Chapter 5: Springs and gravity.

A. Gravity and humans

From birth we develop ways to predict how earth's gravity will affect things and we can instinctively react to its effects. One does not have to think hard about catching a ball or riding a bike, in fact, thinking usually gets in the way of success in these activities.

We have more difficulty in unfamiliar circumstances: in an elevator beginning a rapid accent or decent, in a satellite orbiting the earth, in a car turning rapidly around a curve or navigating on a patch of slippery ice. The rules that we apply in more typical circumstances don't seem to work in these environments. A natural reaction is to develop a set of force rules for each new situation. We find, however, there are too many rules and some are contradictory. It doesn't take long before we start to doubt that a consistent and logical explanation exists. Certain characteristics of gravity make it particularly enigmatic.

The familiar behavior of elastic forces, however, can be used to establish a consistent set of rules that will work for gravity and all other forces. A distorted ideal spring produces an action - reaction pair of forces, one force acting on the mass at each end of the spring. This feature of a spring reflects that fact that one force cannot be generated alone, forces are always generated in pairs. Forces, like shoes or gloves, are always produced in pairs.

B. Gravitational point of action

As shown in Figure 5.1, near a large planet with mass, M, an object with a mass, m, experiences a gravitational force, F, proportional to the product of the two masses. Toward this planet, a mass four times larger ($4m$), of any shape, experiences a gravitational force, four times larger Four gravitational forces with magnitude, F, have the same effect on the motion of a mass as one gravitational force, four times as large, $4F$, acting at its center.

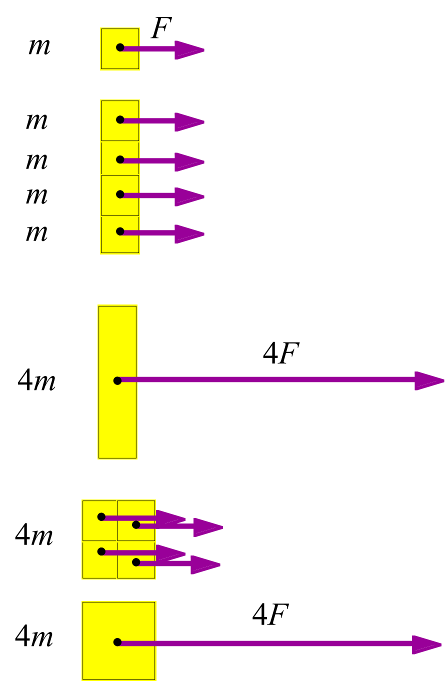

Figure 5.1 Gravitational force on various masses as generated by a planet off the picture, to the right.

Which picture of the gravitational force is closer to the true action of gravity? Gravitational forces act on every atom of the two objects and not just on the surface or on the center. In a figure, it is impossible to draw the gravitational force vectors acting on each atom of an object. Showing the forces acting on four pieces of an object, as in Figure 5.1, is about the best that can be done, but it is still a gross simplification of the true nature of gravity. By convention, only one gravitational force vector is shown acting on each (uniform and symmetrical) solid object at or near its center. It is essential to remember that many small gravitational forces act on the mass, one on each atom of an object.

A gravitational force cannot generate internal tension or compression forces within a small object unless contact is made with another object. *In addition to gravity, there must be another force, usually an elastic force, acting on an object to cause tension or compression forces within the object.* Before an accurate picture of the effects of gravity on human beings can be appreciated, this important feature of gravity must understood.

C. A comparison of gravitational and elastic forces.

Consider two equal masses far away from any planet. The gravitational forces acting between them will attract the masses, as shown in Figure 5.2. Although the forces may vary with distance between the masses, the masses are the same, and therefore the forces must have the same magnitude, F_G. The two masses started at rest and have just been released, so it is reasonable to assume that an instant after their release, each mass starts to move toward the other to make their separation smaller.

Figure 5.2 Gravitational forces between two identical masses

An ideal spring is compressed, as shown in Figure 5.3, to a length that just fits between the two masses. By choosing the proper spring constant, the compression forces, C can be made as large as desired. This spring is then placed between the two masses and the force of the two hands removed, as shown in Figure 5.4. If the spring constant has been chosen to make the spring forces, C, balance the gravitational forces, F_G, the masses will not begin to move toward or away from each other.

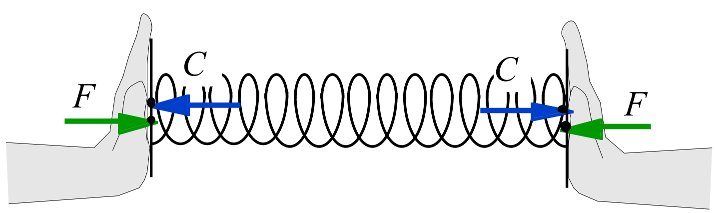

Figure 5.3 An Ideal spring generating compression forces, *C*.

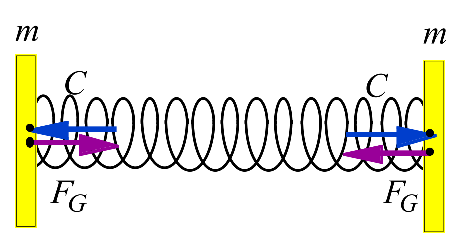

Figure 5.4 Two masses kept apart by a compressed spring.

Newton realized that the forces acting on two gravitationally attracting objects have equal magnitudes. Such a result is certainly believable if the masses are the same, but if the masses are different, say m_2 is bigger than m_1, then the gravitational forces *might* also be different, say F_1 bigger than F_2, as shown in Figure 5.5. How could Newton be so sure of his discovery? The following analysis is presented to show that if the gravitational forces have different magnitudes, as erroneously shown in Figure 5.5, attempts to produce balancing elastic forces, produces an impossible result.

Figure 5.5 Two masses, $m_2 > m_1$, attracted by gravity, assuming $F_1 > F_2$.

If a compressed spring, generating a compression force, C, is inserted between the two masses, the masses move apart if C is larger than either gravitational force and the masses move toward each other if C is smaller than either force. Somewhere between these two extremes, there should be a value of compression keeps the two masses at a constant separation. A logical choice for the compression force that will keep the two masses at their original separation is one that is midway (average) between F_1 and F_2, $C = (F_1 + F_2)/2$. A spring is constructed with the correct spring constant that generates this value of compression when compressed to a length that fits between the two masses as shown in Figure 5.6.

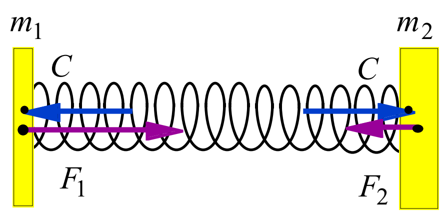

Figure 5.6 Spring compression, C, midway between F_1 and F_2.

Considering the forces acting on m_1, the magnitude of the gravitational force, F_1, is greater than the compression, C, making m_1 begin to move to the right. On m_2, the compression force is greater than the magnitude of the gravitational force, F_2, and it will also begin to move to the right. If the forces are as pictured in Figure 5.6, when released, the masses simultaneously *begin* to move to the right. It does not seem reasonable that the separation would stay constant and the pair of masses moves off to the right. We can therefore surmise that the two gravitational forces must have the same magnitude.

This conclusion follows from Newton's 1st law of forces, stated in Chapter 2: "Balanced forces will not **change** the motion of a mass." Consider the two masses and spring as an object of fixed length. No external forces act on this combined object, so forces on it are balanced. Therefore, when released from rest, the combination must not begin to move and the separation of the masses must not change.

The only interpretation of these results consistent with Newton's 1st law is that the gravitational forces have the ***same magnitude*** on the two masses, *even when the masses are different*! Placing the masses at a separation where the gravitational and spring forces have the same magnitude, $F_G = C$, as shown in Figure 5.7, the masses will not begin to move when released.

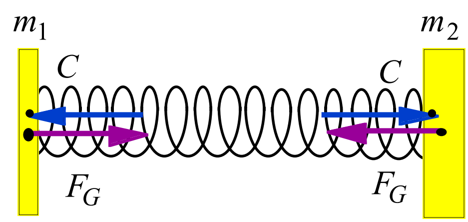

Figure 5.7 The magnitude of the gravitational force, F_G, is the same on both masses.

When one or both masses are increased as shown in Figure 5.8, the two gravitational forces increase by the same amount, though the forces remain labeled, F_G, to simplify the notation. The two masses are kept at a constant separation by increasing the spring constant to make the new compression force again equal to the gravitational force between these masses, $C = F_G$.

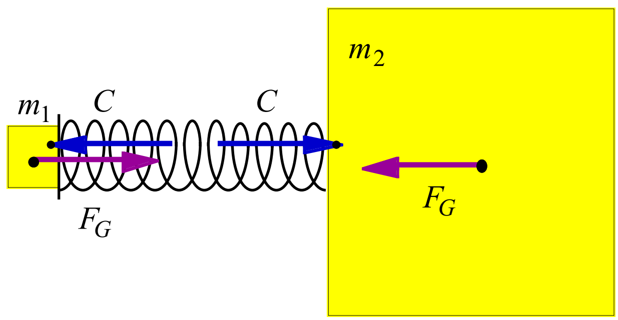

Figure 5.8 Unequal masses separated by an ideal spring

At first, it seems counter-intuitive that the magnitudes of the gravitational forces on two very different masses would be equal. Now consider one small mass m_1 and one extremely large mass m_2, where only a small portion of the surface of the mass can be shown in Figure 5.9. The large mass could be the earth and the small mass an apple.

The spring is compressed between the masses of the earth and apple, and the gravitational force of the earth acting on the apple has the same magnitude as the gravitational force of the apple acting on the earth. These figures are shown in a horizontal orientation to insure that you don't impose any lingering prejudices and misconceptions from your lifelong experience with gravity. When released the apple will not begin to move if the spring compression force C equals the gravitational force F_G. If the spring is removed the apple "falls" to the earth.

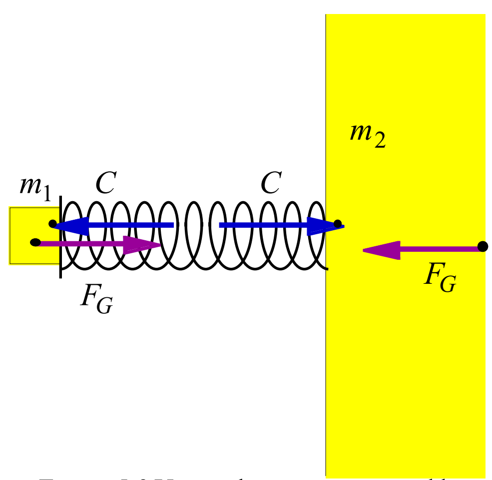

Figure 5.9 Unequal masses separated by an ideal spring

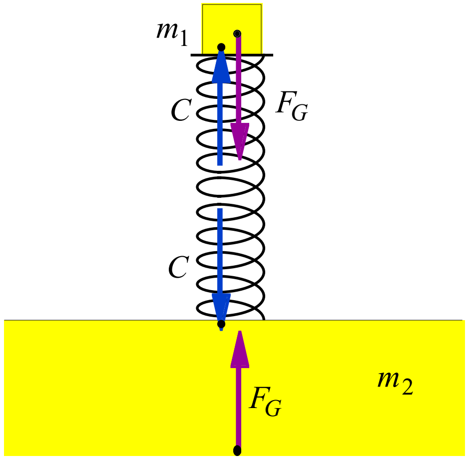

Figure 5.10 Mass supported by a spring

The situation becomes more familiar if rotated to the perspective of someone standing next to the spring, as in Figure 5.10. Now, on the small mass, the gravitational force is shown acting in its normal downward orientation. Also shown is the gravitational force acting *upward* on the earth. The spring is compressed by the masses being pulled together by the gravitational forces, and they are *balanced* by the spring's compression forces pushing the masses apart, i.e., $C = F_G$. The distance the spring compresses, and Hooke's law enables the measurement of the compression force, C, and therefore the force of gravity, F_G, on the mass, also known as its *weight*.

"Weight" must not be confused with how heavy you "feel". Feelings are a measure of the compression or tension forces acting within your body. Gravity, however, acts on the entire body and cannot be felt without another force also acting on the body (this concept will be fully developed in the next chapter).

D. Mass and Weight

On the earth, if a Newton spring (spring constant of 1N/cm) supports a mass, m = 1kg, as shown in fig. 5.10, the spring compresses by 9.81 cm. Using Hooke's law, the force of gravity on that mass (1 kg) is determined to be,

$$F = kx = (1\text{N/cm})(9.81\text{cm}) = 9.81\text{N}.$$

The force of gravity on a mass is its weight, W, and for this 1 kg mass, $W = F_G = 9.81\text{N}$. Inside most scales used to measure weight there will be a spring of one kind or another. The distortion of the spring is translated (mechanically or electrically) into weight with a calibration that uses Hooke's law. The ratio of the weight (near the surface of the Earth) to the mass is always the same: W/m = 9.81 N/kg, and this ratio is called "g". The relationship between mass and the weight can be rewritten in its more common form:

$$\boxed{W = mg, \quad \text{where } g = 9.81\text{N/kg}} \qquad (5.1)$$

The mass and radius of the Earth determine the value of g, and on another planet (or moon) the ratio of weight to mass could be very different. On our moon, this ratio is one-sixth its value on Earth, $g_{moon} = g/6 = 1.6\ \text{N/kg}$.

A mass can be held at a fixed height by a stretched spring. A 1 kg mass, m_1, is hung from a spring (k = 1 N/cm) with the other end attached to the top bar of a frame of negligible mass, as shown in Figure 5.11. The gravitational force acting on this mass is, $W = m_1 g = (1\ \text{kg})(9.8\ \text{N}/\text{kg}) = 9.81\ \text{N}$, and Hooke's law predicts the stretch will be 9.81 cm.

The stretched spring will generate a tension force, T, acting upward on the mass. The mass can remain stationary at a position where the tension force acting upward balances the gravitational force, $F_G = m_1 g$, acting downward on the mass. The spring tension, however, also acts downward on the (massless) top bar and attempts to pull it down to the ground. All the spring has done is *transfer* the force of gravity acting on the mass to the top bar via the spring tension. The tension, $T = m_1 g$, pulls down on the top bar, and the top bar must also be pushed upward to keep the spring stretched.

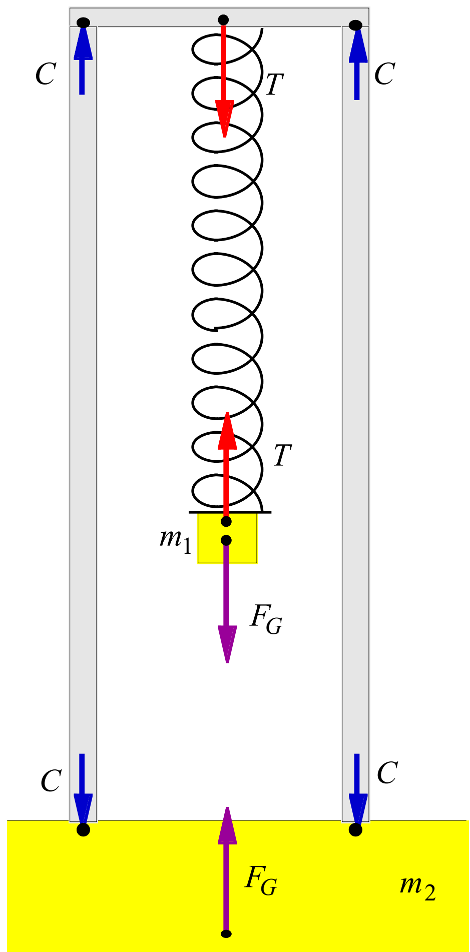

Figure 5.11 A mass stretching a spring. Its tension holds up the mass and pulls down on the roof and it down on the posts.

The (massless) posts are compressed between the top bar and the ground, generating compression forces, C, in the posts that act upward on the top bar and downward on the ground. The net effect of the two posts is to balance the tension force, T, of the spring on the top bar and the gravitational force on the earth. Each post must, therefore, contribute a compression force, $C = T/2$, that is half the force of tension pulling down on the top bar. The tension in the spring is equal to the weight of the mass, $T = m_1 g$. Therefore, the compression, $C = m_1 g/2$, i.e., each wall supports half the weight of the mass.

If instead of the top bar it was your arm that held the end of the spring, then your body would provide the force to maintain the stretch of the spring. To generate the upward force that causes the spring to stretch and prevents the mass from falling, your body must be compressed between the earth and where you hold the spring

E. Weight and materials

Many examples of distorted materials were analyzed earlier but gravity was avoided until this chapter. All tools needed to evaluate the effects of gravity are now available.

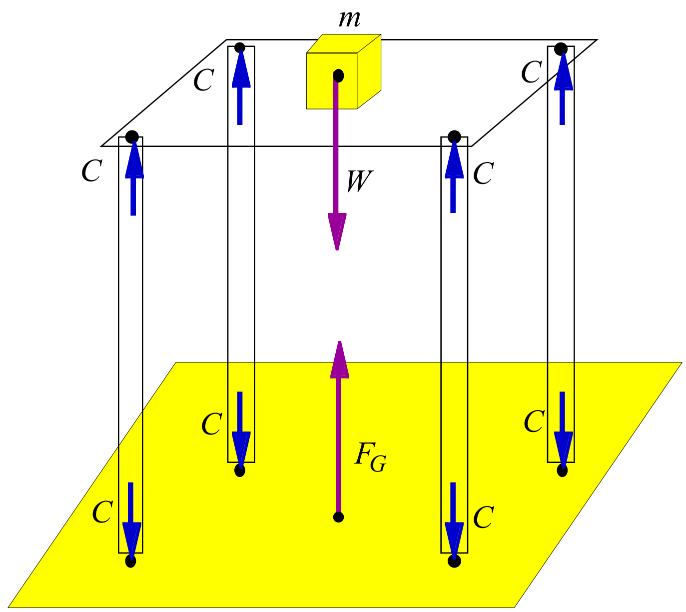

Figure 5.12 Forces caused by a mass, *m*, resting on a table

A mass, *m*, shown in Figure 5.12, is resting on a table with four legs. The tabletop is very strong, but massless, as are the legs. The gravitational forces acting down on the mass, $W = F_G$, and up on the earth, F_G, and the compression forces, *C*, acting at the ends of the legs are all labeled in the figure. The two gravitational forces act on the masses and they in turn squeeze the table legs between them, and create the compression force vectors acting at the ends of the table legs. The gravitational forces act like a stretched spring that is pulling the mass down on the tabletop and pulling the surface of the earth up against the legs.

The compression forces, C balance the weight of the mass, acting upward on the tabletop (and the table top on the mass). These forces are generated by a small decrease in the length of the table legs. Since there are four legs, acting in parallel, the sum of their upward compression forces must balance the downward weight of the mass. This means that the magnitude of the compression force acting in each leg of the table is $C = W/4$. Without the earth pushing upward on the legs, they would not compress. It takes two forces of equal magnitude and opposite direction to compress an ideal spring.

Gravity is the primary cause of these effects but the elastic properties of the table and mass enters into a description of what is happening. Looking deeper, the gravitational force acts on the mass and not on the tabletop. The gravitational force on the mass is transferred to the tabletop through the elastic compression of the mass acting against the tabletop. It is very easy to miss the fact that your body is also elastic and that it stretches and compresses in response to forces. Using the very sensitive nerve endings, the human body senses the distortions caused by forces and not the forces themselves. This very important concept is studied further in the next chapter.

Chapter Summary

- All forces are generated in complementary pairs with equal magnitude and opposite direction.
- A pair of gravitational forces, with equal magnitudes and opposite directions, is generated between, and acts on, any two masses.
- Gravitational force of the earth acts on each component of an object in proportion to that component's mass. The forces on each mass are coupled by electromagnetic forces between the pieces and sum to the total force on the whole object.
- Gravitational forces alone do not generate tension or compression in an object; a tension or compression force from another object must also act to cause a distortion.
- Newton's 1st law of forces stated earlier in Chapter 2, is "Balanced forces will not **change** the motion of an object with mass."
- Elastic forces often balance the gravitational forces acting between two masses.
- The downward gravitational force, F_G, on a mass, m, near the surface of the earth, is called the weight, W, of the mass, and has the value, $W = mg$, where $g = 9.81$ N/kg. With the same magnitude, an upward gravitational force acts on the earth.
- On the surface of a different planet or other astronomical body (e.g., the moon), the value of g' may not be, $g = 9.81$ N/kg. Nevertheless, the weight of the object on that astronomical body is given by, $W = mg'$, with a value for g', appropriate to that body.