Physics 831 - 2002

Statistical Physics

Problem Set 4

1. Calculate the constant
$$C_n = \int_{\sum x_i^2 < 1} \prod_{i=1}^{n} dx_i$$
using mathematical induction and the fact that $C_2 = \pi$ and $C_3 = 4\pi/3$. Compare with the result in the textbook [in particular, calculate $\Gamma(2)$ and $\Gamma(5/2)$]. (5 pt)

2. Problem 6.4 [for the case of different gases] (3 pt)

3. Problem 6.3 (a) - (c) (6 pt)

4. Consider equilibrium of a body in an external time-independent field, for example, in gravitational field. The system is no longer spatially uniform. Still equilibrium with respect to particle exchange requires that $\mu =$ const, but now the chemical potential may depend on coordinates. In a gravitational field, the energy of a particle of a mass m has just an extra term $u(\mathbf{r}) = mgz$. Then the chemical potential per particle has the form
$$\mu(P, T) = \mu_0(P, T) + u(\mathbf{r}),$$
where μ_0 is the chemical potential in the absence of the field. Prove this relation (3 pt) and derive the equation for P as a function of coordinates for a compressible system (3 pt).

5. Problem 6.1 (4 pt)

6. Problem 6.2 (4 pt)

The problems are from Kerson Huang, *Statistical Mechanics*, 2nd edition, (Wiley, NY 1987).