Chapter 4: Springs generate force and store energy

A. Compound Springs

The physical concepts of motion, force, energy, and momentum, are often confused. The distinctions are particularly clear in the spring–model of matter that has been developed over the past few chapters. Combining two or more identical springs, yields a new object with the properties of a spring, but with a combined spring constant that can be larger than, equal to, or smaller than the original. The combined spring changes the amount of distortion that results from a given set of applied forces or, equivalently, increases (or decreases) the reaction forces generated by a given length change.

Examples given in the previous chapters showed springs placed in a side-by-side orientation called a *parallel* combination. Springs can also be attached end-to-end in a *series* combination. The parallel combination will be treated first, followed by the series combination, with its surprisingly different behavior, that will illustrate the difference between the concepts of force and energy.

B. Springs in parallel

Springs placed side by side and joined at the ends by a strong coupling will act in parallel. Springs with the same natural length and spring constant k are considered first.

It makes sense that the parallel connection of springs each with a spring constant k, shown in Figure 4.1, is stronger than any one of the springs. The parallel combination, therefore, exhibits a spring constant that is larger than k. Consider stretching the parallel combination so that each spring provides an inward force, F. To create that stretch, an outward force, $3F$, is required on each side. The force required to change the length of the parallel combination by an amount, x, is three times as large as is needed to make the same change to only one of the springs. This means that the spring constant of the parallel combination is $3k$, or three times the spring constant of a single spring.

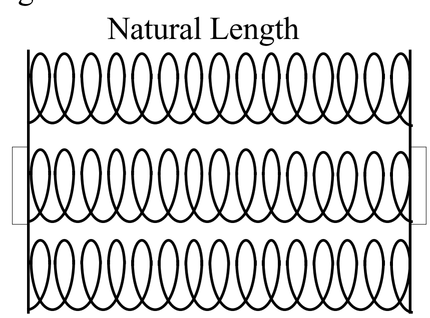

Figure 4.1 Three identical springs, spring constant, k, connected in parallel

The reasoning above can be translated into a derivation using one step of algebra. That derivation, though simple, is an example of what can be done with physics: take a simple law ($F = kx$ for an ideal spring, in this case), apply it to new conditions (springs grouped in parallel and series), and predict the resulting behavior (the combined spring constant).

To illustrate, we start with a natural length ideal spring with spring constant, k, which is then stretched a distance, x, by a force, F (one on each side), as shown in Figure

4.2a. The force and resulting stretch are related through Hooke's Law, $F = kx$. At each end, the tension force generated by the stretched spring has the same magnitude as the applied force, $\underline{T = F = kx}$, as demanded by Newton's 3rd law.

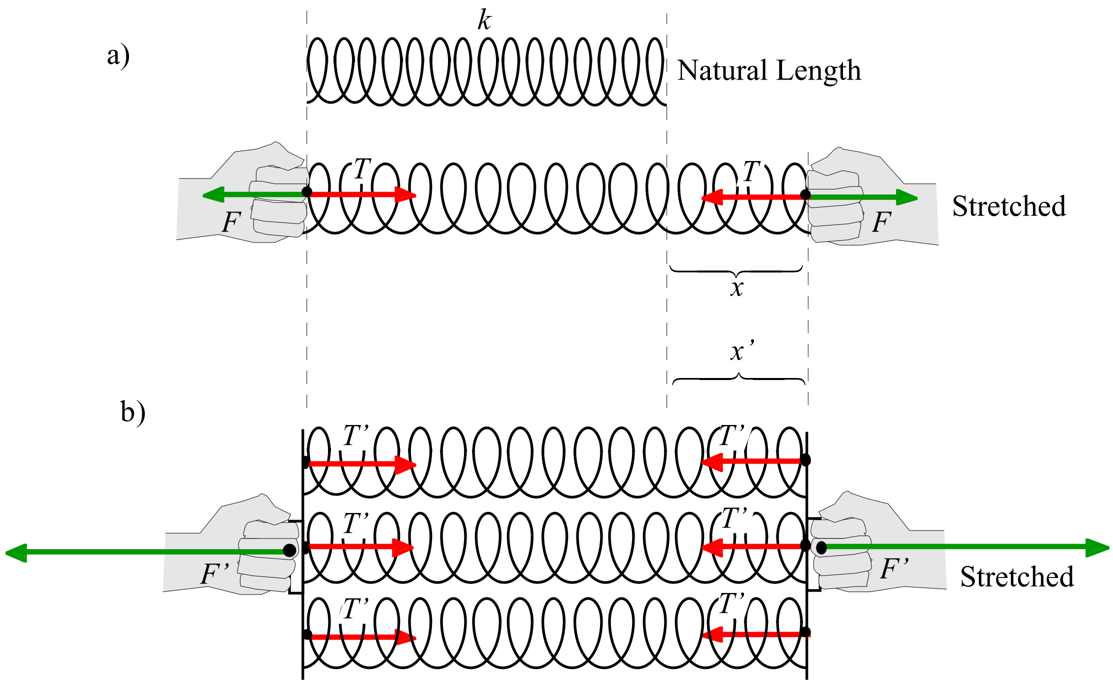

Figure 4.2 a) A spring with spring constant *k*, being stretched a distance, *x*, and b) three of these springs connected in parallel and stretched by the amount, $x' = x$.

Three of these springs are placed side by side, as shown in Figure 4.2b, with their ends connected to form a parallel combination, and stretched by a different force, F', stretched by a distance, x', generating a tension force, T', in each spring. The stretch is adjusted to be equal to the previous stretch of the single spring, $x' = x$. For the three springs in parallel, how does the combined spring constant, k', compare to k of a single spring (a prime, ‘, is attached to a variable if its value may have changed).

The parallel combination of three springs should behave like a new spring, spring constant, k'. The relationship between the applied force, F', and the length change, x', should be, $F' = k'x'$, and therefore,

$$k' = \frac{F'}{x'}. \qquad (4.1)$$

The connected springs, and the single spring, are stretched the same distance $\underline{x' = x}$, and therefore the response of *each* spring to this stretch is to generate the same tension, $\underline{T' = T = kx}$. The net effect of the three tension forces acting inward on the connection bracket, must be *balanced* by the applied force, F', acting outward on the handle. These words translate into the algebraic equivalent, $\underline{F' = 3T' = 3kx}$.

Replacing F' and x' in Equation 4.1 with their equivalents completes the proof,

$$k' = \frac{F'}{x'} = \frac{3kx}{x} = 3k \, . \tag{4.2}$$

A parallel combination of *n* identical springs is *n* times as strong as one spring: $k' = 3k$.

This result agrees with the intuitive discussion at the beginning of this section. In the next section, where the series combination of springs is evaluated, your intuition may fail.

C. Springs in series

A series connection of three ideal springs, as shown in Figure 4.3, is actually *weaker* than any one of the individual springs. The reasoning here is as follows: a single spring stretched a distance *x*, by a force *F*, creates a tension $T = F$. Applying the same force, $F' = F$, to three such springs attached in series, as shown in Figure 4.4, yields a stretch three times as great. This implies a *weaker* spring. There is an action – reaction pair of tension forces at each contact point. The tension in the two outer springs stretches the central spring, and the tension in the central spring provides one of the applied forces on the outer springs.

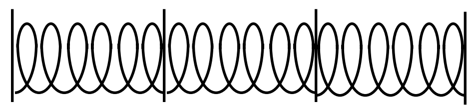

Figure 4.3 Three identical springs, spring constant, *k*, connected in series

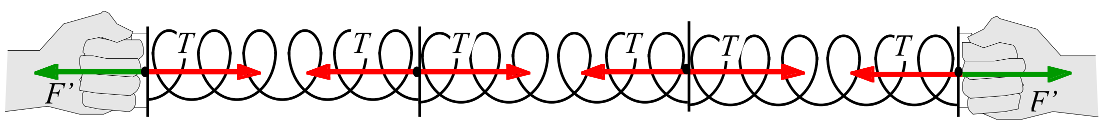

Figure 4.4 The tension *T* in each spring in response to the stretching forces F'.

The spring constant, *k'*, of the series combination is determined by relating the force on the end springs, F', to the full stretch of the combination, x', by Hooke's law, repeated below in Equation 4.3,

$$k' = \frac{F'}{x'} \, . \tag{4.3}$$

The formal solution for the new spring constant, *k'*, starts with three identical stationary springs, spring constant, *k*, each stretched a distance, *x*, by a force, *F*, of a pair of hands, as shown below in Figure 4.5.

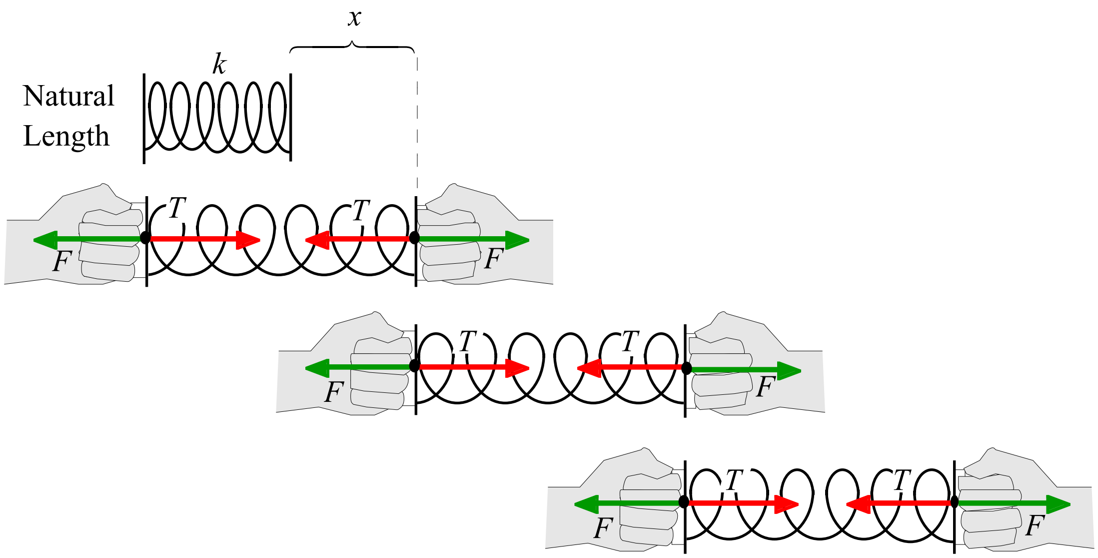

Figure 4.5 Three identical springs each stretched a distance, *x*, by a force, *F*.

By attaching the two outer springs to the central spring, the springs at each joint pull on each other with an action - reaction pair of tension forces, magnitude *T*. The inner hands can remove their forces without affecting the tension forces of the springs, as shown below in Figure 4.6.

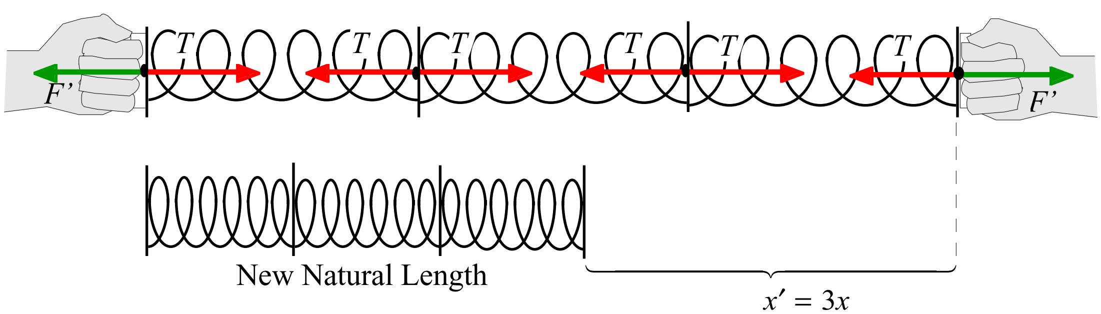

Figure 4.6 A series combination of three identical springs, spring constant, *k*, each stretched a distance, *x*.

The total stretch must be the sum of the stretches of the individual springs $x' = 3x$. The tension $T = kx$ at each end of the springs has not changed, nor has the applied force, $F' = F = kx$. Substituting for both x' and F' in Equation 4.3 yields:

$$k' = \frac{F'}{x'} = \frac{kx}{3x} = \frac{k}{3}.$$

This means that the series combination of three springs is only 1/3 as strong as one of the springs. A series is longer and weaker than one spring.

This picture of an ideal spring with pairs of tension forces acting at each interior point has important implications for all internal forces. Newton's third law of motion is not only a statement regarding the interface between two objects but also refers to any point within a single object where a separation into two pieces could occur. Internal forces are drawn as "back to back" tension vectors (or "head to head" compression vectors) at each joint in a series connection of springs, as shown in Figures 4.6. In the action - reaction pairs, the two forces of each pair *act on a different object*, the objects on each side of the boundary.

Also, the phrase, "a stationary object has a tension, T…" though stated in a singular fashion, implies that two tension forces (an action reaction pair) act with an equal magnitude and opposite direction at each point within the object. At each end of the spring, the tension forces pull inward on the objects providing the stretching forces.

In general, a number, n, of identical springs, spring constant, k, *connected in parallel* make a stronger spring with spring constant, $k' = nk$. A number, n, of identical springs, spring constant, k, connected *in series* makes a weaker spring with spring constant, $k' = k/n$.

These results can be reversed to show that when a long spring with spring constant, k, is cut into a number, n, of equal length pieces, each small piece will be a stronger spring with a larger spring constant, $k' = nk$. For example, a spring cut into three pieces, as shown in Figure 4.7, yields three springs each three times as strong as the original.

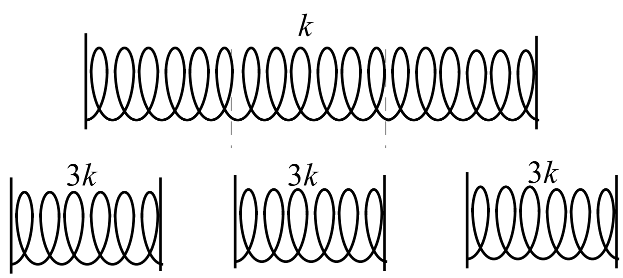

Figure 4.7 A long spring cut into three equal length pieces.

When a wide elastic bar with spring constant, k, is cut (lengthwise) into a number, n, of equal width pieces, each piece will be a weaker spring with a new spring constant, $k' = k/n$. For example, when a bar is cut into three equal width strips, as shown in Figure 4.7, each strip is 1/3 as strong as the original bar.

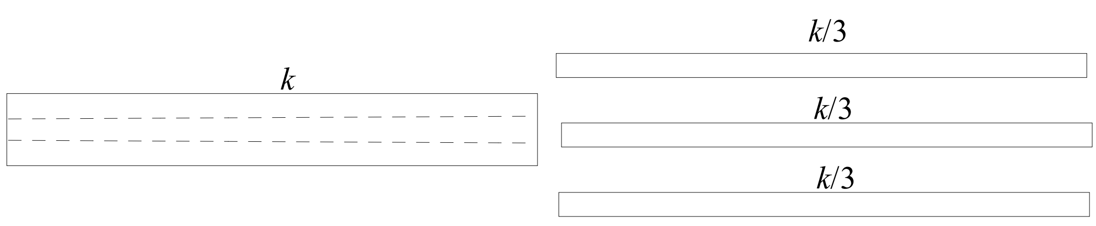

Figure 4.7 A wide bar cut into three equal width pieces.

A parallel combination of three springs, with a different spring constant for the middle spring, can be shown to have an effective spring constant that is the sum of the individual spring constants. It can also be shown (although, with a little more algebra) that the spring constant of a series combination of springs, each with a different spring constant, is smaller than the spring constant of the weakest of the individual springs.

D. Springs do not store force.

A distorted spring *generates* internal reaction forces. The word *generate* should not be confused with the word *store*. When things are stored and then redistributed, one can follow the path of each item. A quantity that is generated does not exist prior to its production and, if the generator is turned off, will cease. For example, light is emitted when a light bulb is on, but turned off, the light ceases. Searching for the light within the bulb or wires is fruitless. The light is generated by the motion of electrons and does not exist prior to its creation. The generated light is a carrier of a quantity called *energy,* which can be stored.

The word *store* comes up often in conversation: store water for a trip across a desert, store money for a rainy day, store wood for a winter fire, store salt to melt winter ice, etc. Synonyms for store are *collect*, *save*, *accumulate*, *archive*, *file*, *hoard*, and many others with the same connotation. These words are inappropriate to use when talking about forces. Also, we often use the word *generate* in conversation: an injury can generate pain, a turbine can generate electricity, the sun generates light, and drums generate noise. Synonyms for generate are *make*, *produce*, *form*, *create* and *spawn*. These are words appropriate to use when talking about forces.

Tension and compression forces are *generated* by a spring and *not stored* in the spring. Stretched springs can be coupled in two distinct ways: in parallel or in series. It was shown above that the net force generated by three identical stretched springs connected in parallel is three times that of a single spring. The force generated at the free ends of stretched springs connected in series, is the same as that provided by any of springs. The property that two force vectors can be generated at points within a series attachment implies that forces are *not stored* in the springs. A good indication that the forces are *generated* by the springs is that the net force available from the same stretched springs depends on how the springs are attached, in parallel or series. A stored quantity would not depend on how the springs were arranged.

The quantity *stored* by a distorted spring is called “energy”. The three stretched springs in Figure 4.5 each have the same amount of stored energy. The total amount of energy stored in the three springs when connected in a series combination, as shown in Figure 4.6, or in a parallel combination, as shown in Figure 4.2, are three times the energy stored by one spring. The forces generated by the three springs have a very different behavior: connected in parallel they provide a force that is three times the force available from a series connection. Energy is stored by a spring but forces are generated by springs.

A dramatic example compares the energy stored in a steel rod, and in a pinball machine spring, when compressed by the same person. A steel rod responds to the external forces of a human with a very small length change, and stores only a small amount of energy. The spring in the pin–ball machine can also respond to external forces but it will compress much more and store a much larger amount of energy. As the spring from the pin–ball machine expands it can make a steel pinball move rapidly but when the steel rod expands (only a tiny amount before its force becomes zero) it cannot give the ball any significant velocity. The storage of energy in a spring will be studied in more detail in future chapters.

Chapter Summary:

- A number, n, of identical springs, each with spring constant k, operating in parallel will have a new spring constant n times larger, $k' = nk$.
- A number, n, of identical springs, each with spring constant k, operating in series will have a new spring constant n times smaller, $k' = k / n$.
- Each piece of an object, spring constant, k, cut into a number, n, of short but equal length pieces, has a new spring constant n times larger, $k' = nk$.
- Each piece of a bar, spring constant, k, cut into a number, n, of equal width but unchanged length strips has a new spring constant n times smaller, $k' = k / n$.
- A stretched or compressed massless object will have the same tension or compression acting at each internal point along its length.
- Springs *generate* forces in reaction to applied forces. Springs *do not store* forces; springs store energy.