Chapter 1: Introduction to forces

A. When is a force ... a force?

In the study of the physical world, it is fair to say that the most important single word in the vocabulary is *force*. A precise definition for a force, the one used in most textbooks, refers to the influence of a force on the motion of objects. Human beings, however, spend most of their lives moving rather slowly, or not at all. From birth, a *concept* of forces is developed in this largely stationary context. That force concept is based on how we *feel*, i.e., how our brain perceives the effects of forces acting on our body. However, applying this comfortable concept to states of motion, often fails. It yields paradoxical explanations of physical phenomena, or makes predictions that are obviously false. When stationary, this force concept works quite well and makes so much sense. How can it be wrong?

To convince you to discard this old concept of forces for a better one is a very difficult task. You have many years of reinforcement behind this incorrect force concept. To varying degrees of success, courses in High School or College Physics attempt to replace the false force concept with a correct one. The fraction of students who make the conversion is disappointingly small. It is my contention that this failure is due, at least in part, to the emphasis on motion in the structure of traditional courses, while misconceptions regarding forces are born in a stationary environment. Before attempting to understand observations made while moving, most people make a transformation to a stationary state to gain some "balance". The structure of this course recognizes that the typical non-science major will think this way. It is, therefore, essential to have a correct interpretation of forces when stationary, before attempting to apply it to motion.

In this text, I develop a *correct* force concept, that explains the experience of forces when stationary, *and* provides a paradox free explanation for our feelings when in motion. To accomplish the transformation in your thinking to this correct force concept, I will challenge you to interpret a simple, stationary object, with your old force concept. Furthermore, I will present a model for the behavior of the human body, based on the properties of this object, which will provide an explanation for your feelings in all situations.

The object is a light but strong coil spring. The spring in a retractable ballpoint pen is a commonly available example. It is very difficult (perhaps impossible) to describe a coil spring without invoking words akin to force, such as “pull” or “push”. It is pretty clear what we mean when we use the word force in this context. A particularly simple force definition can be constructed with this familiar object:

> *Forces* must be applied to a spring to make it stretch or compress.

There is an equivalent definition:

> A stretched or compressed spring exerts *forces* on the objects that touch it.

Electric and magnetic forces are essential for the operation of refrigerators, computers, and televisions. Both of these are now known to be facets of a more general force called the *electromagnetic* force. Another well-known force is *gravity*. Though we have all experienced gravity, it is a force with some peculiar features that make it a poor starting point.

Gravitational and electromagnetic forces are examples of what physicists call "fundamental" forces. The forces acting on, and generated by a coil spring are not usually considered *fundamental*. At the microscopic level, however, it is electromagnetic forces between atoms that determine the properties of a spring, and the macroscopic (large-scale) properties of a spring accurately reflect the underlying electromagnetic forces. For this reason, the spring model of matter presented here will be *fundamental* to understanding all forces.

There are, however, mysterious effects that don't appear to be caused by electromagnetic or gravitational forces, and are often associated with changes in speed or direction of the observing person. For example, in a car going around a sharp turn, a passenger is "pushed" against the car door (or driver) by what appears to be a force, and yet this force has no obvious source. I hope to make clear that these mysterious effects also have their origin in the action of gravitational and electromagnetic forces.

You'll have to excuse me, if I don't discuss "The Force" from the movie *Star Wars*, other "supernatural" forces, telekinesis, metaphysics, or extrasensory perception. On the contrary, I hope the concepts and techniques in this text will help you distinguish between physical science and science fiction.

B. A short history of forces

This section contains a historical perspective on how physicists reached a model of nature with four fundamental forces, and emphasizes that two of these, gravity and electromagnetism, are responsible for nearly all physical phenomena. Also presented are the scientific method's experimental and theoretical requirements for a hypothesis (prospective theory), such as Einstein's relativity, to be accepted as an improvement over existing theory.

Until the mid 1600's, a description of forces by Aristotle, one of the greatest philosophers of all time, had remained unquestioned for nearly two thousand years. Galileo found, however, that Aristotle's seemingly logical description of forces gave incorrect predictions for the results of simple experiments. During and after the Renaissance (1600-1700), natural philosophers (in addition to their other professions), including Copernicus, Galileo, and Newton, found the errors in Aristotle's logic. Their descriptions of planetary motion, the effects of forces, and the behavior of the gravitational force, gave correct predictions for nearly all terrestrial and astronomical observations, and has been known ever since as "Newtonian mechanics". In particular, Newton was the first to recognize that gravity is a force created by, and acting on *mass*.

In the 19th century, Faraday, Maxwell, and others, were remarkably successful in describing the forces of electricity and magnetism. These forces are related to a feature of matter called electric charge, of which there are three: positive (+), negative (–), and neutral. Two isolated objects with a charge of the same sign (+ + or – –) will be pushed apart by a repulsive electric force, and with opposite signs (+ –) will be pulled together by an attractive electric force. With equal amounts of positive and negative charge, immobile and uniformly distributed, an object will appear neutral and will not generate an electric force on a nearby charged or neutral object. Neutral objects in which charge can move, however, will be affected by a nearby charged object. In 1865, Maxwell uncovered an underlying theoretical connection between electricity, magnetism, and light (electromagnetic waves) and thereby "unified" these forces into the *electromagnetic* force.

> The electromagnetic force is created by, and acts on charge, and is capable of producing interactions involving light.

Phenomena as diverse as a candle burning in air, or a sugar cube dissolving in water, and those listed in Figure 1.2 and most other physical processes, are effects caused (primarily) by the electromagnetic force. Atoms interact with other atoms solely through electromagnetic forces between their charged components, electrons and protons.

> Any chemical reaction, or phase change in matter, e.g., solid to liquid, reflects the properties of the electromagnetic force between atoms.

- pull of a stretched bungie cord
- color of flower
- smell of fresh paint
- melting of ice
- thunder
- bubbles in beer
- causes of a sunburn
- pain of a bad sunburn
- flame of a match
- friction
- effects of aspirin
- perception by a human brain
- explosion of TNT

Figure 1.2 Processes caused by electromagnetic forces.

The principles of Newtonian mechanics and the behavior of the forces of gravity and electromagnetism provided the foundation for the industrial and scientific revolutions that followed. Near the end of the 19th century, however, the results of two experiments shook the confidence of the scientific community. Michelson and Morley found that:

> The speed of light is *unaffected* by the motion of an observer.

In Newtonian physics, for observers in two trains travelling at the same constant speed and direction, it can appear to them that they were not moving, yet a stationary observer clearly sees them move. Thus the apparent speed of an object *is affected* by the speed of the observer. That the speed of light seemed to be the same no matter how the observer moved, implied that aspects of the Newtonian theory must be flawed.

This situation was rectified during the period 1905-1915 when Einstein obtained revolutionary insights into the nature of space, time, and gravity. Einstein's special and general theories of relativity provided correct predictions in the case of high speeds (near the speed of light) or strong gravity. An often misunderstood feature of the scientific method, is that before a hypothesis, such as Einstein's, becomes accepted theory it must pass a rigorous test:

> A new theory must (a) predict those phenomena incorrectly described by the old theory, and (b) show why the old and flawed theory, in normal circumstances, could give apparently correct predictions.

Usually, the latter condition is the most difficult to satisfy. In the case of relativity, at speeds much less than the speed of light, the theory of Einstein predicts extremely small differences between it and the Newtonian theory. As more precise measurements became available these small differences were always found.

Many further expansions of scientific knowledge were made during the first 30 years of the 20^{th} century. Atomic structure (see the next section) was realized, radioactivity was discovered, and the quantum nature of light and matter (quantum mechanics) was uncovered. A leading example was the discovery that in addition to its wave properties, light had the properties of a neutral particle called the *photon*. Also found were two new forces, with effects observable only at distances comparable to the size of the atomic nucleus, about 10^{-14} m. One of these forces is the *strong nuclear* force:

> The strong nuclear force acts between protons and neutrons to hold the nucleus of an atom together, against the electric repulsion of the positively charged protons.

Nuclear forces are responsible for the energy released in the *fission* (splitting) of uranium nuclei in nuclear power plants, and the energy released in stars from *fusion* reactions forming helium nuclei from two protons and two neutrons.

The other, the *weak nuclear* force, is responsible for radioactive *beta decay*, the transformation of a neutron into a proton, electron, and an antineutrino.

> The weak nuclear force is characterized by the emission or interaction of a nearly massless neutral particle, the neutrino (or antineutrino), and identified by the Greek letter "nu", ν (or $\bar{\nu}$).

The beta decay reaction can be written as, $n \rightarrow p + e^- + \bar{\nu}$ where the emitted electrons are called "beta" radiation. In the sun, the weak nuclear force plays an essential role in the production of neutrons from protons, $p \rightarrow n + e^+ + \nu$, so that fusion reactions, driven by the strong nuclear force can proceed. After a few billion years in the life of a large star, in an explosion called a supernova, the weak nuclear force, in less than a second, most of a star's protons and electrons into neutrons, emitting a truly astronomical number of neutrinos ($p + e^- \rightarrow n + \nu$). Except for very rare instances, neutrinos pass completely through the earth without interaction. Nevertheless, in 1987, a few neutrinos from a supernova explosion in a neighboring galaxy were detected in an experiment.

Significant progress has been made by physicists in recent years to *unify* the weak nuclear force with the electromagnetic force and work continues today to include the strong nuclear force and gravity in this unification. If achieved, this would create a unified theory of all forces, showing them to be manifestations of a single, but complex force.

The *weak* and *strong nuclear* forces are responsible for just a few important processes. The forces of *gravity* and *electromagnetism*, however, are responsible for most common phenomena. Particularly important are the electric forces generated between atoms of stretched or compressed solid objects. A review of atomic structure is given in the next sections leading to a description of elastic forces.

C. Structure of the atom

It is remarkable that Newton's laws, Maxwell's electromagnetism, and Einstein's relativity were conceived *before* the basic structure of the atom was uncovered by Ernest Rutherford and his colleagues in 1911. Their experiments showed that the atom's negatively charged electrons (discovered by J. J. Thomson, 1897) and positively charged protons were not spread uniformly over the volume of the atom.

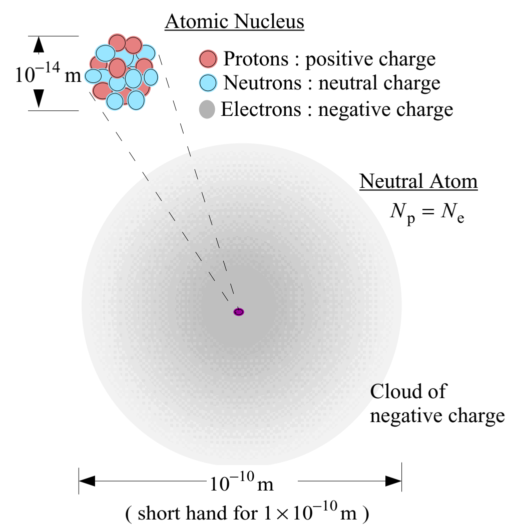

Figure 1.3 A model for the structure of the atom

Instead, as sketched in Figure 1.3, a measurement of the size of an atom determines the approximate extent of a diffuse cloud of electrons held, by the attractive electric force, in the vicinity of the protons. The protons and neutrons (first detected by Chadwick, in 1937) are confined (bound) by the strong nuclear force to the *atomic nucleus*. The nucleus is a very small region with a diameter of about 1×10^{-14} m at the center of the atom; and for this reason, protons and neutrons are called nuclear particles (*beware*, the central part of a living cell is also called the nucleus. Atoms with the same number of protons but differing number of neutrons in their nuclei are called isotopes of that element.). An atom interacts with other atoms only by means of their electrons in *chemical* reactions.

Atomic electrons determine the chemical properties of an element.

For an object as small as, or smaller than an atom, a precise description is possible only in the language of quantum mechanics. The artist's rendition shown in Figure 1.3, however, can be used as a model to discuss the type, number, charge, and size, of the components of an atom. As will be described later, the model can also provide a reasonable interpretation for the interaction between neighboring atoms in a solid object,

In an undisturbed atom, the number of electrons and protons, and amounts of negative and positive charge, are equal. Therefore, when viewed from afar, an undisturbed atom appears electrically neutral (net charge zero). Up close, however, the extremely light (1/2000 the weight of a proton) electrons are comparatively mobile, are in constant motion, and are occasionally found well beyond the nominal size of the atom. Only the negatively charged electrons can be easily transferred from one neutral object to another, as shown in Figure 1.4. This leaves the former with a positive charge and gives the latter a negative charge (the transfer the very small mass of the electrons is relatively unimportant). A positively charged object can be easily created, *only* by transferring to other objects the same amount of negative charge. These easily removed (or added) electrons determine all the chemical properties of matter.

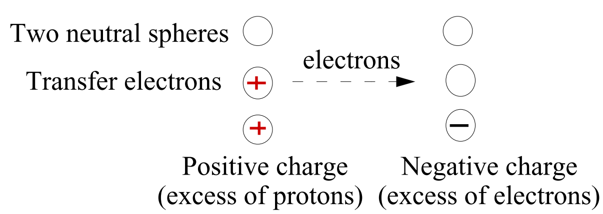

Figure 1.4 Creating a positive charge on a neutral object

Nearly all matter is electrically neutral, even matter within a battery. To emphasize this point, I will dispel a common misconception about batteries. In a battery, reversible chemical reactions create electric forces that move electrons from atoms at one pole (labeled +) to the other pole (labeled –), as shown in Figure 1.4. The buildup of charge on the two poles prevents the chemical reaction from increasing the charge beyond some limit. In this process, the electric forces that separated the charges have increased the "energy" of the electrons. A wire between the poles provides a path for electrons to return to the positive pole, and while doing so, deposit their energy as heat in the wire. Further chemical reactions then quickly move more electrons across the battery to create a continuous flow (current) of electrons. A battery is "dead", its energy depleted, when the chemicals, needed to move electrons across the battery, are completely reacted.

"Charging" a battery is a misnomer because a battery does not store charge. To charge a dead battery, the original chemical mixture in the battery must be restored. In a car, an electric generator pushes electrons in the opposite direction through the battery, thus reversing the chemical reactions and "recharging" the battery.

D. Molecules

If atoms behaved like neutral billiard balls, nothing special would happen when two atoms were brought close to each other. The electrons of an atom, however, are in constant motion, and near the periphery of the atom they can be influenced by the charge of electrons and protons in a neighboring atom. The atomic model of the previous section will be used to model the forces on two atoms moved close to each other, however, as with the single atom, only a quantum mechanical description can be accurate.

Shown in Figure 1.5 are two oxygen atoms (or atoms of some other common gases) at four separations. At the top, the atoms are far enough apart so that no interaction occurs between them. Next they are closer, where there is a small overlap of the electron clouds, causing electrons to redistribute around both atoms. Between the atoms, due to sharing (interchange) of electrons, the density of electrons is a little larger than just the sum. The result is an electric attraction of the two nuclei toward this concentration of electrons. Skipping the third state for a moment, when the atomic nuclei are much closer (as shown at the bottom) the density of electrons in the region between them becomes smaller than present in an overlap. Here, repulsion of the two atomic nuclei dominates, and the nuclei are pushed toward a larger separation.

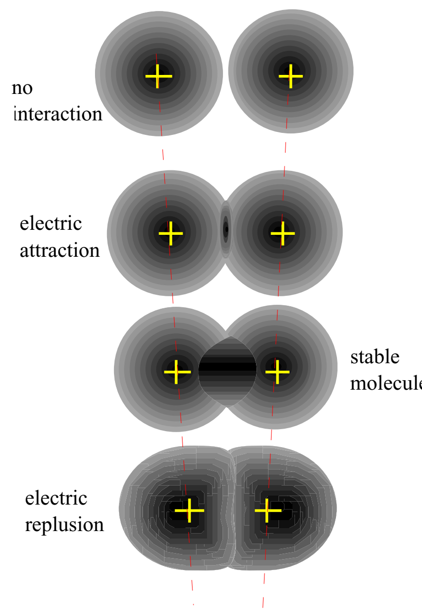

Figure 1.5 A model for the formation of molecules.

Between the two extremes of electric attraction and electric repulsion there is a stable separation, where no net attraction or repulsion exists between the two nuclei. The atoms are "bound": increasing their separation slightly causes an attractive force, and decreasing their separation generates a repulsive force. The two atoms considered permanently separated only at large separations, where the interactions with distant atoms begin to dominate.

At the stable, or equilibrium separation, the atoms form a molecule with a "covalent chemical bond". The words bond, bound, and binding bring to mind images of ropes tied tightly around objects. For bound atoms, however, the ropes are loose until the separation of the atoms is changed. The binding forces exist only when the separation between atoms is larger than the equilibrium separation. Redistribution of electrons around the atoms, as governed by quantum mechanics, creates *two* states where *no force* (none or zero) acts on either atom: 1) at the equilibrium separation, and 2) when the atoms are far apart. More than two atoms and atoms of different elements can form molecules. Common examples are molecules of water (H_2O) and carbon dioxide (CO_2). Within the molecule, the force acting on each atom is zero. However, changing the separation of the atoms generates restoring forces. Electromagnetic forces of similar origin form collections of molecules (matter), that are found in a particular phase (solid, liquid, or gas) at a temperature and pressure determined by the details of the forces generated between them.

E. Elastic forces in matter

Most solid objects exhibit the properties of a spring from a retractable ballpoint pen. The forces *generated by* these objects are known as "elastic" forces. To stretch or compress the objects from their normal shape, distorting forces produced by other objects must be *applied to* them. Without applied forces the object has a "normal shape".

Elastic objects return to their normal shape if the distorting forces are slowly removed.

Most solid matter, including a concrete walkway, a steel beam, or a piece of plywood, will react elastically when distorting forces (that are not too large) are *applied to* it.

As in the system of two atoms of the previous section, each atom or molecule in a solid object is bound to its neighbors by electromagnetic forces that arise *only* if they are moved from their equilibrium separation and natural locations. In an undistorted (normal or naturally shaped) object containing many atoms, consider only three neighboring atoms, as shown at the top of Figure 1.6. Each atom is separated from its neighbors by the equilibrium or "natural" spacing, and in the following, consider the end atoms to be fixed permanently at their current locations.

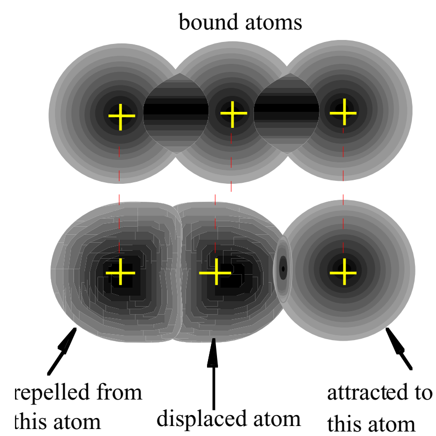

Figure 1.6 Forces on a displaced atom in a solid object.

To illustrate the forces that maintain the structure of a solid object, the central atom is moved *to the left* by an external force (e.g., with your hand) and then released, as shown at the bottom of Figure 1.6. The forces acting on the displaced atom act in a direction that will return it to the original location: the atom on the left pushes (repels) the displaced central atom *to the right*, while the atom on the right pulls (attracts) the displaced atom, also *to the right*. Each force acts in a direction "restoring" the displaced atom to its "natural" separation from its neighbors, as shown at the top of Figure 1.6.

> In an undistorted object, the "equilibrium" separation and the "natural" separation of atoms are the same. In a distorted object, however, the atoms find a new equilibrium separation.

In the direction an object is stretched, the spacing of the atoms will be larger than the natural separation. In a squeezed object, the atoms find a new *equilibrium* spacing that is smaller than the natural spacing. In distored objects, a single displaced atom will be subjected to restoring forces from the neighbors that cause the displaced atom to recover the equilibrium spacing.

To describe the state of an atom in an equilibrium position within an undistorted object, consider that you stand between two friends who hold your arms out to the sides and prepare, but don't yet apply forces. In this state, no force is applied to either arm, and is analogous to the natural state of an object.

To obtain a state analogous to a stretched object, the friends pull on your arms with external forces of equal magnitude, as shown in Figure 1.7. This results in a *balanced* state of forces: the amount of force acting to the right is balanced by an equal amount of force acting to the left. If you try to move toward one friend and away from other, the forces will likely become unbalanced in a way that will pull you back toward the original position. The friends can, perhaps, do things with their bodies that will modify these results somewhat. The electric forces on atoms, however, have no choice but to try to move an atom back toward its equilibrium position.

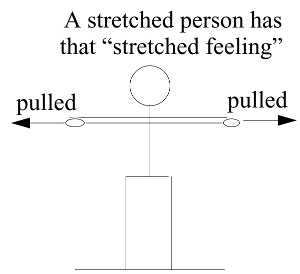

Figure 1.7 A person being stretched by two friends.

You are quite aware that two external forces act on your body because you *feel* your arms being pulled outward from your body. When both friends simultaneously stop pulling, you loose the stretched feeling. If both friends apply equal *inward* forces, as shown in Figure 1.8, you experience a squashed, squeezed or crushed sensation. At no time will these forces cause your body to *begin* to move in either direction. The forces here are also *balanced* so that the location of the center of your body does not move, but the body is pressed inward from both sides.

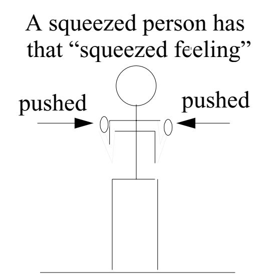

Figure 1.8 A person being squeezed by two friends

It is not hard to translate these *sensations* into words that describe the same forces acting on atoms *within* an inanimate object The external forces, as shown in Figure 1.9, are those stretching (pulling on) or crushing (pushing on) the object. The *feeling* of being stretched translates to the word *tension* in the solid object, while the *feeling* of being squeezed translates to the word *compression*. *Tension* and *compression* refer to the forces acting on atoms *within* the distorted object.

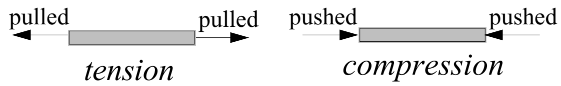

Figure 1.9 Matter, distorted external forces (arrows), generates internal forces called *tension* or *compression*, acting within the object.

> Tension forces act on atoms *within* a stretched object and compression forces act on atoms *within* a compressed (squeezed) object.

The spaces between atoms in a solid object are small, so that the separation distance between their centers is close to an atomic diameter, approximately 10^{-10} m. When a stiff object is stretched, the distance between atomic centers is changed by only a very small amount. In the direction of the distortion, a large number of microscopic changes in the separation between atomic centers add up, causing a macroscopic (large-scale) change in the length of the object.

To emphasize the mechanism that generates internal forces, the movement of each atom relative to the neighbors must be exaggerated. This is accomplished by representing atoms as small hard spheres, as shown in Figure 1.10, attached to each other by coil springs. The elastic springs define the natural spacing of atoms and model the behavior of electromagnetic forces between the atoms. At this natural separation, there are no forces acting on the atoms. Please note that a slinky toy or a bungie cord can be stretched but not compressed, therefore*, these objects do not have the characteristics of the (ideal) strong coil springs used here*.

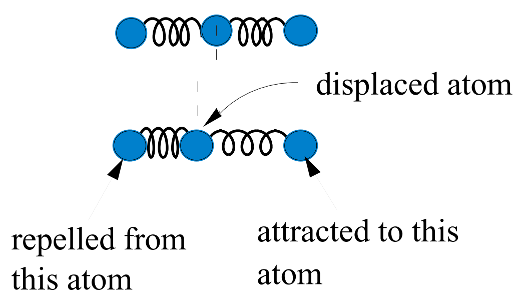

Figure 1.10 Forces on a displaced atom

Keeping the atoms on either end fixed at the natural positions, and moving the central atom to the left, as shown in the lower portion of Figure 1.10, the spring on the left compresses, and the spring on the right stretches. The stretched spring pulls the displaced atom to the right, and the compressed spring pushes it to the right. Both forces restore the displaced atom to its equilibrium position, in agreement with the effects of the electromagnetic forces between atomic electron clouds described earlier.

A model for solid materials, constructed using these components, can accurately reflect the behavior of the solid when it is stretched or compressed. Of course, an atom in the interior of a solid will have neighboring atoms on all sides. The type of atom and the process that formed the solid define the natural spacing throughout the object. A particularly simple solid has the natural locations of its atoms in a rectangular pattern, as shown in Figure 1.11. This pattern appears in magnifications of a small region on the surface of solid bars in the three states: compressed, natural length, and stretched. The compressed state (top) has *external* forces squeezing the bar (represented by arrows pointing inward), the natural length state (middle) with no external forces affecting the bar, and a stretched state (bottom) with external outward forces stretching the bar.

The atoms in the natural length bar are in their natural and equilibrium locations. The springs have their natural length, and apply no forces to the atoms.

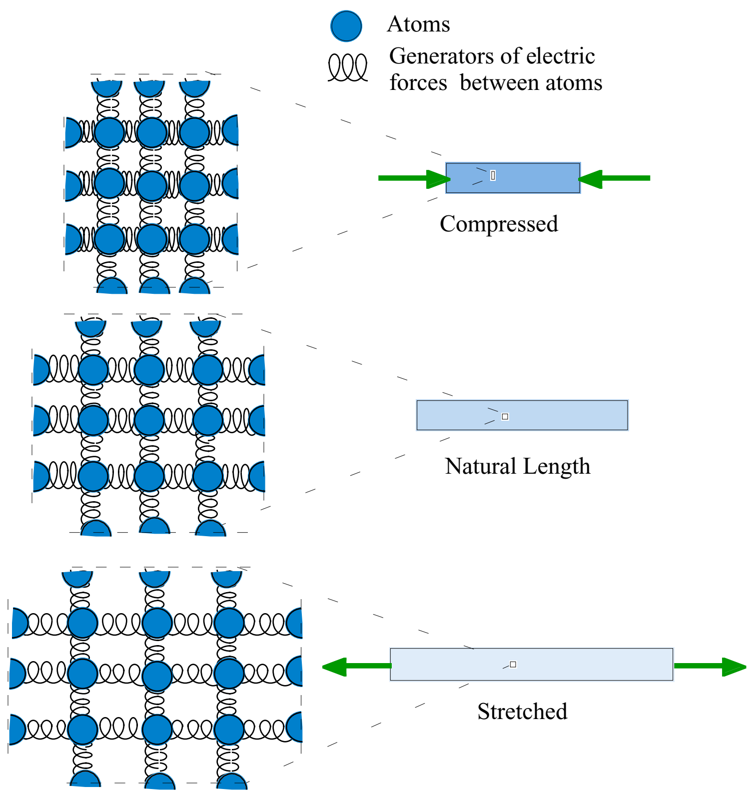

Figure 1.11 Functional model for the structure of solid matter.

The bar compressed by external forces has a reduced length and atoms have moved to a new smaller equilibrium separation along the length. "Compression" forces (modeled by the compressed springs) are generated between the atoms that attempt to push the atoms back toward their natural spacing. Squeezed harder, the bar's compression forces will increase each time the length decreases.

When the bar is stretched, as shown in the bottom picture of Figure 1.11, the length increases, and the atoms move farther apart along the length of the object to new equilibrium separations. At the same time, between atoms, attractive *tension* forces (modeled by stretched springs) attempt to pull the atoms back toward their natural spacing. The tension forces will increase each time the length is increased by increasing the stretching forces. The internal forces of tension and compression are called "elastic" if the object returns to its original length when the applied forces are removed. In the next few chapters only the elastic forces, tension and compression, will concern us.

> A simple model using common springs to model the microscopic forces between atoms provides a good description of the macroscopic forces generated by an elastically deformed object.

The behavior of a spring is very familiar to us, and this familiarity will be used to develop confidence in a quantitative description of forces and related quantities before tackling the effects of gravity, a much less intuitive force.

Review of Scientific Notation

(1) Powers of Ten:

$$0.01 = 10^{-2}$$
$$0.1 = 10^{-1}$$
$$1 = 10^{0}$$
$$10 = 10^{1}$$
$$100 = 10^{2}$$
$$200 = 2\times 100 = 2\times 10^{2}$$
$$314159 = 3.14159\times 10^{5}$$
$$\frac{1}{100} = \frac{1}{10^2} = 10^{-2} = 0.01$$

(2) Multiplying and dividing:

$$\frac{1}{10^{-n}} = 10^{n}$$
$$10^{m}10^{n} = 10^{m+n}$$
$$\frac{10^{m}}{10^{n}} = (10^{m})(10^{-n}) = 10^{m-n}$$

Chapter Summary:

- Only four fundamental forces are known: gravitational, electromagnetic, weak nuclear and strong nuclear forces.
- Unification of electric and magnetic phenomena was accomplished when light was found to have properties of an electromagnetic wave, or of particles called photons. Electric or magnetic forces are an expression of the more general electromagnetic force.
- A force is electromagnetic in origin if gravity or nuclear forces are eliminated as candidates. All chemical reactions and phase changes are due to the electromagnetic forces between atoms.
- In an atomic nucleus, the *weak nuclear force* causes radioactive beta decay, and is always accompanied by the emission of a neutrino. It is the *strong nuclear force*, however, that binds protons and neutrons to form the atomic nucleus.
- Objects with electric charge experience electric forces that are attractive for charges of opposite signs and repulsive for charges with the same sign. Two neutral objects are charged by removing electrons from one object, that leaves it with a positive charge, and depositing those electrons on the other object, giving it a negative charge.
- In a neutral atom, protons (+ charge) reside in the nucleus (size 10^{-14} m), with a surrounding cloud (size 10^{-10} m) containing an equal number of electrons (– charge). Forces between neutral atoms are determined by interaction of their electron clouds.
- A hypothesis explaining new phenomena becomes an accepted theory only when it passes two stringent tests: a) it correctly predicts new phenomena, and b) explains the correct predictions of the old theory for other phenomena.
- In an undistorted object, the atoms locate at natural separations where electromagnetic forces acting on each atom are absent (none, zero).
- For an atom shifted in location relative to its neighbors, electromagnetic forces act in a direction to restore the atom to its original location. For the atoms in an undistorted object, the natural and equilibrium positions are the same.
- In a stretched or compressed object, atoms find a new equilibrium separation, where balanced electromagnetic forces (non-zero) act on each atom. These internal forces are called tension forces when generated by stretching, and compression forces when generated by squeezing.
- A distortion of a *solid* object is "elastic" if it quickly returns to its original length when the applied stretching or compressing forces are removed.
- In an investigation of fundamental forces, the wide variation in their strength requires the use of scientific notation.