Lecture 7

Electric Potential – Chapter 25
Review

- Electric field near an infinite non-conducting sheet of uniform charge does NOT depend on distance from sheet.
- How can that be?
  - Proved equation using Gauss’ law.
  - Get same relation by painful integration:
    - Section 23-7 the field due to a charged disk.

\[
E = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right)
\]

\[
R \to \infty, E = \frac{\sigma}{2\varepsilon_0}
\]
Electric Potential (1)

• When electrostatic force acts between charged particles assign an electric potential energy, $U$

• Difference in $U$ of a charge at two different points, initial $i$ and final $f$ is

$$\Delta U = U_f - U_i$$
Electric Potential (2)

• If change system from initial state $i$ to final state $f$ electrostatic force does work

\[ \Delta U = U_f - U_i = -W \]

• Electrostatic force is conservative
• Work done by force is path independent
  – Work is same for all paths between points $i$ and $f$
Electric Potential (3)

• Potential energy, $U$, is a scalar

• Need to choose a reference point where $U = 0$
  – Choose sea level to be zero altitude
  – What if we define Denver to be zero altitude?
  – Does the difference in altitude change?

• Choose $U = 0$ at $i = \infty$ for electric potential
Electric Potential (4)

- Have several charges initially at $\infty$ so $U_i=0$

- Move charges close together to state $f$

- $W_\infty$ is work done by force to move particles together from infinity

\[ \Delta U = U_f - U_i = U_f \]

\[ \Delta U = -W \]

\[ U_f = -W_\infty \]
Electric Potential (5)

• Checkpoint #1 – A proton moves from point \( i \) to point \( f \) in a uniform electric field.

• Does the electric field do positive or negative on the proton?
**Electric Potential (6)**

- What is the work done by an electric field?

\[
W = \vec{F} \cdot \vec{d} \quad \vec{F} = q\vec{E}
\]

\[
W = q\vec{E} \cdot \vec{d} = qEd \cos \theta
\]

\[
W = qEd \cos(180) = -qEd
\]

Negative work
Electric Potential (7)

• Does the electric potential energy of the proton increase or decrease?

\[ \Delta U = U_f - U_i = -W \]

\[ \Delta U = -(-qEd) = qEd \]

Increases
Electric Potential (8)

• Electric potential, $V$, is defined as the electric potential energy, $U$, per unit charge

\[ V = \frac{U}{q} \]

• Potential, $V$, is characteristic of the electric field only
  – Unique value at any point in an electric field
Electric Potential (9)

- Electrostatic potential difference, $\Delta V$, between points $i$ and $f$

\[
\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q}
\]

- $V$ is a scalar and can be $+$, $-$, or 0
- Using reference point of $U_i=0$ at infinity

\[
V = -\frac{W_\infty}{q}
\]
Electric Potential (10)

- Important difference between $U$ and $V$
- **Electric Potential Energy**, $U$, is energy of a charged object in an external $E$ field
  - Measured in Joules (J)
- **Electric Potential**, $V$, is property of $E$ field
  - Doesn’t care if charged object is placed in $E$ field or not
  - Measured in Joules per Coulomb (J/C)
Electric Potential (11)

- Define new SI unit for \( V \), the volt

- 1 volt = 1 joule per coulomb

- Define \( E \) field in volts per meter (V/m)

\[
E = \frac{F}{q}
\]

\[
\frac{N}{C} = \left( \frac{N}{C} \right) \left( \frac{V \cdot C}{J} \right) \left( \frac{J}{N \cdot m} \right) = \frac{V}{m}
\]
Electric Potential (12)

- **Equipotential surface** - all points are at same electric potential, $V$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

- $W = 0$ if move between points $i$ and $f$ which lie on same potential surface
  - True regardless of path taken between points
Electric Potential (13)

• For paths I and II

\[ W = -q \Delta V = -q(V_1 - V_1) = 0 \]
\[ W = -q \Delta V = -q(V_3 - V_3) = 0 \]

• For paths III and IV

\[ W = -q \Delta V = -q(V_2 - V_1) \]
Electric Potential (14)

- Draw equipotential surfaces for distributions of charges.
- Equipotential surfaces are always ⊥ to electric field lines and to $E$. 
Electric Potential (15)

- Calculate $\Delta V$ between points $i$ and $f$ in an electric field $E$

$$\Delta V = V_f - V_i = -\frac{W}{q_0}$$

- Need to find $W$
Electric Potential (16)

- Calculate differential amount of work

\[ dW = \vec{F} \cdot d\vec{s} \]

- Remember

\[ \vec{F} = q\vec{E} \]

\[ dW = q_0\vec{E} \cdot d\vec{s} \]
Electric Potential (17)

• Work is

\[ W = q_0 \int_i^f \bar{E} \cdot d\bar{s} \]

• Substitute to find \( \Delta V \)

\[ \Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \bar{E} \cdot d\bar{s} \]

• Potential decreases if path is in the direction of the electric field