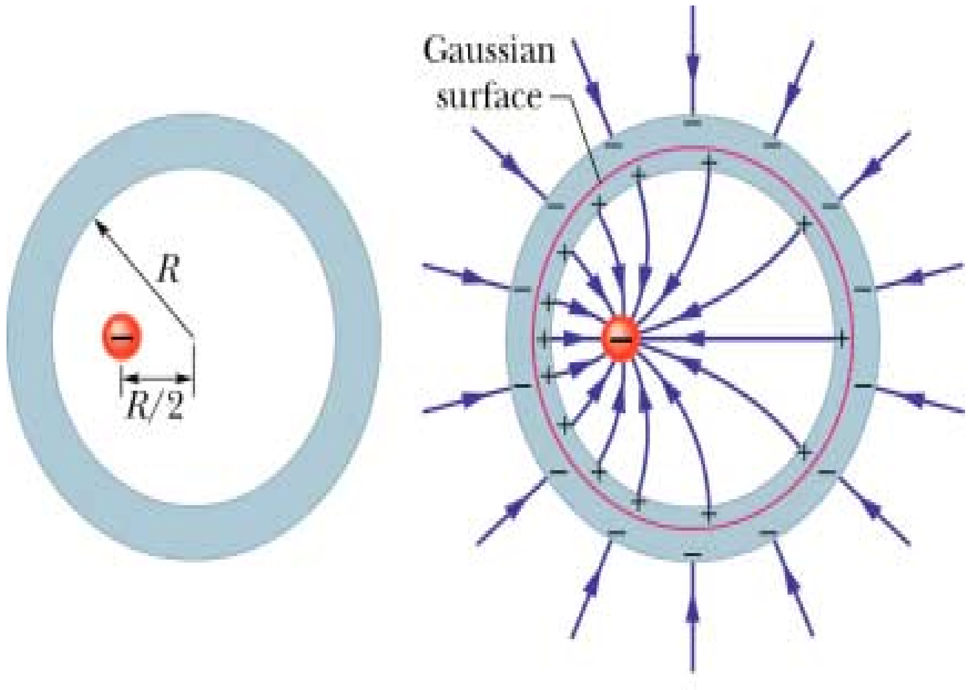
$$\boxed{q_{enc} = \sigma A}$$

- So E for a conducting surface is

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Gauss' Law (16)

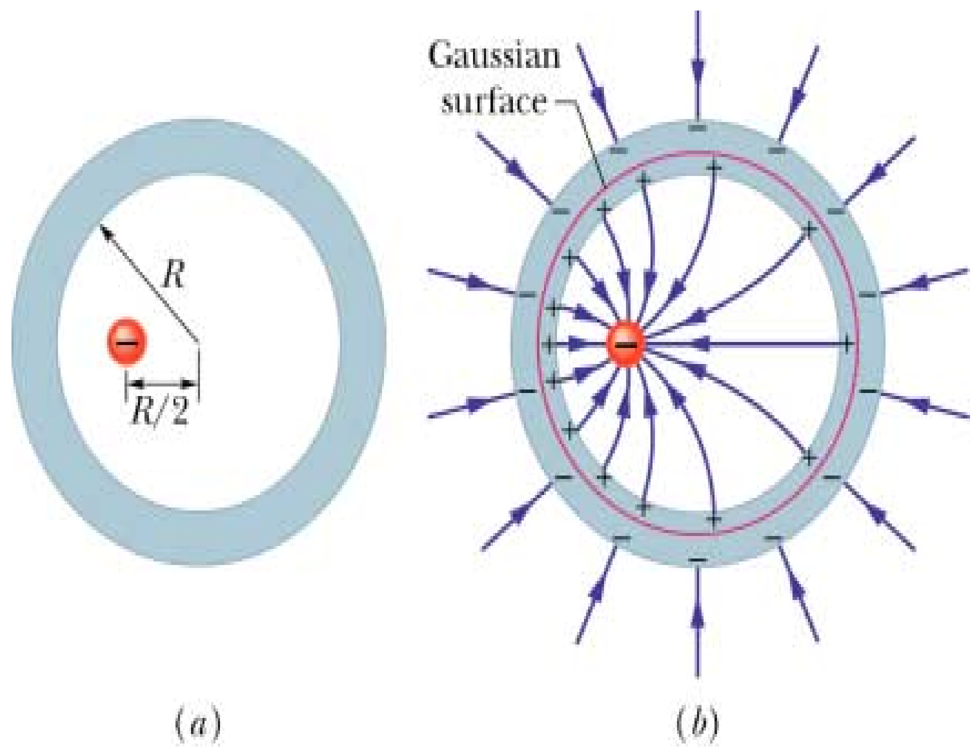
- Have point charge of -5.0μC not centered inside an electrically neutral spherical metal shell
- What are the induced charges on the inner and outer surfaces of the shell?



(a) (b)

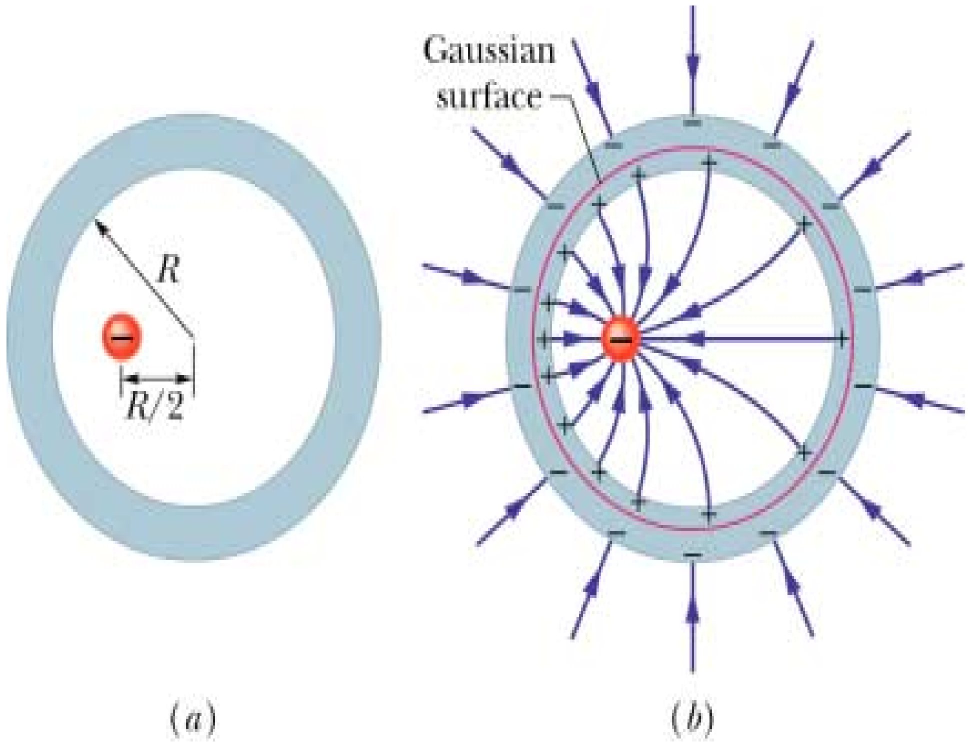
Gauss' Law (17)

- E=0 inside conductor
- Thus Φ=0 for Gaussian surface
- So net charge enclosed must be 0
- Induced charge of +5.0μC lies on inner wall of sphere
- Shell is neutral so charge of -5.0μC on outer wall



Gauss' Law (18)

- Are the charges on the sphere surfaces uniform?
- Charge is off-center so more + charge collects on inner wall nearest point charge
- Outer wall the charge is uniform
  - No E inside shell to affect distribution
  - Spherical shape



Gauss' Law (19)

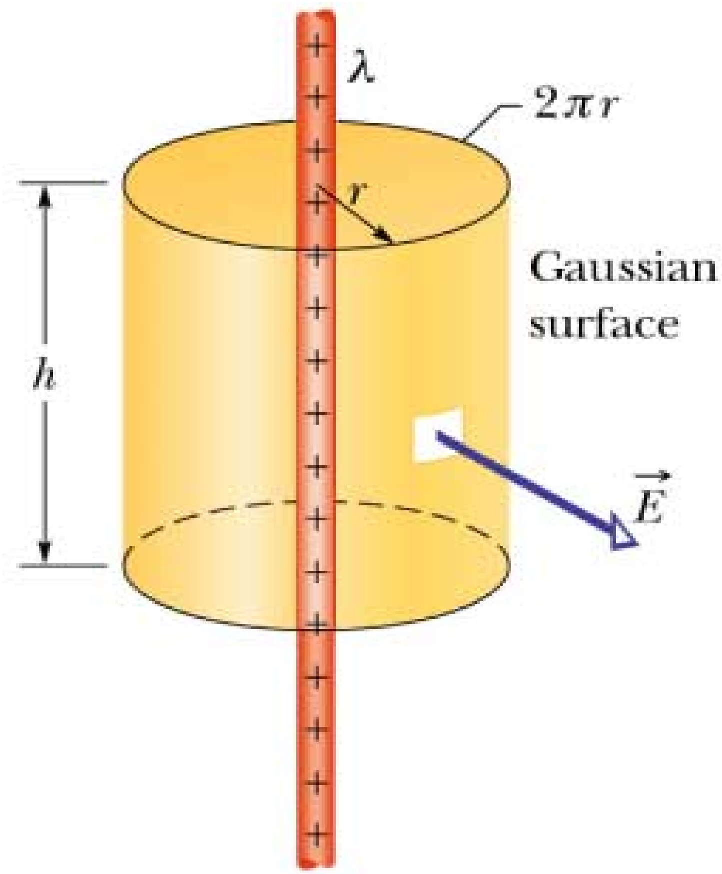
- Checkpoint #4 – A ball of charge -50e lies at the center of a hollow spherical metal shell that has a net charge of -100e. What is the charge on a) the shell's inner surface and b) its outer surface?

a) +50e

b) -150e

Gauss' Law (20)

- Infinitely long insulating rod with linear charge density λ
- Pick Gaussian surface of cylinder coaxial with rod
- What does *E* look like?
- Φ = 0 for the endcaps
- Φ = *EA* for cylinder

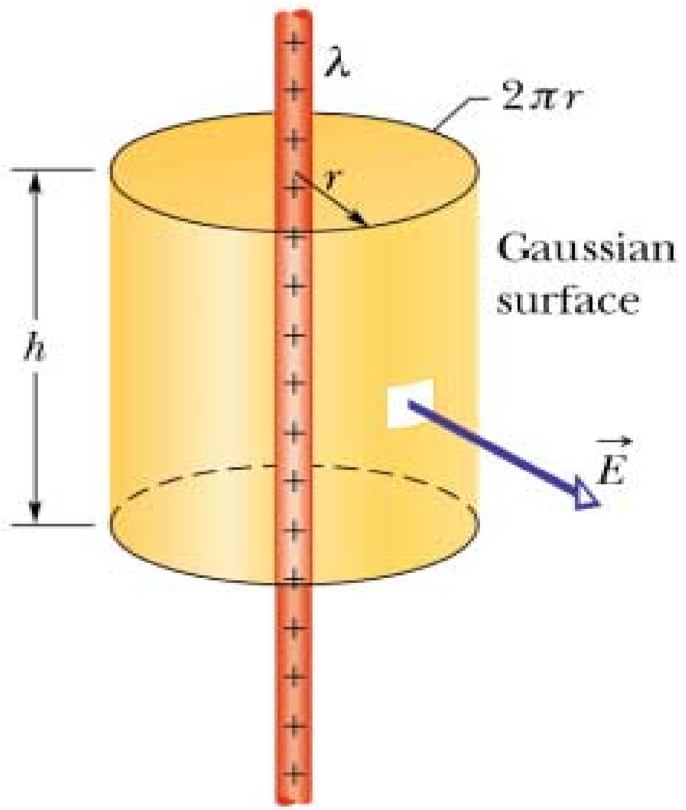


Gauss' Law (21)

- Substituting in Gauss' law gives

$$\varepsilon_0 \Phi = \varepsilon_0 E A = q_{enc}$$

$$A = 2\pi r h \qquad q_{enc} = \lambda h$$



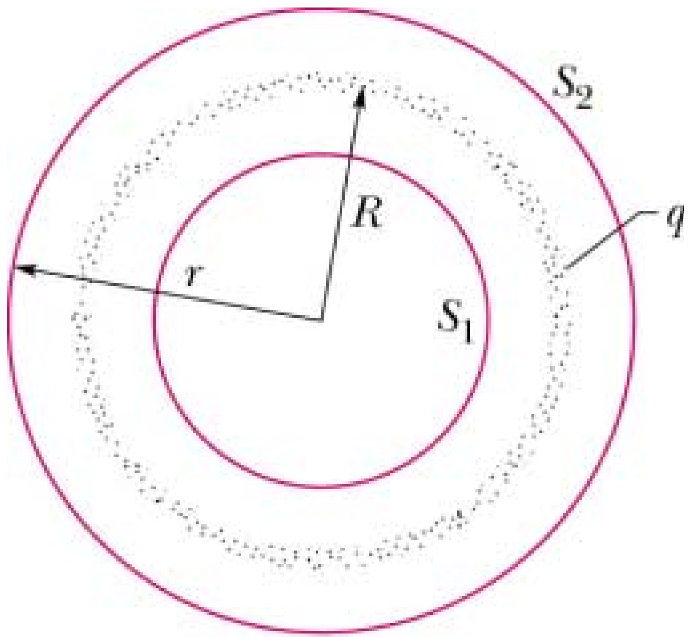
- *E* for a line of charge is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Gauss' Law (22)

- Apply Gauss' law to a spherical shell

$$\varepsilon_0 \Phi = q_{enc} = \varepsilon_0 \oint \vec{E} \bullet d\vec{A}$$



- E radiates out || to A so

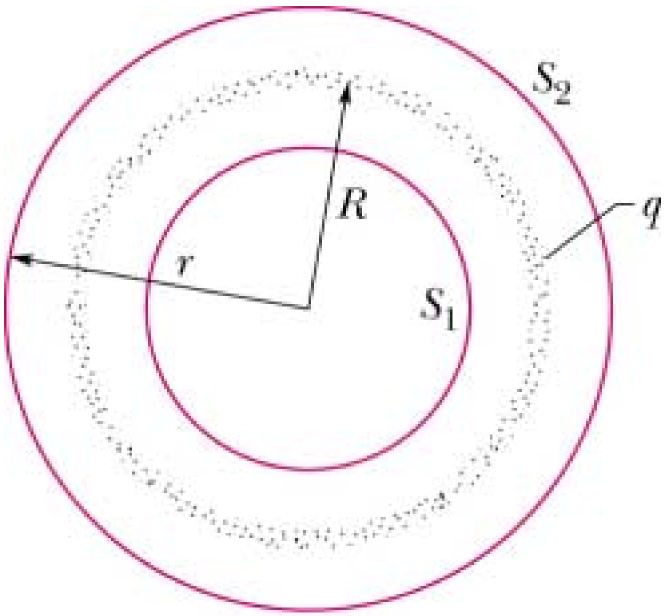
$$\oint \vec{E} \bullet d\vec{A} = \oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$A = 4\pi r^2$$

- Substitute to find E

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}, r \geq R$$

Gauss' Law (23)



$$E = k\frac{q}{r^2}$$

- *E* outside of a charged spherical shell is same as *E* of point charge at center of shell
- Shell of uniform charge exerts no *F* on a charged particle inside shell
  - By Gauss' law E=0 inside shell