Lecture 6

Gauss' Law – Chapter 24

Review

- Coulomb's law
  - Like charges repel, *F* is away from other charge
  - Unlike charges attract, *F* is toward other charge

$$F = k\frac{|q_1||q_2|}{r^2}$$

- Electric field, *E*, felt by positive test charge, q_0

$$\vec{E} = \frac{\vec{F}}{q_0} = k\frac{|q|}{r^2}$$

- Conversely *F* on a charged particle in an *E* field is
  - *F* is in dir. of *E* if charge is +, opposite dir. of *E* if charge is -

$$\vec{F} = q\vec{E}$$

Gauss' Law (Review)

- Gauss' law – form of Coulomb's law
  - q_{enc} is the total charge enclosed by a Gaussian surface

$$\varepsilon_0 \Phi = q_{enc}$$

- Flux is proportional to # of *E* field lines passing through a Gaussian surface

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

Gauss' Law (Review)

- For ***conductors***
  - Excess charge resides on the surface
  - E field is $\perp$ to surface of conductor
  - $E = 0$ inside a conductor

Gauss' Law (24)

- *E* just outside a conductor is proportional to surface charge density at that location

$$E = \frac{\sigma}{\varepsilon_0}$$

  - If – charge on conductor, *E* toward conductor
  - If + charge on conductor, *E* directed away

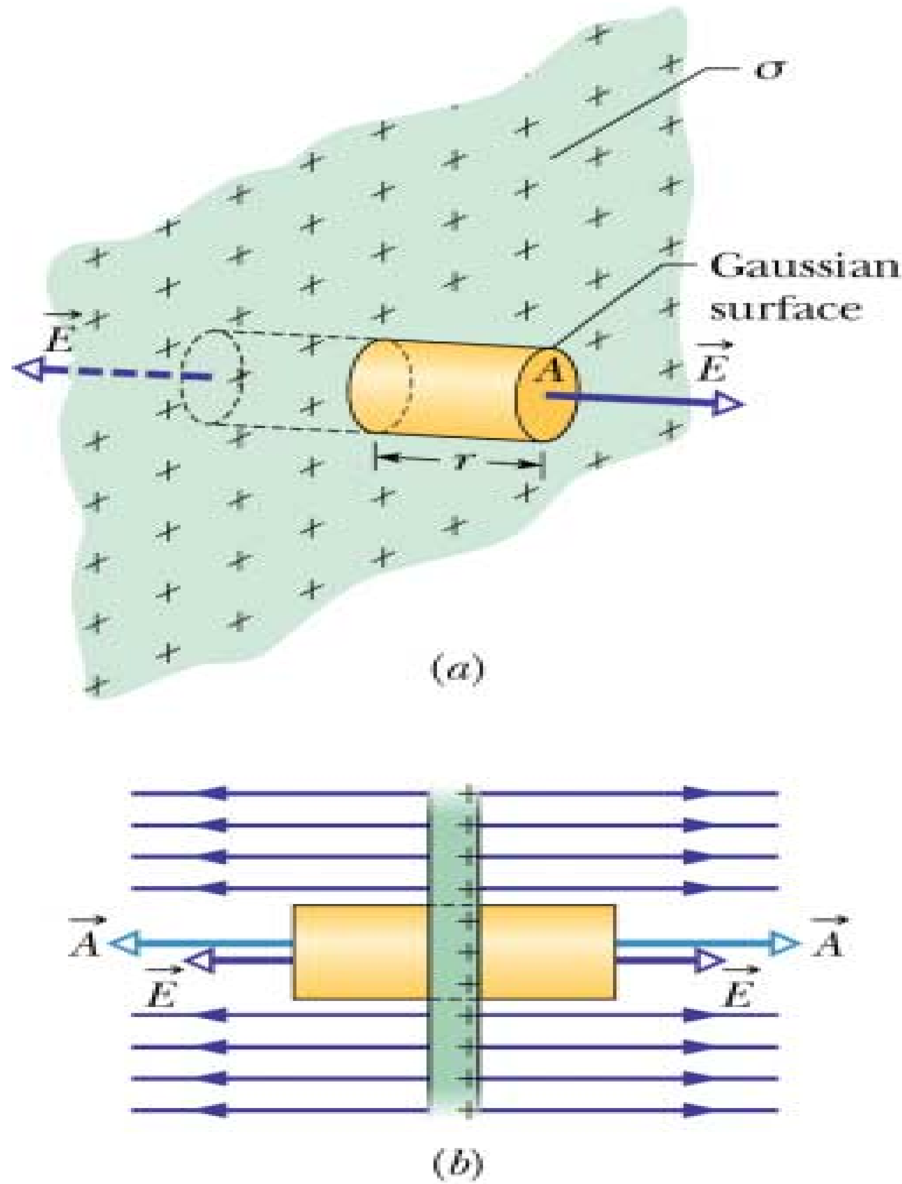
Gauss' Law (25)

- Non-conducting sheet of charge σ

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

$$\varepsilon_0 (EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

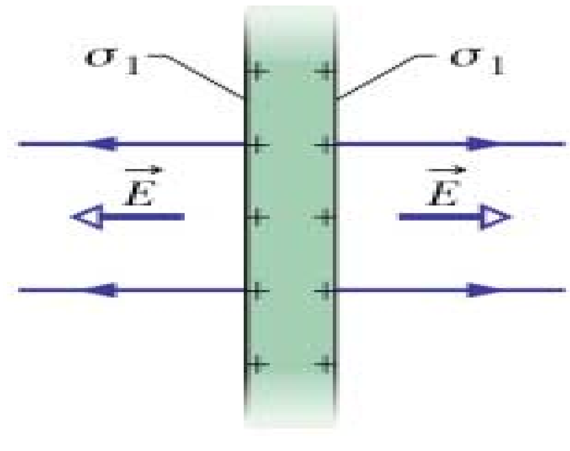


Gauss' Law (26)

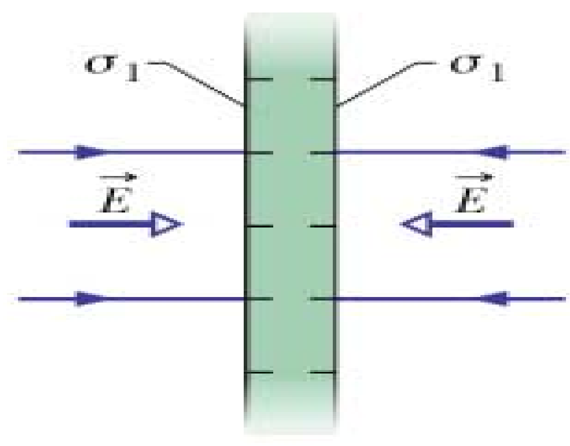
- Conducting sheet of charge
  - Total charge spreads over 2 surfaces
  - σ_1 is charge on one surface

$$E = \frac{\sigma_1}{\varepsilon_0}$$

- a) *E* away from plate
- b) *E* toward the plate



(a)



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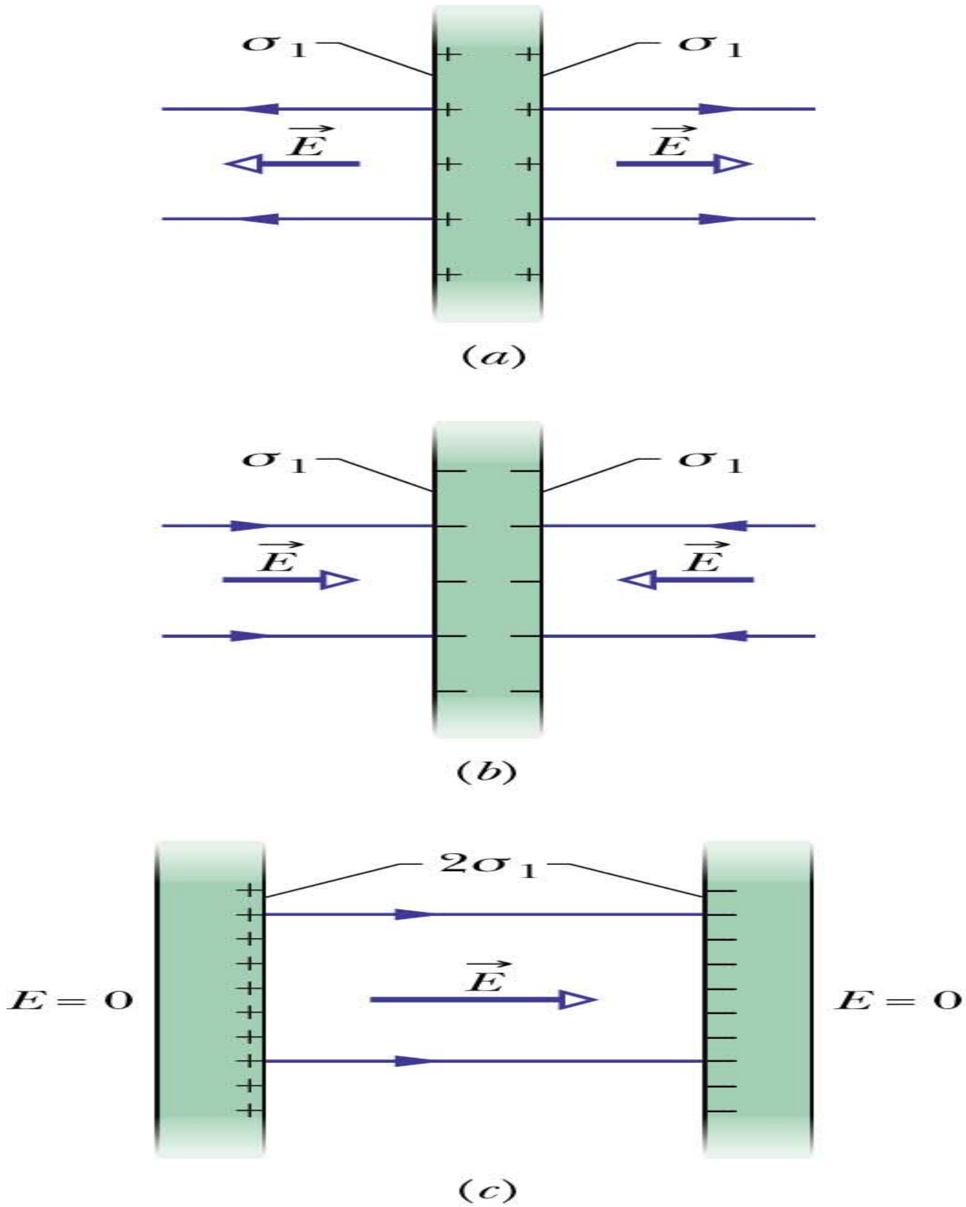
Gauss' Law (27)

- Positive and negative charged conducting plates
  - Excess charges moves to inner faces
  - New total surface density, σ, is equal to $2\sigma_1$

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

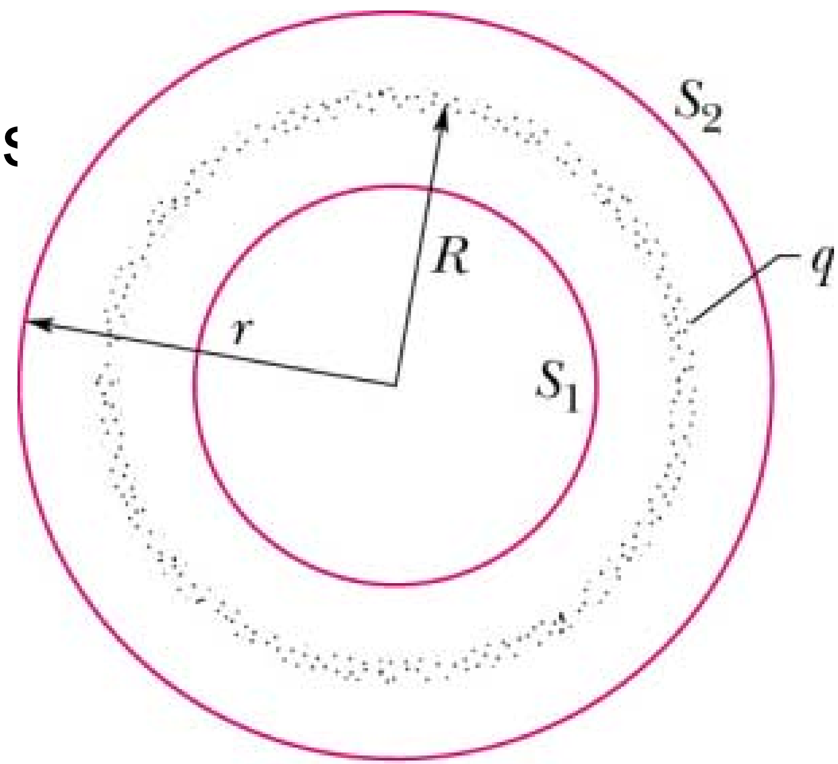


Gauss' Law (28)

- Uniform thin spherical shell of charge with radius R
  - *E* outside shell acts as all charge at center

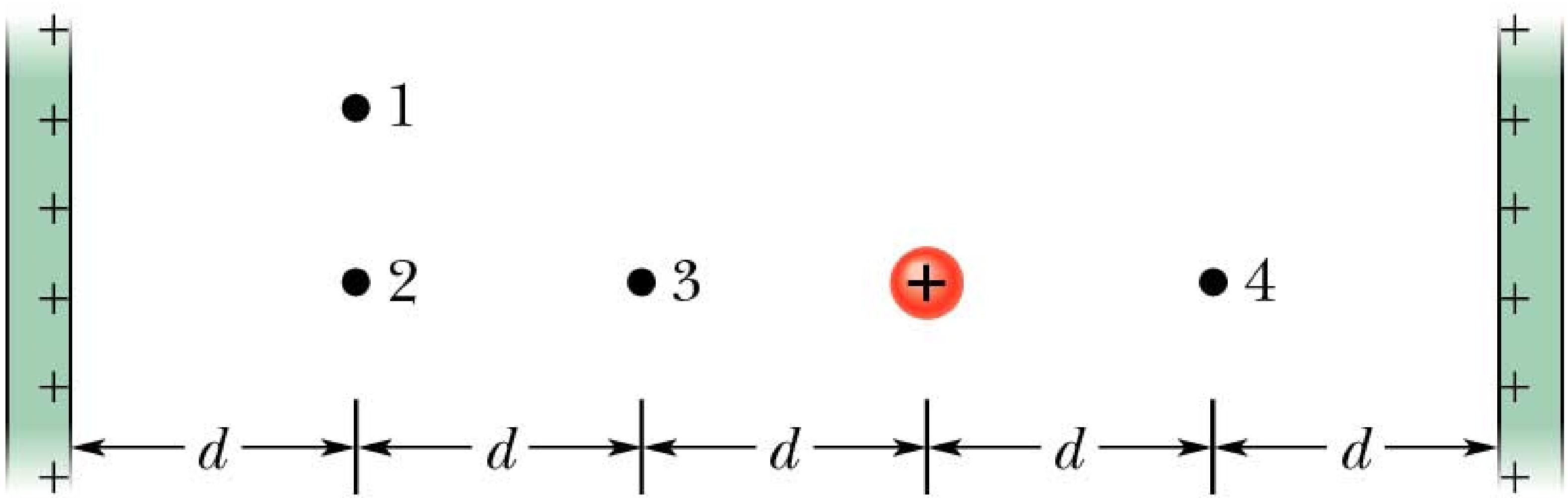
$$E = k\frac{q}{r^2}, r \geq R$$

  - *E* = 0 inside spherical shell



Gauss' Law (29)

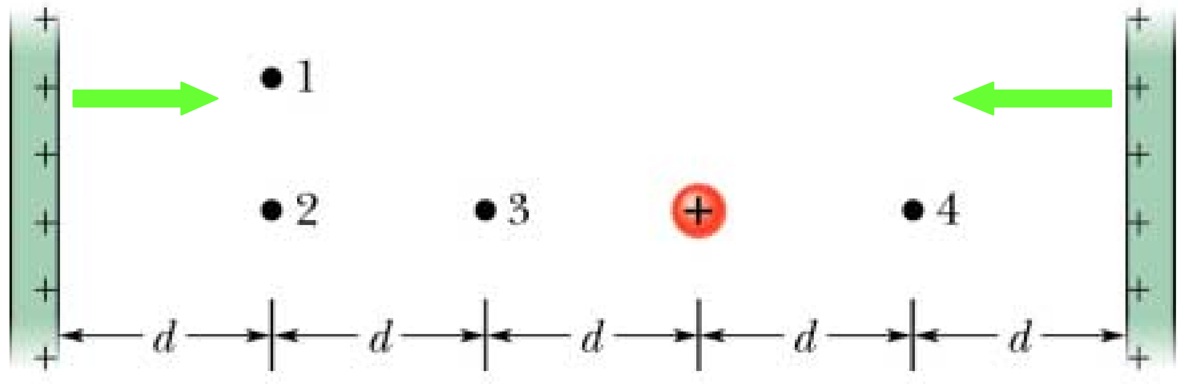
- Checkpoint #5 – 2 large, parallel, non-conducting sheets with identical + charge and a sphere of uniform + charge. Rank magnitude of net *E* field for 4 points (greatest first).



Gauss' Law (30)

- *E* due to sheets

$$E = \frac{\sigma}{2\varepsilon_0}$$



- *E* due to point charge

$$E = k\frac{q}{r^2}$$

- Magnitude depends on distance r from point charge

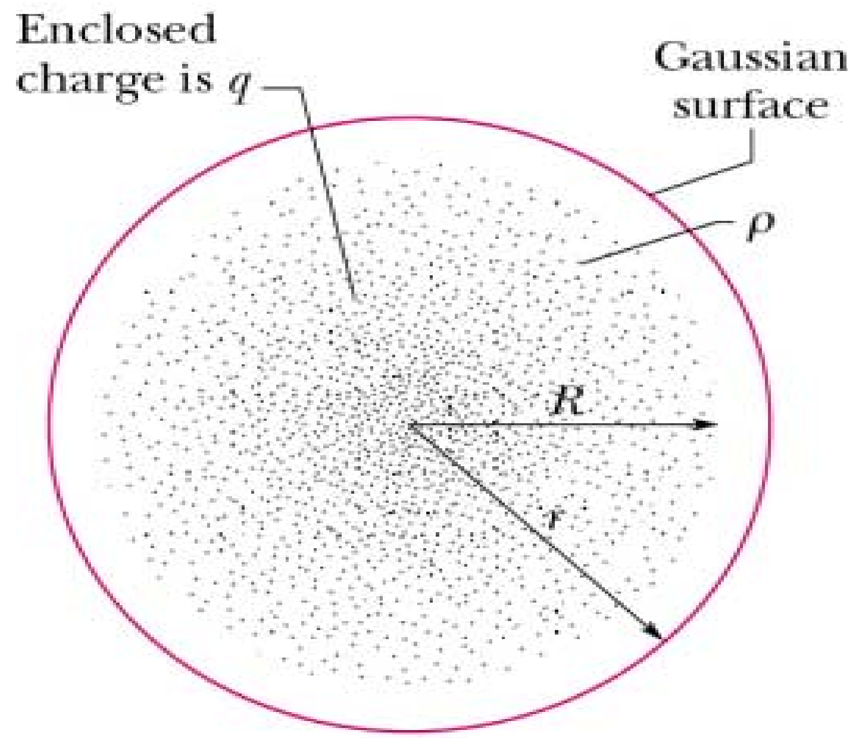
3 and 4 tie, then 2, then 1

Gauss' Law (31)

- Non-conducting solid sphere of radius R and total charge q
- Gaussian sphere outside sphere

$$E = k\frac{q}{r^2}, r \geq R$$

- Same as shell



(a)

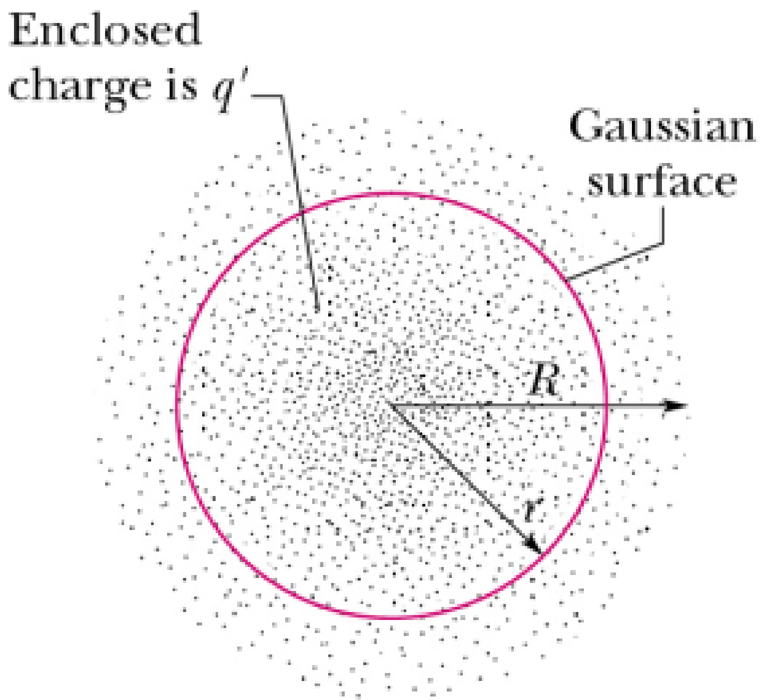
Gauss' Law (32)

- Use series of Gaussian spheres for inside

$$E = k\frac{q'}{r^2}$$

- Full charge enclosed within R is uniform so q' within r is proportional to q

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}$$



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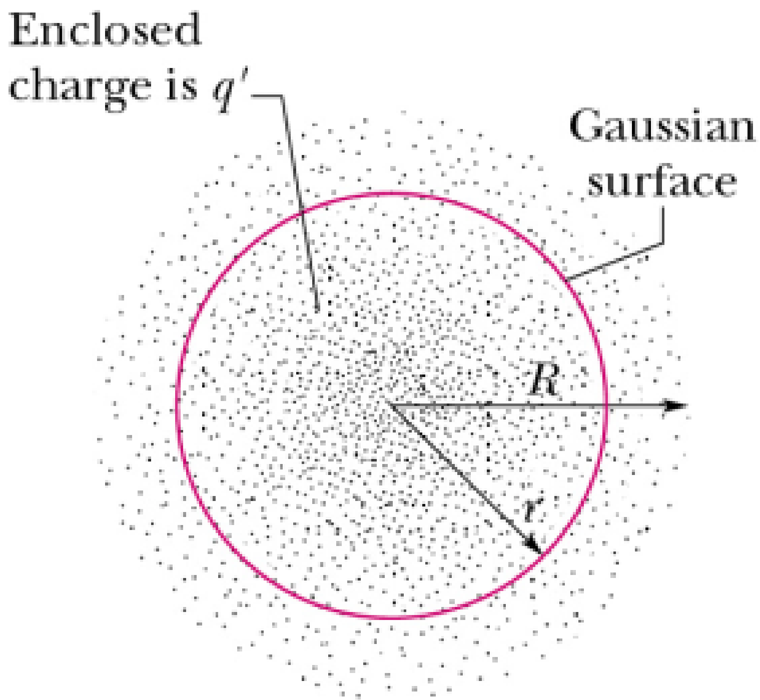
Gauss' Law (33)

- Enclosed charge at r is

$$q' = q\frac{r^3}{R^3}$$

- *E* field inside sphere

$$E = \frac{kqr}{R^3}, r \le R$$



(b)