Physics 831 - 2002

Statistical Physics

Problem Set 7

1. Show that the chemical potential of an ideal gas can be written as $\mu(P,T) = kT \log P + \chi(T)$ and find the function χ for an atomic gas (4 pt)

2. Starting with the expression for the partition function, show that the total pressure of a mixture of ideal gases is the sum of partial pressures of each gas. (4 pt)

3. Consider a system in chemical equilibrium. There are several species X_i which react with each other, so that the reaction can be written in the form $\sum_i \nu_i X_i = 0$ (for example, for the reaction $2H_2 + O_2 \rightleftharpoons 2H_2O$ the coeeficients are $\nu_{H_2} = 2, \nu_{O_2} = 1, \nu_{H_2O} = -2$). From the condition that the Gibbs potential of the mixture is minimal find the interrelation between the chemical potentials of the species in equilibrium. (4 pt)

4. Show that, if reacting species are ideal gases with densities n_i, then there holds a relation $\prod_i n_i^{\nu_i} = K(P,T)$, where P is the total pressure (the law of mass action). Calculate the function K in terms of P and the functions $\chi(T)$ of the Problem 1. (4 pt)

5. For a quantum system with a density operator ρ, find the probability distribution over momenta $p \equiv p_1, \ldots, p_{3N}$ (5 pt)

6. Find the energy spectrum of a particle of a mass m in a large 3-dimensional box, with the potential $U(\mathbf{r}) = 0$ for $\mathbf{r}$ inside the box ($0 \le x \le L_x$, $0 \le y \le L_y$, $0 \le z \le L_z$), and $U(\mathbf{r}) = \infty$ for $\mathbf{r}$ outside the box. Find the number of states within an energy interval ΔE around the energy $E \gg (\hbar^2/2m)\sum_i L_i^{-2}$ (assume that there are many states within ΔE, but that $\Delta E \ll E$). (5)

The problems are from Kerson Huang, *Statistical Mechanics*, 2nd edition, (Wiley, NY 1987).